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A Model for the Integration of Prioritized Knowledge Bases through Subjective Belief Games

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Abstract. Belief merging is concerned with the integration of several belief bases such that a coherent belief base is developed. Various belief merging models have been developed that use a belief negotiation game to address this problem. These models often consist of two key functions, namely: negotiation, and weakening. A negotiation function finds and selects the weakest belief bases among the available belief bases, and then a weakening function removes the least valuable set of information from the selected belief base. This process is iteratively repeated until a consistent belief base is developed. In this paper, we extend the current game-based belief merging models by introducing the Subjective belief game model. The Subjective belief game model operates over a subjective belief profile, which consists of belief bases with subjectively annotated formulae. The subjective information attached to each formula enables the proposed model to prioritize the formulae in the merging process. One of the advantages of our proposed game is that it provides room for enhancing the content of the weak belief bases, instead of enforcing their further weakening. Trustworthiness in the form of the reliability of the information provided by the belief bases is also considered. We provide several instantiations of the model by introducing suitable functions. Furthermore, we provide insight into a new field of application of belief games: *collaborative conceptual modeling* and formally investigate how the outcome of such a process can be obtained as a solution of the Subjective belief game model.

Keywords: Belief Merging, Collaborative Modeling, Formal Conflict Resolution, Social Contraction, Negotiation

1. Introduction

Belief is what an intelligent believes to be true about the state of the world; however, not every thing that an intelligent believes to be true is considered to be true by other peer agents. As a matter of fact, different agents may possess different or even conflicting views towards reality. Belief merging deals with the issue of acquiring and developing a consistent set of beliefs (a belief base) from several different sources of belief, i.e., given a set of information sources that may contain mutual inconsistencies, a coherent set of information must be developed which is a fair representative of the content conveyed by all the information sources [1]. Several researchers have approached this issue as a multi-stage game [2, 3, 4]. The game is a competition between the information sources where the weakest information sources are considered as the losers of the game and should make appropriate concessions. The basic idea of the merging game is to form some sort of coalition between like-minded information sources, and penalize the information sources that are furthest away from the coalition. The rationale for such an approach under the reliability assumption for the information sources is based on *Condorcet's Jury theorem* [5]. This theorem argues that in most cases listening to the majority of the members of the jury (coalition) is the most rational decision.

Each stage of the merging game is known as a round of negotiation or competition. Konieczny's *Belief Game Model (BGM)* [2] and Booth's *Belief Negotiation Model (BGM)*

[3, 6] are among the most popular frameworks for belief merging, which are defined for a pure propositional logic setting. In these models, in each round of negotiation, some of the information sources are selected by the negotiation (choice) function. The selected sources should then weaken their beliefs using a weakening (contraction) function. Focusing on a pure propositional logic formalism makes the definition of flexible negotiation and weakening functions more difficult in these frameworks.

Several models have been proposed which exploit priority relationships between the belief bases and their constituting formulae in the process of merger [7, 8, 9]. The priorities provide essential information with regards to the degree of formula importance. They can be helpful in deciding which formulae need to be discarded in order to restore consistency. Possibilistic logic has been extensively used for this purpose where the formulae in the belief bases are annotated with a weight denoting their degree of necessity. One of the key factors that needs to be incorporated into the priority rankings is the degree of uncertainty of the information sources about their expressed opinions. Possibility theory addresses uncertainty through the necessity and possibility measures. *Subjective logic* [10], an extension to *Dempster-Shafer theory of evidence* [11], is also able to explicitly address uncertainty. It defines uncertainty as one of the dimensions of belief in its three dimensional belief structure.

Logic-based merging approaches have been employed in different fields such as multi-agent systems, and distributed computing. Similar to these areas, collaborative software or knowledge modeling (conceptual modeling) is concerned with the employment of information from different sources. Here, the sources of information are human experts that are typically cooperating to develop a unique model for a given domain of discourse [12]. The model developed by each expert can be considered as an information source and their integration can be viewed as a belief merging process. The factor distinguishing collaborative modeling from the above mentioned fields is that it directly interacts with *human participants*. Humans usually make conceptual errors due to risk aversion, short-term memory or even framing and perceptual problems [13]. This implies that not all asserted information from the sources are correct or equally reliable. Furthermore, epistemic uncertainty (also known as partial ignorance) is an indispensable element of human judgments that makes them even more susceptible to inaccuracy and imprecision [14]. There is no guarantee that the received information coming from various human sources be consonant or consistent. In many cases they may be arbitrarily (with very few common elements) or disjointedly (no common elements) distributed [14]. Hence collaborative modeling needs to consider the significant role of uncertainty and imprecision in the manipulation of the experts' expressions.

In this paper, we propose a *Subjective belief game model*, where the significance of each formula is addressed through the framework of Subjective logic using subjective opinions. Each belief source is represented by a Subjective belief base (SBB). The process of integrating various subjective belief sources is accomplished in two steps. In the first step, called social contraction, some of the inconsistent belief bases selected by the choice function are both syntactically and semantically enhanced using an enhancement function to make them mutually coherent. For this step, some choice and enhancement functions are defined based on the subjective opinions of the belief bases. In the second step, known as the combination step, the coherent belief bases acquired from the first step are combined

using the Subjective consensus operator, which can have a reinforcement effect on the final subjective belief of the formulae. We will further formally show how the outcome of a collaborative conceptual modeling process can be obtained as a solution of the Subjective belief game model.

