Mobile positioning based on relaying capability of mobile stations in hybrid wireless networks

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Abstract: A novel mobile position location procedure, based on the relaying capability of the mobile stations in a hybrid cellular/peer-to-peer ad hoc network to improve positioning coverage and accuracy, is proposed. The proposed mobile location scheme is a hierarchical-structured flooding procedure, making use of the relaying capability of mobile stations to achieve location diversity. Simulation results show that the proposed positioning scheme offers improvement in coverage and accuracy compared to conventional positioning methods reported in the literature. With the increase of the mobiles in the system, the coverage of the conventional system does not change. However, the coverage of the proposed discovery mechanism monotonically increases towards 100% by exploiting location diversity. The gain of the coverage is also promising when cell size is large, where the conventional methods suffer the hearability problem.

1 Introduction

Because of the requirement for location-aware services, mobile position location has been a hot research topic in recent years. Mobile positioning is especially necessary for emergency services, e.g. Enhanced 911 (E-911) is required by the Federal Communications Commission (FCC) in the United States [1–4]. Positioning information is also required for mobility management, lawful interception, battle against cellular phone fraud, wireless system design and for efficient radio resource management, fleet operations [3, 5, 6], etc.

The available basic techniques for accurate position location may be roughly categorised as handset-based technologies such as Global Positioning System (GPS) and network-based location systems that exploit the cellular infrastructure to obtain geolocation information [2]. The integration of radio location into cellular network could exploit the existing infrastructure and utilise the communication channels for position as well as data transmission. Accordingly, it would then be feasible for the pervasive location-aware applications to be implemented everywhere at minimum cost. In this paper, we will focus on network-based location techniques.

Conventional cellular positioning is based on a single-hop cellular network. In terms of mobile positioning, the cellular network and the peer-to-peer ad hoc network are two distinct environments. In recent years, integrated cellular and ad hoc networking has attracted a lot of research attention. Ad hoc relay station (ARS) has been proposed for relaying traffic to achieve load balancing [7–9], where an overlay ad hoc network is employed to dynamically balance the load of the hot spots in the cellular network. The system capacity and system performance can be improved. The relaying capability of mobile stations (relay stations) has also been extensively investigated (see for example, [10–16]). In these systems, a mobile can relay data traffic when the channel between the target mobile and the home base station is in deep fade or in non-line-of-sight (NLOS). Using relay can improve system coverage and robustness against network or radio link failure, and to increase capacity by lowering transmission powers and associated intercell interference with negligible increase to the mobile station’s complexity or cost [10]. In [13], the performance tradeoffs between the cellular and peer-to-peer network are discussed.

A hybrid network model which offers the performance benefits of peer-to-peer networks while not exhibiting the associated problems is also proposed. This kind of peer-to-peer network model (or peer-to-peer relaying) integrated with the cellular architecture has been considered as an attractive candidate for future wireless networks because of its ability to operate using small transmission powers and to increase the spatial reuse in the network [11]. Consequently, the integration of the cellular and mobile/wireless relaying technologies makes a good candidate for future wireless packet data networks because of the significant tradeoffs between the cellular and the peer-to-peer networks. It is conceivable that the hybrid system can offer performance improvement over the individual systems.

Conventional positioning methods require that the target mobile can hear at least three adjacent base stations (BS). However, in cellular communication systems, one of the main design philosophies is to make the link loss between the target mobile and the home BS as small as possible, while the other link loss as large as possible to reduce the interference and to increase signal-to-interference ratio for the desired communication link. This design philosophy is not favourable to position location (PL), and leads to the main problems in the current PL technologies, i.e. hearability and accuracy.

In this paper, we investigate another possible application of the mobile’s relaying capability assisting mobile positioning. We propose a PL scheme based on a hierarchical management of the BSs and MSs in a hybrid cellular/peer-to-peer network to improve positioning coverage.
2 Conventional position location (PL) techniques

Positioning uses the signals emitted, reflected, or diffracted from the target object to locate the object. Accordingly, cellular positioning is to utilise the radio signals transmitted between a mobile station (MS) and a set of BSs with known positions in a cellular network to locate the MS by measurement and data processing. In this section, the conventional positioning methods are first reviewed, followed by an analysis of the impairment and a discussion of the challenges in cellular networks for accurate positioning.

2.1 Positioning methods

A variety of basic methods and their variants are available in the literature for cellular positioning. Each category is associated with the measurement of a specific set of signal characteristics. Conventional positioning methods can be classified as follows: received signal strength (RSS), angle of arrival (AOA)/direction finding (DF), time of arrival (TOA), time difference of arrival (TDOA), frequency difference of arrival (FDOA)/doppler difference (DD), carrier phase of arrival (POA), pattern recognition, and hybrid techniques comprised of two or more of the above methods.

RSS method, which has been well studied since early stages of the research on radiolocation, is based on measurement of the signal strength, i.e. signal amplitude [14, 15]. The measured RSS provides ranging information for estimating the distance between the MS and the BS. The MS lies on the circumference of a circle, with the BS as centre and a radius equal to the distance estimate. The intersection of the circles arising from multiple measurements determines the location of the MS. RSS method is comparatively simple for analysis and implementation but very sensitive to interference caused by either fast multipath fading or slow shadow fading in a hostile wireless environment. Furthermore, in cellular networks, power control techniques are generally adopted to enhance system capacity and performance. Therefore, current RSS is usually applied under circumstances of coarse-grained localisation, e.g. indoor geolocation [16]. It has also been pointed out in [16] that AOA and/or TDOA have been found to be promising in outdoor environments, their effectiveness in indoor environments is limited by the multiple reflections suffered by the RF signals.

The AOA/DF method is based on the measurement of signal direction through the use of antenna arrays. The desired location of the MS lies at the intersection of multiple lines of bearing pointing from the BSs to the MS [1–3]. The AOA method has long and widely been studied in many areas, especially in radar and sonar technologies for military applications. Using complicated adaptive phased array antennas, high-resolution angle measurement would be obtainable for AOA. In cellular systems, the deployment of smart antenna makes AOA practical. However, the drawback of AOA includes (i) complexity and cost for the deployment of antennas at the BS side and impractical implementation at the MS side; (ii) susceptibility to linear orientation of BSs; (iii) accuracy deterioration with the increase in distance between the MS and the BS owing to fundamental limitations of the devices used to measure the arrival angles and changing scattering characteristics [3].

The FDOA/DD method is based on the measurement of frequency shift in the signals of interest when there is a relative motion between the MS and the spatially separated BSs. The received frequency difference of the same signal transmitted from a moving MS to a pair of BSs is a function of the location coordinates of BSs, and the location and velocity of movement of the MS. Multiple pairs (greater than 3) of BSs in the same plane form an equation set which determines the location of the MS. However, owing to inadequate Doppler effects resulting from low speed mobiles, FDOA is impractical in terrestrial cellular systems but is commonly used for satellite or aerial military applications [17, 18].

The phase difference of carrier signals between MS and BSs can assist in location estimation as well. The PDOA method measures the phase difference of the same signal arrived at different BSs. Obviously, the phase shift is caused by path difference and defines a hyperbola. The intersection of hyperbolas determines the location estimate. The phase-based technique could achieve very high resolution, less than one wavelength, for location estimation [19]. However, the challenges in the PDOA scheme include: (i) a large number of ambiguities in the solution resulted from multiple times of integer wavelengths at the receiver; (ii) the requirement to maintain a continuous lock of the carrier signal. Hence it is only commonly applied in short-range location [20].

Pattern recognising, which is also widely known as location fingerprinting (LF), belongs to another classification of synthetic positioning method, which locates mobiles by recognising the wave pattern of the radio signals radiating from the MSs [21, 22]. Pattern recognising stores the synthesised modelling of radio propagation environment covering the entire interested zone in a database with a field training process. This is the first phase for data collection (off-line phase). The location estimate of the target MS could be derived from a comparison of signal measurements and the information stored in the database. This is the second phase of locating an MS in real-time (real-time phase). In this phase, a pattern-matching algorithm is then used to identify the most likely recorded fingerprinting to the measured one and hence to infer the corresponding user’s location [21, 22].

Pattern recognising method could obtain high resolution as long as the synthesised modelling process collects adequate information. However, the high cost for deployment and maintenance is obvious and unavoidable. As a result, it is a promising technique but not a mainstream option for the time being.
Time of arrival (TOA) and time difference of arrival (TDOA) methods are based on time measurements [1–3]. Since radio waves travel at the speed of light, the TOA measurement directly provides a distance estimate from an MS to a BS. Several of these measurements can thus be used to locate the target MS. The MS lies on a circle with radius equal to the distance estimate and centred at the BS. The desired MS location is determined by the intersection of at least three circles formed by multiple measurements between the MS and several BSs. TOA measurement needs to timestamp the signals and requires synchronisation among the BSs. The TDOA method is a variation of TOA and it measures the time difference of signals travelled from an MS to a pair of BSs, or vice versa. The MS lies on a hyperbola defined by constant distance difference to the two BSs with the foci at the BSs. The desired location of the MS is determined at the intersection of the hyperbolas produced by multiple measurements. TDOA method requires no timestamp and only the synchronisation among the BSs is required.