Building on the Subjective belief game model, we further show that a collaborative design process can be modeled by extending the functionality of such games. Basically, any collaborative process can be viewed as the exchange of information between human participants, each of which is providing some information from their perspective. The information coming from each participant can be represented as a belief base. Now, since the individual participants information are given from their own point of view, they can contain contradicting, or inconsistent information when they are put together. We explain how these differences can be resolved through the employment of the Subjective belief game models.

The rest of the paper proceeds as follows. We give some preliminaries and an introduction to the belief game model by Konieczny and the belief negotiation model by Booth in the next section. In Section 3, our Subjective belief game model is introduced. Specific choice and enhancement functions for the Subjective belief game model are given in Section 4. We instantiate the proposed model and discuss its properties in Section 5. The application of the Subjective belief game model to collaborative conceptual modeling and an illustrative example are given in Section 6. Finally some related work are reviewed and the paper is concluded in Section 8.

2. Preliminaries

2.1. Propositional Belief Profiles

Throughout this paper, we let $\mathcal{L}$ be a propositional language over a finite alphabet $\mathcal{P}$ of propositional symbols. Ω represents the set of possible interpretations. An interpretation is a function from $\mathcal{P}$ to $\{\bot, \top\}$, where $\bot$, and $\top$ denote falsehood and truth, respectively. An interpretation ω is a model of formula ϕ, noted as $\omega \models \phi$ which makes the formula true. Furthermore, let ϕ be a formula, then $mod(\phi)$ is employed to denote the set of models of ϕ, i.e. $mod(\phi) = \{\omega \in \Omega \mid \omega \models \phi\}$. Classical propositional logic deduction is represented using $\vdash$. Two formula such as ϕ and φ are equivalent, expressed $\phi \equiv \varphi$, if and only if $\phi \vdash \varphi$ and $\varphi \vdash \phi$. A formula ϕ satisfying $mod(\phi) \neq \bot$ is considered to be consistent.

A belief base φ is a consistent propositional formula (or, equivalently, a finite consistent set of propositional formulae $\{\phi_1, \phi_2, ..., \phi_n\}$, considered conjunctively: $\phi_1 \wedge \phi_2 \wedge .... \wedge \phi_n$). Let $\varphi_1, ..., \varphi_n$ be n not necessarily different belief bases, then we define a belief profile as a multi-set Ψ consisting of those n belief bases, i.e. $\Psi = (\varphi_1, ..., \varphi_n)$. This definition permits two or more different sources to be identical. The conjunction of the belief bases of Ψ is represented as $\bigwedge \Psi = \varphi_1 \wedge ... \wedge \varphi_n$. Belief profile inclusion is denoted as $\sqsubseteq$, and belief profile union is represented by $\sqcup$. The cardinality of a finite belief profile Ψ is noted as $\#(\Psi)$.

A belief profile Ψ is consistent if and only if $\bigwedge \Psi$ is consistent. Let $\mathcal{E}$ be the set of all finite non-empty belief profiles. Two belief profiles Ψ_1 and Ψ_2 are equivalent ($\Psi_1 \equiv \Psi_2$)

if and only if there is a bijection (f) between Ψ_1 and Ψ_2 such that each belief base of Ψ_1 is logically equivalent to its image in Ψ_2 (i.e., $\forall\varphi \in \Psi_1, f(\varphi) \in \Psi_2, \varphi \equiv f(\varphi)$).

2.2. Belief Game Model

Richard Booch has proposed a framework for incrementally merging sources of information, which contain inconsistencies [3]. This framework, the *Belief Negotiation Model (BNM)*, provides several rounds of negotiation through which some of the information sources make concessions to reach an agreeable state. Suppose the belief profile Ψ is developed by several information sources. The negotiation process commences in the first round of negotiation by analyzing Ψ^0 (Ψ^0 represents the initial state of the belief profile). If Ψ^0 is consistent, it is considered as the solution. But if Ψ^0 is inconsistent, a round of negotiation between the information sources is performed. The operationalization of this process is performed using two base functions, namely: *choice* and *weakening* functions. In each round, the choice function identifies the losers of the negotiation process. The losers must then make some concessions by conforming to other beliefs through the employment of the weakening functions. BNM possess two important features. First, it does not require all information sources to make concessions in each round. This means that even if two information sources reveal equivalent belief bases, one may need to weaken while the other does not. Second, each information source can possess its own individual weakening (contraction) function. The belief negotiation model has its roots in *belief revision*, which can be viewed in terms of two primary operations: belief contraction, and belief expansion [15].

2.3. Basics of Evidence Theory

Evidence theory is a theoretical models which is able to numerically quantify the lack of knowledge with regards to a certain phenomenon in an effective manner [16]. It is a potentially useful tool for the evaluation of the reliability of information sources. Dempster-Shafer's (DS) theory of evidence is one of the most widely used models that provides means for approximate and collective reasoning under uncertainty [11]. It is basically an extension to probability theory where probabilities are assigned to sets as opposed to singleton elements. The employment of DS theory requires the definition of the set of all possible states in a given setting, referred to as the frame of discernment represented by θ. The powerset of θ, denoted 2^θ, incorporates all possible unions of the sets in θ that can receive belief mass.

The truthful subsets of the powerset can receive a degree of belief mass; therefore, the belief mass assigned to an atomic set such as $\psi \in 2^\theta$ is taken as the belief that the given set is true. Moreover, the belief mass ascribed to a non-atomic set such as $\psi \in 2^\theta$ is interpreted as the belief that one of the atomic sets in ψ is true, but uncertainty rules out the possibility of pinpointing the exact atomic set.

Definition 1 *A belief mass assignment is a mapping $m : 2^\theta \to [0, 1]$ that assigns $m_\theta(\psi)$ to each subset $\psi \in 2^\theta$ such that:*

(1) $m_\theta(\psi) \geq 0$,

(2) $m_\theta(\varnothing) = 0$,

(3) $\sum_{\psi \in 2^\theta} m_\theta(\psi) = 1$.

$m_\theta(\psi)$ is then called the belief mass of ψ. A belief mass assignment is called dogmatic if $m_\theta(\theta) = 0$, since all the possible belief masses have been spent on the subsets of θ.

The belief in ψ is interpreted as the absolute faith in the truthfulness of ψ, which not only relies on the belief mass assigned to ψ, but also to belief masses assigned to subsets of ψ.

Definition 2 *A belief function corresponding with m_θ, a belief mass assignment on θ, is a function $b : 2^\theta \to [0, 1]$ defined as:*

$$b(\psi) = \sum_{\varphi \subseteq \psi} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta. \tag{1}$$

Analogously, disbelief is the total belief that a set is not true.

Definition 3 *A disbelief function corresponding with m_θ, a belief mass assignment on θ, is a function $d : 2^\theta \to [0, 1]$ defined as:*

$$d(\psi) = \sum_{\varphi \cap \psi = \varnothing} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta. \tag{2}$$

To address the degree of uncertainty which is inherent in the above definitions, Jøsang provides a complementary definition, uncertainty, which computes the degree of possible confusion in belief assignment [10].

Definition 4 *An uncertainty function corresponding with m_θ, a belief mass assignment on θ, is a function $u : 2^\theta \to [0, 1]$ defined as:*

$$u(\psi) = \sum_{\varphi \cap \psi \neq \varnothing, \varphi \not\subseteq \psi} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta. \tag{3}$$

With the above definitions, Subjective logic[1] [17] is related to Dempster-Shafer theory of evidence. A belief expression in Subjective logic is defined as a 3-tuple $\chi_x^A = (b_x^A, d_x^A, u_x^A)$ also known as the opinion of expert A about hypothesis x (χ_x^A). It can be shown with this definition that belief (b_x^A), disbelief (d_x^A), and uncertainty (u_x^A) elements of an opinion should satisfy:

$$b_x^A + d_x^A + u_x^A = 1. \tag{4}$$

Josang discusses that the above condition restricts the possible values that can be expressed as an opinion by an expert only to the points placed in the interior surface of an equilateral triangle [18]. The three constituent elements determine the position of an opinion within the triangular space. Figure 1 shows the three axis that can be used to identify the position of an opinion point in the triangle. In the opinion triangle, the line connecting

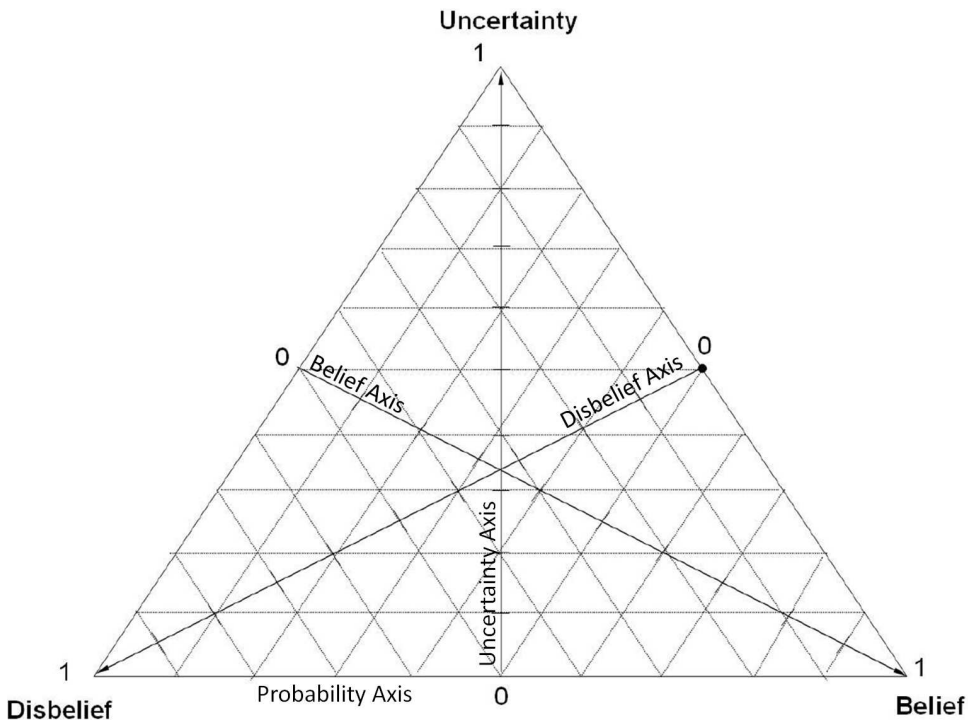

Figure 1. The opinion space can be mapped into the interior of an equilateral triangle.

absolute belief and absolute disbelief corners (right and left corners) is called the probability axis. This is because the removal of uncertainty from Subjective logic will result in a pure probabilistic interpretation of belief (i.e. $b_x^A + d_x^A = 1.$), which respects the additivity condition. The opinions which are situated on this axis are named dogmatic opinions since they do not contain any degree of uncertainty. Among dogmatic beliefs, the two opinions located on the extreme ends of the probability axis are called absolute opinions and represent inflexible agreement or disagreement with a given hypothesis.