Since the above reviewed location methods complement each other, hybrid techniques, which use a combination of available range, range-difference, or angle measurements, or other methods to solve for locations, have been extensively investigated (see for example, [23, 24]). Hybrid techniques are also studied to combat the challenges in positioning problems, such as hearability, accuracy, NLOS problems, which will be discussed in the next subsection. Hybrid methods are especially useful in hearability-challenged conditions when the number of available BSs is limited. This may be the case in CDMA-based systems where power control is employed or in areas where the density of the BSs is low [24]. Most typical hybrid method combines TOA (TDOA) location with AOA location [23–25]. The scheme proposed in [26] combines TDOA with RSS measurements. RSS combined with location fingerprinting method is studied in [27].

For the proposed positioning approach in Section 3, we select the TDOA method to construct topology equations between the known and the unknown coordinates. This selection is based on the fact that time-based methods are more practical for cellular positioning and have been standardised by commercial cellular systems. The observed time difference of arrival (OTDOA) technique has been chosen to position the user equipment (UE) in the UMTS [28], TDOA has been applied in IS-95 CDMA [2, 5] and GSM [19, 5]. Furthermore, TDOA has several advantages over TOA or AOA, e.g. it does not require the knowledge of absolute time of transmission, nor special phase array antenna. It has been the most popular technique employed in practical PL methods. It is noted that the proposed position location discovery procedure is not limited to TDOA method. More advanced algorithms, for example, hybrid TDOA/AOA, can also be integrated into the position discovery procedures.

### 2.2 Challenges in cellular networks for positioning

The main problems in PL are the inaccuracy of the estimated position and the hearability of the BS, i.e. the ability to receive sufficient signals from at least 3 BSs to estimate its position. This is because typical positioning techniques require multiple BSs to locate a unique MS position [5]. The hearability problem may result from two causes: one is due to the distance which leads to the attenuated signal strength being lower than the acceptable threshold. Another cause is the paths between the MS and the BSs are obstructed, for example, by hilly terrain or high buildings. In rural areas, lack of enough BSs for cellular positioning is quite common owing to the large cell size.

Since the basic purpose of cellular network is for communications, therefore, in the cellular structure, one basic design principle is to make the signal-to-interference ratio (SIR) high only for the link between the mobile and the home BS, while the signal strength seen at other BSs is designed to be low to reduce inter-cell interference. The communication design philosophy requires minimum acceptable signal power in order to reduce interference to other nearby MSs, making it unfavourable to PL. This is more severe when the MS is located close to the serving BS or when the size of the cell is large. The number of neighbouring BSs that receive the signal at a sufficient power level for the duration necessary to make measurements may be less than the minimum number required for positioning. As a result, the coverage (the percentage of the successfully position located mobiles) deteriorates. This is particularly a problem in a CDMA system where the serving BS maintains the MS transmission power just high enough for proper detection [5, 24]. The coverage is mainly restricted by the hearability problem. Therefore, many measures have to be taken to alleviate the hearability problem such as network planning and power control mechanisms. Hearability is one of the major concerns for positioning.

Accuracy requirement is another challenge for positioning, which is the focus of this paper. For the proposed PL scheme described in Section 3, we will use the TDOA approach to solve the location problem. Since the TDOA method measures the time difference of the signals travelled from an MS to a pair of BSs, or vice versa, signal strength plays a crucial role in the accuracy of position determination. In the case of very low signal strength, the ability to obtain acceptable TDOA estimates could be greatly impaired.

Other obstacles to accurate estimation include non-line-of-sight (NLOS), multipath, co-channel interference [4, 24, 29], and the application of cellular power control techniques. The NLOS problem tends to be the main source of error in the range measurements, and the resulting range measurements are positively biased [29]. Here NLOS refers to the signal traversing through the direct path between the transmitter and the receiver being attenuated or blocked by obstacles. In [29], a classic method that uses historical measurements to identify NLOS and recover the estimates was proposed. It is possible to correct the NLOS ranging error by exploiting a priori knowledge of the statistical characteristics of the system’s standard measurement noise [29]. In [4], a constrained nonlinear least squares solution was proposed for TOA-based location estimation based on the assumption that NLOS range errors are always positively biased. In [30], true ranges are represented as a scaled version of the LOS range and a nonlinear optimisation is applied to estimate the scale factors. In [31], the distribution function of the TOAs are used to mitigate the NLOS errors. In [24] and [32], hybrid positioning techniques and the residuals formed from several sets of BSs are utilised to reduce or mitigate the effect of NLOS error.