A Subjective Belief Base (SBB) is a set of n formulae annotated with subjective opinions in the form of $B = \{(\phi_i^*, \chi_{\phi_i}^A) : i = \{1, .., n\}$, where $\chi_{\phi_i}^A$ is a subjective opinion such that $\chi_{\phi_i}^A = (b_{\phi_i}^A, d_{\phi_i}^A, u_{\phi_i}^A)$. The classical propositional form of B is represented by B^*:

$$B^* = \left\{ \begin{array}{ll} \phi_i \in B^* & \text{iff } (\phi_i^*, \chi_{\phi_i}^A) \in B \text{ and } b_{\phi_i}^A > d_{\phi_i}^A \\ \neg\phi_i \in B^* & \text{iff } (\phi_i^*, \chi_{\phi_i}^A) \in B \text{ and } b_{\phi_i}^A < d_{\phi_i}^A \\ \phi_i \wedge \neg\phi_i \notin B^* & \text{iff } (\phi_i^*, \chi_{\phi_i}^A) \in B \text{ and } u_{\phi_i}^A > b_{\phi_i}^A \text{ and } u_{\phi_i}^A > d_{\phi_i}^A \end{array} \right\}$$

A subjective belief profile $\mathcal{SP}$ consists of multiple SBBs. $\mathcal{SP} = (B_1, ..., B_n)$ is consistent if and only if $B_1^* \cup ... \cup B_n^* \cup \mu$ is consistent. $\mathcal{SE}$, $\mathcal{F}_B$, and $\mathcal{K}$ are employed to denote the set of all finite non-empty subjective belief profiles, the set of all formulae in belief base B and the set of all subjective belief bases, respectively.

Definition 5 *Let B be a SBB, and $\alpha \in (0,1]$. The α-cut of B is $B_{\succeq\alpha} = \{\phi \in B^* | (\phi, \chi) \in B$ and $(b_\chi \geq \alpha$ or $d_\chi \geq \alpha)\}$.*

An α-cut of a Subjective belief base removes all of the formulas of the Subjective belief base that do not have a belief or disbelief value of more than or equal to α. Based on the definition of α-cut, two subjective belief bases are equivalent, shown as $B \equiv_s B'$, if and only if $\forall\chi \in (0,1],\ B_{\succeq\chi} \equiv B'_{\succeq\chi}$. Furthermore, two $\mathcal{SE}$s are equivalent, denoted $\mathcal{SE} \equiv_s \mathcal{SE}'$, if and only if there is a bijection between them.

One of the major difficulties with subjective belief bases is that they require the experts to provide three annotation values for each formula (i.e. belief, disbelief, uncertainty), which is a cumbersome task. Experts are usually more comfortable to work with linguistic variables or singular values in the range of $[0,1]$. It has previously been shown that fuzzy linguistic variables representing subjective opinions can be developed for the ease of use of the experts [19, 20]. Furthermore, we show in the following that experts can also use possibilistic values to annotate their statements and that it is possible to convert such prioritized (possibilistic) belief bases into subjective belief bases. The necessity measure in possibility theory can be used to easily annotate belief bases with a value from the range of [0,1]. The resulting annotated belief base can then be converted into a subjective belief base for the required analysis.

Proposition 1 *[21] For any given possibility distribution such as π, a basic belief mass assignment function within the context of DS theory can be always derived such that the corresponding belief function is consonant.*

This proposition shows that the necessity and plausibility measures in possibility theory are associated with the belief and plausibility functions in DS theory; therefore, a corresponding consonant belief function can always be derived to represent any given possibility distribution.

Proposition 2 *Let $\mathcal{A} = \{(\phi_1, a_1)..., (\phi_n, a_n)\}$ be a prioritized belief base such that the propositional formulae (ϕ_i) are augmented with their degree of necessity (a_i) in possibilistic logic. A corresponding subjective belief base (image of $\mathcal{A}$), denoted $\mathcal{A}_s$, can be derived to represent $\mathcal{A}$.*

Proposition 2 is an important result of Definitions 1-4 and Proposition 1, and shows that any possibilistically prioritized belief base can be converted into a subjective belief base. The consequence of this proposition is that subjective belief bases are a generalization of prioritized belief bases and any of the properties defined for a subjective belief base are applicable to a prioritized belief base through its image in subjective form.

2.4. Semantics of Consensus Formation

The intention of combining information is to meaningfully summarize and simplify a corpus of data elements coming from various sources in order to develop a consensus between the participating sources [18]. There exist several possible techniques for aggregating multiple belief mass assignments in Dempster-Shafer theory of evidence. The base combination rule for multiple mass assignment functions is Dempster's rule of combination, which

is a generalization of Bayes' rule [22]. This combination operator ignores the conflicts between the functions and emphasizes their agreements.