The multipath signals resulting from multiple reflections and diffractions can lead to several undesirable consequences. (i) Amplitude fading: the received signal may exhibit Rayleigh fading, Rician fading, or Gaussian distribution depending on the relative levels of the LOS component to multipath component. (ii) Time dispersion: multipath components reflected from different objects would arrive at the MS or BS with time delays. The time
delay at the level of milliseconds would result in serious inaccuracy of location estimate, while the time delay at the level of nanoseconds that is less than a chip period would be difficult to resolve for time-based estimation methods. (iii) Angle change: the multipath components reflected from objects could arrive at the receiver via a changed direction. This would result in errors for AOA method. Efficient computing and signal processing algorithms have been designed to mitigate the effect of multipath, for example, the database correlation method (DCM) enables positioning with fewer measurements and avoids multipath problems [33], super resolution methods for AOA such as Root-MUSIC [34].

The ratio of the received signal strength to co-channel interference is referred as signal-to-interference ratio (SIR); the value of SIR has a direct impact on the quality of position location estimation. An increase in co-channel interference directly reduces the SIR of the signal and thus degrade location estimation. This is a lesser problem in frequency division multiple access (FDMA) and time division multiple access (TDMA) compared to code division multiple access (CDMA), which is an interference-limited multiple access technology [35].

For CDMA communication systems, no matter what the distance of the MS to the serving home BS is, a power control mechanism has been adopted to enable signals from the MSs to the serving BSs to be maintained at the same level to combat the ‘near-far problem’. This power level is designed just high enough for proper detection. However, for cellular positioning, the conventional power control mechanism is not desirable owing to the need for multiple BS to assist in positioning at the same time. One possible technique is to power up the target MS so that it would be heard at distant assisting BSs. Another possible technique is to lower the emitted power of other BSs so that the assisting BS could be heard at the MS. The former technique is used in network-based solution while the latter is used in terminal-based solution [36].

In this paper, we assume that errors owing to NLOS propagation, multipath and co-channel interference have been factored out prior to employing the position estimator. We further assume the existence of the pilot channel, which is not power controlled. Our focus is to improve the coverage and estimation accuracy by making use of the relaying capability of the mobiles to exploit as much as possible the short distance transmission. In addition to the obstacles mentioned above, the accuracy problem associated with the TDOA method depends to a large degree on the efficiency of the computational algorithm used. In the next subsection, we discuss two computing algorithms that are appropriate for the TDOA method.

2.3 Computational algorithms

In general, the computation process of cellular positioning can be divided into two steps. The first step is to obtain values of measurement estimate and form a set of mathematical equations with respect to different positioning methods introduced above, e.g. RSS, AOA, TDOA. The second step is to solve these equations and derive the location estimate. A number of computing algorithms have been developed for a specific positioning method. Since we will use the TDOA method to construct positioning equations, in this subsection we will focus attention on the algorithms for TDOA.

When the TDOA measurement is available, a set of hyperbolic equations can be established. The target MS is supposed to lie at the coordinates where the hyperbolas intersect. Theoretically, the coordinates could be directly derived from the solution of the equation set. However, there are two major challenges: first, owing to the measurement deviation caused by noise, interference, and imprecision in measurement, the equations are inconsistent and the locus curves defined by the equations do not intersect at a point in most cases, i.e. the equation set is nonlinear and usually do not have a solution; second, in some situations, the number of equations is more than the number of variables in the equation set to make use of the redundancy for more accurate location estimation. Therefore, appropriate measures have to be taken. The first is to linearise the nonlinear equation set. Secondly, a least squares (LS) fitting is then used to combine all measurements to exploit the redundant information. Weighted least squares (WLS) is commonly used to minimise the errors by giving different weight to different measurement [37].

There is a number of well-developed linearising algorithms. The classic algorithms can be grouped into recursive and non-recursive. A typical recursive algorithm uses the Taylor-series expansion [38] to linearize the equation set while a typical non-recursive algorithm is that developed by Chan and Ho [39]. These two algorithms offer the best performance among most of the candidates and much of the recent works have been carried out based on them. The Taylor-series method starts with an initial guess and improves the estimate at each step by determining the local linear least-squares. Convergence of the iterative process will depend on the initial value selection. Moreover, the recursive computation results in high computational cost as least-squares computation is required in each iteration. Alternatively, with Chan and Ho’s algorithm, computational cost is relatively low, convergence problem is eliminated and it works well with Gaussian distributed noise [39].