Definition 6 *Let m_1 and m_2 be two belief mass assignments defined on a frame of discernments θ which stem from two distinct sources. Let the combined resulting belief mass assignment from Dempster's rule of combination be $m_{\bigodot} = m_1 \bigodot m_2$ where $\bigodot$ denotes the operator of combination.*

$$m_{\bigodot} = m_1 \bigodot m_2 = \frac{\sum_{\varphi,\psi\subseteq\theta,\varphi\cap\psi=\beta} m_1(\varphi)m_2(\psi)}{1-\sum_{\varphi,\psi\subseteq\theta,\varphi\cap\psi=\emptyset} m_1(\varphi)m_2(\psi)}, \forall\beta\subseteq\theta, \beta\neq\emptyset \tag{5}$$

when

$$\sum_{\varphi,\psi\subseteq\theta,\varphi\cap\psi=\emptyset} m_1(\varphi)m_2(\psi) \neq 1. \tag{6}$$

Dempster's rule redistributes the conflicting masses over the non-conflicting masses and therefore insists on the mutual agreements and removes conflicts [23]. This approach to belief integration has been criticized due to its counter-intuitive results under highly conflicting belief expressions [24]. Several authors have proposed models to overcome this problem. For instance, Smets has proposed the assignment of the conflicting masses to ϕ. His interpretation of conflicts is that they occur when the hypothesis space is not exhaustive [25]. In a different approach, Yager proposes the assignment of conflict masses to θ, and interprets it as the degree of overall ignorance [26]. Within the framework of Subjective logic, the combination operation is performed through the application of the Consensus operator.

Definition 7 *Let $\chi_x^A = (b_x^A, d_x^A, u_x^A)$ and $\chi_x^B = (b_x^B, d_x^B, u_x^B)$ be two opinions about a common fact x stated by two different information sources A and B, and let $\kappa = u_x^A + u_x^B - u_x^A u_x^B$. When $u_x^B \to 0$, and $u_x^A \to 0$, the relative dogmatism between the two opinions are defined using $\gamma = u_x^B / u_x^A$. Now $\chi_x^{A,B} = (b_x^{A,B}, d_x^{A,B}, u_x^{A,B})$ is a fair representative of both opinions, and the outcome of the Consensus operator $\oplus$ is as follows:*

when $\kappa \neq 0$	when $\kappa = 0$
$b_x^{A,B} = (b_x^A u_x^B + b_x^B u_x^A)/\kappa$	$b_x^{A,B} = \frac{\gamma b_x^A + b_x^B}{\gamma+1}$
$d_x^{A,B} = (d_x^A u_x^B + d_x^B u_x^A)/\kappa$	$d_x^{A,B} = \frac{\gamma d_x^A + d_x^B}{\gamma+1}$
$u_x^{A,B} = (u_x^A u_x^B)/\kappa$	$u_x^{A,B} = 0$

The Consensus operator has been shown to have a stable behavior under various conditions and even while merging conflicting dogmatic beliefs [27]. It satisfies two important algebraic properties i.e. commutativity ($A \oplus B = B \oplus A$), and associativity ($A \oplus [B \oplus C] = [A \oplus B] \oplus C$), which are of great significance while merging two peer information sources.

3. An Subjective Belief Game Model

In this section, we propose a Subjective belief game model that incorporates features from both BGM [2], and BNM [3]. In the proposed model, the significance of the formulae is specified through subjective belief annotation values. Each source of belief in this model is represented using a SBB. It is assumed in our model that each SBB is internally consistent, i.e., no two conflicting formulae can be found in a single belief base.

Definition 8 *A choice function under the integrity constraint μ is a function $g_\mu : \mathcal{SE} \rightarrow \mathcal{SE}$ such that:*

(ec1) $g_\mu(\mathcal{SP}) \sqsubseteq \mathcal{SP}$,

(ec2) *If* $\bigwedge \mathcal{SP} \wedge \mu \not\equiv \top$, $\exists B_i, B_j$ *such that* $B_i^* \wedge B_j^* \wedge \mu \not\equiv \top$, *then* $(B_i$ *or* $B_j) \in g_\mu(\mathcal{SP})$,

(ec3) *If* $\mathcal{SP} \equiv_s \mathcal{SP}'$ *then* $g_\mu(\mathcal{SP}) \equiv_s g_\mu(\mathcal{SP}')$.

Briefly, integrity constraints are semantic information about the relationship of the formula. For instance, ϕ_1 and ϕ_2 are not syntactically in conflict with each other, but if we define $\mu = \{\phi_1 \equiv \not\phi_2\}$, then ϕ_1 and ϕ_2 will be conflicting formula. In the above definition, condition $ec1$ is a direct generalization from $c1$ in BGM and ensures that the selected belief bases are a subset of the belief profile. The second condition states that the choice function does not select those SBBs which are not inconsistent with any other SBBs. Therefore, it minimally includes any of the two SBBs whose conjunction produces an inconsistent belief base with regards to the integrity constraint μ. $ec2$ is required to ensure that the proposed Subjective belief game model properly terminates. It can also be seen in $ec1$ that the choice function is dependent on the syntactical format of the SBBs, due to the need to perform consistency checking on the belief base. Finally, a choice function g is syntax-independant if it satisfies $ec3$.