Furthermore, Fang’s algorithm [40] avoids the inefficiency and convergence problem of the iterative algorithm and it is shown that the computation of the hyperbolic position fixes could be reduced to the solution of a quadratic equation. Friedlander [37] derived optimal weighting matrix for the least-squares estimator from noisy measurements of range differences. Smith and Abel [41] presented the spherical interpolation method to obtain the closed-form least-squares approximates for localisation from TDOA measurements. Their location formula is derived from least squares ‘equation-error’ minimisation.

In our simulation work, we use the algorithm of Chan and Ho to obtain the MS coordinates by solving the TDOA equations. This is motivated by the fact that the method of Chan and Ho performs well and has substantially lower computational complexity than the Taylor-series method, and is able to tolerate a higher noise threshold [1].

3 Proposed positioning based on relaying capability

Because of the inherent obstacles associated with mobile positioning in cellular systems, we propose a novel positioning approach by utilising the relaying capability of the MS in a hybrid cellular/peer-to-peer network to assist location estimation.

3.1 Proposed PL

The BSs and MSs are arranged in a hierarchical structure. Those mobiles that can receive signals above a given threshold from more than 3 BSs are selected as level-1 mobiles. They are ready to serve as relays for lower level mobiles after their position coordinates are calculated by using the known coordinates of the BSs with good fidelity.
Then the BSs, together with these level-1 mobiles, are employed to assist the PL for the remaining mobiles. This hierarchical arrangement of BSs and MSs to make use of the relaying capability is tantamount to a flooding procedure.

This flooding process is illustrated in Fig. 1. The top level consists of the fixed based stations, whose coordinates are known a priori. The middle level is the level-1 MS/relays. It will be shown in simulation that most of the level-1 mobiles are located near the 3-way cross-sections of the cells and have acceptable signal strength from at least 3 BSs. The lowest level contains the level-2 MSs, whose positions are to be determined making use of one or more level-1 relays. It will be shown that most of the level-2 mobiles locate close to the centre of the home cell. They can have strong signal strength only to/from the home BS, but poor signal strength to/from other neighbouring BSs. This flooding procedure can continue further. The problem with this flooding procedure is ‘error accumulation’. To avoid error accumulation, we only consider 1-hop relaying, as shown in Fig. 1. It will be shown that by using relaying, the coverage for positioning can be extended, the hearability problem can be alleviated, and the estimation accuracy can be improved.

3.2 Analysis procedure

In the conventional TDOA method, each mobile estimates the three (or more) strongest probe signals from the nearby base stations. When the received probe signal at the $i$th-MS from the $j$th-BS is lower than the prescribed threshold, the $i$th-MS cannot hear from the $j$th-BS, or equivalently, the $j$th-BS cannot assist the $i$th-MS for its positioning.

In the proposed procedure, the first step is the same as the conventional TDOA method, i.e. to find the mobiles which can hear at least three nearby BSs, whose probe signals are received by the MS above the threshold. These mobiles are categorised as level-1 mobiles, or relays, since they can serve as relays to assist locating the remaining mobiles. The coordinates of these level-1 mobiles can be immediately determined by using either the computing algorithm of [38] or [39]. After this step, the remaining mobiles, which are categorised as level-2 MSs, continue to monitor the signals from the BSs and all the level-1 mobiles (relays) and select the strongest three probe signals. If all three selected signals are above the threshold, the coordinates of the mobile can be immediately obtained using the same algorithm as level-1 mobiles’ positioning. If the number of the probe signals above the threshold is less than three, we claim that this mobile cannot perform desirable positioning location. The coverage of the PL then deteriorates.

It is not hard to see that the threshold is an important parameter that has an impact on the performance of the proposed positioning algorithm. When the threshold is set very low, the proposed algorithm becomes the conventional location method. The level-2 mobiles merge into the level-1 relays, as shown in Fig. 1. The flooding procedure becomes the conventional positioning method, i.e. no relays. All mobiles’ positioning requires the assistance from the BSs. When the threshold is set very high, then nearly no mobile can be selected as relay. The middle level mobiles in Fig. 1 are switched to the lower level (level-2) and all the lower level mobiles have to depend on the BSs for positioning, which again returns to the conventional positioning method.

The threshold will also have an effect on the tradeoff between coverage and accuracy. If we set a lower threshold, more mobiles can be selected as relays. In addition, more level-2 mobiles can detect more than three signals from either BSs or known relays above the threshold. As a result, the coverage can be improved. However, for both level-1 and level-2 mobiles, lower threshold leads to lower accuracy. Furthermore, the positioning error occurred in level-1 relay further propagates to level-2 positioning when using the relay’s coordinates, which further exacerbates the overall accuracy.

The proposed location finding procedure is also motivated from the fact that GPS achieves high accuracy by utilising LOS signals because there are few obstacles blocking the LOS path between MSs and the satellites. However, as for cellular positioning, LOS path is easily blocked by hilly terrain, vehicles, and buildings. In such a hostile radio propagation environment, the channel impairment deteriorates the estimation accuracy. In the proposed algorithm, we exploit the possibility of the short distance transmission as much as possible by making use of the relatively short distance between the level-2 mobiles and the nearby relays. By this means, an MS always attempts to discover the best radio signal source to obtain more accurate location estimates. As a result, location diversity can be achieved. In addition, the coverage of effective positioning in cellular networks will be extended with the increase in the number of mobiles in the system owing to less dependence on the fixed BSs. The large number of MSs provides a high degree of flexibility and forms a location diversity option.

3.3 Formulation of the TDOA equations

The position of an unknown mobile is resolved by measuring the TDOA from three known mobiles/relays (referred to as references in the sequel) with known coordinates: $(x_1, y_1), \ i = 1, 2, 3$. The unknown mobile is located at $(x, y)$, as shown in Fig. 2. For completeness, the

![Fig. 1 Mobile location flooding procedure](image)

![Fig. 2 Geometry for establishing the TDOA equations](image)
avoid folding/boundary effect. We further assume that the related part to solve the TDOA equations using Chan and Ho’s algorithm [39] is described below. Let \( d_i, i = 1, 2, 3 \) be the distance between the unknown mobile at \((x, y)\) and the known reference at \((x_i, y_i)\). The square distance is

\[
d_i^2 = (x - x_i)^2 + (y - y_i)^2 = k_i^2 - 2x_i x - 2y_i y + x^2 + y^2 \quad i = 1, 2, 3
\]

where

\[
k_i^2 = x_i^2 + y_i^2
\]

Let \( c \) be the speed of light, \( \delta_i \) be the distance difference estimate between the target mobile to \((x, y)\), \( d_i, i = 2, 3 \), and that to \((x_1, y_1)\), \( d_1 \), and \( \Delta t_i \) be the corresponding time difference of arrival estimate. The relation between \( \delta_i \) and \( \Delta t_i \) is

\[
\delta_i = c \cdot \Delta t_i = d_i - d_1 \quad i = 2, 3
\]

The TDOA estimate, \( \Delta t_i \), can be expressed as

\[
\Delta t_i = \Delta t_i + n_i \quad i = 2, 3
\]

where \( \Delta t_i \) denotes the noise free value and \( n_i \) the estimation noise. Assume that the TDOA measurement errors are independent Gaussian random variables with zero-mean and variance \( \sigma_i^2 \). From (2), we have \( d_i^2 = (\delta_i + d_1)^2, i = 1, 2, 3 \), and \( \delta_1 = 0 \). Substituting this relation into (1) yields

\[
\delta_i^2 + 2\delta_i d_1 + d_1^2 = k_i^2 - 2x_i x - 2y_i y + x^2 + y^2 \quad i = 2, 3
\]

When \( i = 1, (1) \) can be written as

\[
d_1^2 = k_1^2 - 2x_1 x - 2y_1 y + x^2 + y^2
\]

Subtracting (5) from (4), we have

\[
\delta_i^2 + 2\delta_i d_1 + d_1^2 = k_i^2 - k_1^2 - 2(x_i - x_1)x - 2(y_i - y_1)y \quad i = 2, 3
\]

which can be written in a matrix form as

\[
\begin{bmatrix} 2(x_2 - x_1) \\
2(x_3 - x_1) \\
2(x_3 - x_2) \\
2(y_2 - y_1) \\
2(y_3 - y_1) \\
2(y_3 - y_2)
\end{bmatrix} \begin{bmatrix} x \\
y \\
2\delta_2 \\
2\delta_3 \\
d_1 \\
\delta_i^2 + k_i^2
\end{bmatrix} = \begin{bmatrix} \delta_2^2 + K_1^2 - K_i^2 \\
\delta_3^2 + K_1^2 - K_i^2
\end{bmatrix}
\]