In the following, we introduce the structure of the enhancement functions. These family of functions extend the weakening functions of BGM and BNM in such a way that the weakest belief bases are not just expected to concede, but can also provide justifications for their weak formula.

Definition 9 *An enhancement function is a function* $\blacktriangledown : \mathcal{K} \times \mathcal{SE} \times \mathcal{SE} \rightarrow \mathcal{K}$ *where for a SBB B, and two subjective belief profiles* $\mathcal{SP}, \mathcal{SP}'$, *if* $\mathcal{SP}' \sqsubseteq \mathcal{SP}$ *and* $B \in \mathcal{SP}'$ *then* $\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B)$ *satisfies:*

(ee1) $\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B) - \blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B) \subseteq B$,

(ee2) *If* $\exists B_i \in \mathcal{SP}$ *such that* $B^* \wedge B_i^* \wedge \mu \not\equiv \top$ *then* $\blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B)^* \wedge B_i^* \equiv \top$,

(ee3) *If* $B = \blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B)$, *then* ($\nexists B_i \in \mathcal{SP}$ *such that* $B^* \wedge B_i^* \wedge \mu \not\equiv \top$ *and* $\blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B) = \varnothing$).

where $\blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B)$ *denotes the justifications on* B.

The enhancement function can be extended to subjective belief profiles by allowing $\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(\mathcal{SP}) = \{\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B) : B \in \mathcal{SP}\}$.

The enhancement function promotes the contents of the information sources. Analogous to the weakening function in BGM, this function weakens the belief bases in order to remove inconsistencies, but additionally allows the information sources to add extra consistent information to the belief base to justify or extend their standpoint. For instance, when an agent's belief base is selected for concession, the agent may notice the reason why other agents are disagreeing with it's beliefs. To provide further justifications on its standpoint, the enhancement function allows the agent to add more information to its belief base under the condition that these new information do not create new conflicts with the already expressed information of the other agents. These newly added information that allow more room for justification for the weakest belief base, are called *justifications* and are denoted by $\blacktriangledown^{a}_{\mathcal{SP},\mathcal{SP}'}(B)$.

Here, $ee1$ ensures that the result of the enhancement function has been weakened except for the additional consistent information that have been added by the information source for standpoint justification. $ee2$ states that the additional information should be consistent with the information in the peer belief base with which the inconsistencies had initially occurred. Furthermore, the result of the enhancement function is only equivalent to the initial subjective belief base if a peer inconsistent belief base does not exist and no further justifications have been added to the belief base ($ee3$).

Definition 10 *A Subjective belief game model is a pair $\mathcal{N} = \langle g, \blacktriangledown \rangle$ where g is a choice function and $\blacktriangledown$ is an enhancement function. The final solution to a subjective belief profile $\mathcal{SP}$ for a Subjective belief game model $\mathcal{N} = \langle g, \blacktriangledown \rangle$, under the integrity constraint μ, denoted $\mathcal{N}_\mu(\mathcal{SP})$, is the subjective belief profile $\mathcal{SP}^{\mu}_{\mathcal{N}}$ defined as:*

(sp1) $\mathcal{SP}_0 = \mathcal{SP}$,

(sp2) $\mathcal{SP}_{i+1} = \blacktriangledown_{\mathcal{SP}_i, g(\mathcal{SP}_i)}(\mathcal{SP}_i)$,

(sp3) $\mathcal{SP}^{\mu}_{\mathcal{N}}$ *is the first* $\mathcal{SP}_i$ *that is consistent with* μ.

Let $\mathcal{SP} = \{B_1, ..., B_n\}$ be a subjective belief profile. The integration of the subjective belief bases in the Subjective belief game model is achieved through several rounds of negotiation. In each round, the belief bases which create inconsistencies are weakened and justified through the enhancement function in order to obtain a consistent subjective belief profile ($\mathcal{SP}^{\mu}_{\mathcal{N}}$)(social contraction step). Once a consistent $\mathcal{SP}^{\mu}_{\mathcal{N}}$ has been created, the SBBs in $\mathcal{SP}^{\mu}_{\mathcal{N}}$ are combined using the subjective consensus operator which creates a fair trade-off between the information of different sources (combination step) as a result of which an ultimately consistent subjective belief base is obtained.

4. Choice and Enhancement Functions

4.1. Choice Function

The structure of the choice function is based on a *fitness measure* (analogous to belief entropy functions, smaller values are more desirable) that defines the quality of the subjective belief bases (SBBs) i.e. those belief bases which are less eminent than the others

are selected as suitable candidates in the choice function. We define the fitness measure based on *intrinsic* and *extrinsic* properties of the belief bases. The intrinsic properties of a belief base address the internal features inferred from the subjective structure of the belief base, while the extrinsic properties relate to the behavior of the belief base with regards to the $\mathcal{SP}$ as a whole. Intrinsic properties of subjective belief bases are apparent from the belief base's individualistic attributes without attention to the content of other peer belief bases. Extrinsic properties of a subjective belief base are defined based on the context of the belief profile where a given subjective belief base is positioned; therefore, belief bases in the same belief profile have mutual effects on each others' contingent properties.