Equation (7) is a set of linear equations with unknowns \( x, y \) and \( d_1 \). The solution to \((x, y)\) as a function of \( d_1 \) can be obtained as

\[
\begin{bmatrix} x \\
y
\end{bmatrix} = - \begin{bmatrix} (x_2 - x_1) \\
(x_3 - x_1) \\
(x_3 - x_2) \\
(y_2 - y_1) \\
(y_3 - y_1) \\
(y_3 - y_2)
\end{bmatrix}^{-1} \begin{bmatrix} \delta_2^2 + K_1^2 - K_i^2 \\
\delta_3^2 + K_1^2 - K_i^2
\end{bmatrix}
\]

Inserting this result into (5) gives a quadratic in \( d_1 \). Substitution of the positive root back into (8) produces the solution for the unknown coordinates \((x, y)\).

4 Simulation results and discussions

In the simulation, we model a hybrid cellular system shown as in Fig. 3, where the number of mobiles in the system is 80 in this example. We assume that there are 27 BSs, located at the centres of the 27 hexagonal cells. The radius of the cells is \( r = 200 \text{ m} \) (the cell size can be scaled to any other length), except when explicitly specified for other values. Mobile stations are uniformly distributed within a circle with a radius of \( R = 5r \cdot \sin(\pi/3) \text{ m} \) \((R = 870 \text{ m} \text{ if } r = 200 \text{ m})\) to avoid folding/boundary effect. We further assume that the transmitted probe signal strength is \( P_r = 0 \text{ dB} \), the received signal strength at the target MS is given by

\[
P_r = P_t + L_p(x, d) + \mu_r(\sigma_x) \quad \text{(dB)}
\]

where \( \mu_r(\sigma_x) \) is the shadow fading, which is a normal random variable with zero mean and standard deviation (STD) \( \sigma_x \) typically in the range 6–12 dB for terrestrial radio environment; \( L_p(x, d) \) is the path gain given by

\[
L_p(x, d) = 10 \cdot \log_{10}(d^2)
\]

where \( z \) is the path gain exponent, and \( d \) is the distance between the transmitter and the receiver, i.e. the distance between the target mobile and the reference station. In the simulation, we select \( z = -4 \). Furthermore, the TDOA estimation error is assumed to have a Gaussian distribution with zero mean. Instead of directly working with the time difference measurement error, we focus on the distance difference measurement error. The difference of these two kind of errors is a multiplication factor, \( c \), the speed of light, as given by (2). The variance of the measurement noise is assumed linearly proportional to the square distance difference

\[
\sigma^2 = k \cdot \delta^2
\]

where \( k \) is the proportional constant which reflects the sensitivity of the TDOA measurement error to the distance. A large value of \( k \) implies the measurement is more sensitive to the distance, or equivalently to the energy, since energy is a monotonic function of distance. As a result, \( k \) is referred as the sensitivity factor in the sequel. The presented simulation results on coverage and accuracy are obtained by averaging 1000 simulation runs.

Figure 4 shows the discovered relays (level-1 mobiles) and level-2 mobiles in the flooding procedure, shown as plus (+) markers and large dots, respectively. The small dots represent the mobiles that cannot successfully detect at least three probe signals above the threshold from the known stations. Therefore, we cannot specify the positions of these small-dot-marked mobiles. We can observe that the relays tend to locate near the cross-sections of three adjacent cells, where they can receive probe signals from three BSs, all above the threshold. The level-2 mobiles are more likely to locate near the home BSs than that of the level-1 mobiles.

Figure 5 shows the coverage against the threshold by using the proposed algorithm when shadow fading is ignored \((\sigma_x = 0 \text{ dB})\). The coverage represents the percentage
of mobiles that the algorithm is able to resolve their locations. The solid and dashed curves are the results when the total number of users is 60 and 120, respectively. The two lower circle-marked curves (that coincide with each other) are the coverage for level-1 mobiles, and the two upper plus-marked curves are for the total coverage. Since the conventional method only locates level-1 mobiles, i.e. positioning is completed by the assistance from the BSs. The lowest circle-marked curve also denotes the coverage by using the conventional method. The difference between the plus-marked curves and the circle-marked curves is the corresponding gain in coverage by using mobile relays.

It is shown that the coverage for the conventional cellular single hop positioning technique does not change when the number of users changes. With an increasing number of mobile users in the system, the total coverage percentage by using mobile relays improves significantly. We can also observe that when the threshold is either very high or very low, there is no benefit by using the proposed hybrid cellular and ad hoc structure. In this case, the coverage tends to 0% or 100% with an unacceptable accuracy. Therefore, the threshold can be selected to tradeoff between coverage and accuracy, providing the proposed hybrid positioning technique superior performance compared to conventional techniques.