Definition 11 *The fitness measure of a subjective belief base B under integrity constraints μ is a function $fm : \mathcal{K} \times \mathcal{SE} \rightarrow [0, \infty)$. Given two SBBs B and $B' \in \mathcal{SP}$, B is considered more competent than B' if and only if $fm^{\mu}_{\mathcal{SP}}(B) < fm^{\mu}_{\mathcal{SP}}(B')$.*

Now we introduce the building blocks of the fitness measure that can be employed for evaluating the quality of each individual belief base.

4.1.1. Intrinsic Properties of Belief Bases in $\mathcal{SP}$ The intrinsic properties of each belief base is closely related with the subjective annotation values of its constituting formulae. These properties represent the strength and validity of the stated formulae, and the fortitude of beliefs of the information source that is providing the belief base. Here, two major intrinsic features are considered, namely: *Ambiguity*, and *Indecisiveness*.

Before introducing these properties, an aggregation function needs to be introduced [4, 28]. The aggregation function is required in order to accumulate the overall value of each property for a subjective belief base.

Definition 12 *An aggregation function is a total function f that assigns a non-negative integer to every finite tuple of integers and satisfies non-decreasingness (if $x \leq y$ then $f(x_1, ..., x_n) \leq f(x_1, ..., y, ..., x_n)$), minimality ($f(x_1, ..., x_n) = 0$, iff $x_1 = ... = x_n = 0$), and identity ($\forall x \in \mathbb{N},\ f(x, ..., x) = x$).*

Now, we can define the intrinsic properties. First, the ambiguity property provides the basis to calculate the degree of confusion in the subjective belief base about the exact fraction of belief that should be assigned to each formulae.

Definition 13 *Let $\phi = (\phi^*, \chi_\phi)$ be a subjective formula in a subjective belief base, and ϵ be a normalization factor. Ambiguity is a function $\zeta : \mathcal{F} \rightarrow [0, 1]$, defined as:*

$$\zeta(\phi) = -\left(\frac{1}{e^{1-u_\phi}} - 1\right)/\epsilon. \tag{7}$$

Ambiguity is similar to the belief entropy metric in the generalized entropy criterion [29]. The normalization factor, ϵ, can also be derived from the definition of the Ambiguity function. The lower limit of the function is reached when $b_\phi + d_\phi = 1$. In such a case, regardless of the value of ϵ, the function produces zero. The function outputs its upper limit when $b_\phi + d_\phi = 0$. In this case, the final value needs to be normalized into one; therefore

$\epsilon = 1 - \frac{1}{e}$. The normalization factors of the rest of the functions can also be simply calculated accordingly. Ambiguity provides the basis to calculate the degree of confusion of the belief base with regards to the exact fraction of belief that should be assigned to a given formula. An ambiguous formula shows that the belief base (agent) is still unsure about the accuracy and correctness of the formula.

Further, the indecisiveness property is a measure of the ability of the information source to firmly state a given formula. The further away the degree of belief and disbelief of a given formula are, the stronger and more decisive the formula is.

Definition 14 *Let $\phi = (\phi^*, \chi_\phi)$ be a subjective formula in a subjective belief base, and ϵ' be a normalization factor. Indecisiveness is a function $\vartheta : \mathcal{F} \to [0, 1]$, defined as:*

$$\vartheta(\phi) = \left(\frac{1}{e^{|b_\phi - d_\phi|}} - \frac{1}{e} \right) / \epsilon'. \tag{8}$$

Indecisiveness is a measure of the ability of the belief base to firmly assert a given formula. The further away the degrees of belief and disbelief for a given formula are, the stronger and more decisive the formula is. A completely decisive statement is one that either possesses $b_\chi = 1$ or $d_\chi = 1$, which means that the belief base either completely agrees with this statement or fully disagrees with it.

The intrinsic properties of the formulae can be extended to a subjective belief base $B = \{\phi_1, ..., \phi_n\}$, by allowing $\zeta(B) = f\left(\zeta(\phi_1), ..., \zeta(\phi_n)\right)$, and $\vartheta(B) = f\left(\vartheta(\phi_1), ..., \vartheta(\phi_n)\right)$, where f is an aggregation function (e.g. sum). These properties are called intrinsic since they can be computed within the context of a single belief base. Therefore, the setting of the other belief bases in the subjective belief profile is not important in the behavior of the intrinsic properties.

Proposition 3 *Let B be a Subjective belief base, $\phi \in B$ be a formula in B where $\chi = (b_\chi, d_\chi, u_\chi)$, and $\zeta(\phi)$ and $\vartheta(\phi)$ be the degrees of ambiguity and indecisiveness of ϕ, respectively. The following two relations always hold between these two properties:*

- *if $\vartheta(\phi) \to 1$ then $\zeta(\phi) \to 1$*
- *if $\zeta(\phi) \to 0$ then $\vartheta(\phi) \to 0$*

This proposition asserts that a highly indecisive statement is also highly ambiguous, whereas a statement with a low degree of ambiguity is always highly decisive. The most desirable state for a statement of any given belief base is unambiguity and decisiveness, which is achieved when all the possible belief mass is either assigned to b_χ or d_χ. The difference between indecisiveness and ambiguity is quite subtle in that ambiguity measures the ignorance of the belief base towards the possibilities of the formula, while indecisiveness evaluates the inability of the belief base to select the best of the possibilities in a tradeoff situation.