Figure 6 has the same legend as in Fig. 5, except that shadowing with a standard deviation of 8 dB is included in the simulation. It is interesting to observe that shadowing essentially improves the coverage. Since shadowing is modelled as zero mean, the gain of the positive shadowing values outweighs the negative values. This can be explained as a kind of space diversity gain. With the coverage improvement of level-1 mobiles, the total coverage of the proposed discovery mechanism improves significantly over the case without shadowing.

Figure 7 shows the coverage against the number of mobile users in the system when the cell radius is 200 m, threshold is $-62$ dBm. The lower curve represents the results of coverage for level-1 mobiles, and the upper curve for the total coverage. It is shown that with the increase in the number of users, the coverage of level-1 mobile, i.e. the conventional approach, is kept constant, which is consistent with the results presented in Figs. 5 and 6. However, with the increase in the number of mobile users, the coverage of the proposed discovery mechanism increases monotonically towards 1.

Figure 8 depicts the same trend with cell radius 800 m, and threshold $-85$ dBm. We can observe that with an increase in the cell size, in order to maintain a comparable coverage for level-1 mobiles, the threshold has to be decreased, which leads to a sacrificed estimation accuracy. The advantage by using the proposed discovery mechanism
is more promising in terms of the absolute increase of the overall coverage over the conventional discovery method.

In this paper, we use mean estimation error (MEE) as a performance measure of the positioning method. The mean estimation error (MEE) is defined as the mean distance of the estimated mobile location to the true location, given as

\[ \text{MEE} = E \left[ \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2} \right] \]

where \((x, y)\) and \((\hat{x}, \hat{y})\) denote the true and the estimated mobile locations, respectively.

Figures 9 and 10 are the MEE against the threshold for various sensitive factors \(k\), when the number of users in the system is 60. In these figures, the square-marked curves are the results for the level-1 mobiles. For mobiles in this level, there is no difference between the conventional single-hop and the proposed hybrid discovery method. Therefore, we do not distinguish the results for the conventional and the hybrid systems. The plus-marked and the circle-marked curves are the MEE results for the level-2 mobiles with relay and without relay, respectively. We can observe that with the increase of the threshold, all of the level-1 and level-2 mobiles exhibit improved positioning accuracy. The improvement is obtained at a price of a reduced coverage. In the listed range of sensitivity factor, \(k\), level-2 mobiles with relays always perform better than their counterpart mobiles without relays. With a large sensitivity factor, \(k\), the improvement in accuracy of the hybrid method over the conventional method is more significant. When \(k\) is small and the threshold is relatively high, e.g. \(k = 0.005\), the accuracy of the positioning for the level-2 mobiles is similar for the schemes with and without relays. We have also found from simulation that with the increasing number of mobiles, the MEE performance does not have obvious changes with Figs. 9 and 10.

It is expected that when the NLOS effect is included, the proposed scheme will exhibit significant improvement over the conventional method owing to the fact that the relays provide location diversity.

5 Conclusions

In this paper, we briefly reviewed the conventional mobile location technologies and assessed the advantages and disadvantages of the conventional approaches. A new hierarchical location method is proposed. Those mobiles heard from at least three BSs are selected as level-1 MS/relays to assist level-2 MS positioning. Simulation results show that the proposed method offers improved coverage and positioning accuracy compared to the conventional methods. With an increase in the number of mobiles in the system, the coverage of the conventional system does not change. However, the proposed discovery mechanism can achieve monotonic increase towards 100% by exploiting the position diversity provided by the relays. With an increase in the cell size, the application of the proposed method is also promising in terms of coverage improvement.

The proposed integration of the cellular and mobile/wireless relaying technologies can effectively flood the positioning information in the hybrid system, and increase the coverage of the positioning algorithm. Together with the relaying capability used in load balancing, traffic forwarding, the integration technologies may lead to the convergence between the next generation wireless cellular systems and ad hoc networking.

Other advantages by using mobile relays may be the exploitation of the line-of-sight links as much as possible. In this paper, we did not explicitly model the NLOS phenomenon. It is conjectured that when considering NLOS, the improvement by using mobile relay will be significant. The improved location information can be used in aid of, for example, soft handover at optimum time, location-based resource management, adaptive cell sizing, adaptive load balancing, etc.
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7 References

28 UMTS: ‘Stage 2, functional specification of UE positioning in UTRAN’ (3GPP TS 25.305 version 5.4.0)