4.1.2. Extrinsic Properties of Belief Bases in $\mathcal{SP}$ These properties show the behavior of the belief bases and their sources in relationship with the others. They are also somewhat

dependant on the past behavior of the information sources. These extrinsic properties are only meaningful in comparison and within the context of a specific $\mathcal{SP}$, and not in vacuum. We introduce two extrinsic properties, namely: *Disagreement* and *Trustworthiness*.

The Disagreement property defines the degree of discrepancy between the belief of a single belief base towards one formula in comparison with another belief base's opinion on some formula. Disagreement can be taken as the extent of divergence of the two belief bases.

Definition 15 *Let $\phi = (\phi^*, \chi_\phi)$, and $\phi' = (\phi'^*, \chi_{\phi'})$ be two subjective formula, and ϵ'' be a normalization factor. Disagreement is a function $\delta : \mathcal{F} \times \mathcal{F} \rightarrow [0, 1]$, defined as:*

$$\delta(\phi, \phi') = \frac{(b_\phi \times d_{\phi'} + b_{\phi'} \times d_\phi)}{\epsilon''}. \tag{9}$$

Analogous to the intrinsic properties, disagreement can be extended to two subjective belief bases $B = \{\phi_1, ..., \phi_n\}$ and $B' = \{\phi'_1, ..., \phi'_n\}$ by letting $\delta(B, B') = f\left(\delta(\phi_1, \phi'_1), ..., \delta(\phi_n, \phi'_n)\right)$, where f is an aggregation function (e.g. $\sum$, or max) and ϕ'_i is the image of ϕ_i. Now based on this extension, we can define the overall degree of disagreement of a single belief base i.e. $\delta(B_j) = f\left(\delta(B_j, B_1), ..., \delta(B_j, B_{(\#\mathcal{SP})})\right)$. The disagreement property of a belief base shows the degree of divergence of a given belief base from the overall opinion of the other belief bases in the same belief profile.

The second extrinsic property of a belief base is related to overall behavior of the information source throughout a period of observation called *Trustworthiness*. In different contexts researchers have employed terms such as trustworthiness [30], reputation [31], confidence [32], and others to refer to the same issue. We formalize the trustworthiness of the source of a belief base as a subjective opinion that represents the degree of belief in the fact that the information source is going to reveal a correct and consistent belief base.

Definition 16 *Let $\chi_B = (b_B, d_B, u_B)$ be a subjective opinion about the trustworthiness of a given belief base B. Trustworthiness is a function $\Re : \mathcal{K} \rightarrow [0, 1]$ such that:*

$$\Re(B) = \frac{b_B + \tau}{d_B}. \tag{10}$$

where τ is a small correction value. It is used to prevent divide by zero when $\Re^{-1}$ is employed, since $\Re^{-1}$ is only employed in the following.

4.1.3. Fitness Measure-based Choice Function The formalization of the intrinsic and extrinsic properties of belief bases provides the basis for the characterization of an ordering mechanism based on the following fitness measure.

Definition 17 *[Extends Definition 11] The fitness measure of a subjective belief base B under integrity constraints μ is a function $fm : \mathcal{K} \times \mathcal{SE} \rightarrow [0, \infty)$ such that:*

$$fm^{\mu}_{\mathcal{SP}}(B) = \left|\overrightarrow{\left(\zeta(B), \vartheta(B), \delta(B), \Re^{-1}(B)\right)}\right|. \tag{11}$$

This fitness measure is one of the possible ways to create a measure based on the intrinsic and extrinsic properties of subjective belief bases. It is one of the viable choices since the intrinsic and extrinsic properties are mutually independent; therefore, creating a vector model of these properties can represent the relative fitness of the corresponding belief bases. For other fitness measures defined for subjective belief bases see [33].

Definition 18 *Let $\mathcal{SP} = \{B_1, ..., B_n\}$ be a subjective belief profile. A fitness measure-based ordering, $\prec_{fm}$, of two SBBs B_i, and $B_j \in \mathcal{SP}$ is defined as:*

$B_i \prec_{fm} B_j$ *iff* $(B_i^* \equiv \top$ *and* $B_j^* \not\equiv \top)$
or $(B_i^* \not\equiv \top$ *and* $B_j^* \not\equiv \top$ *but* $fm^{\mu}_{\mathcal{SP}}(B_i) < fm^{\mu}_{\mathcal{SP}}(B_j))$.

Simply stated, a consistent belief base is more fit than an inconsistent belief base, and for two inconsistent belief bases, the one with a better fitness measure is more fit.

Theorem 1 *Let $B_1 \equiv_s B_1'$, $B_2 \equiv_s B_2'$, $\Re_{B_1} = \Re_{B_1'}$, and $\Re_{B_2} = \Re_{B_2'}$. If $B_1 \prec_{fm} B_2$ then $B_1' \prec_{fm} B_2'$.*

Proof: The ordering of subjective belief bases is directly dependent on the fitness measure which is itself reliant on intrinsic and extrinsic properties of the belief bases. So, if the properties of two belief bases are equivalent, it can be concluded that $fm^{\mu}_{\mathcal{SP}}(B) = fm^{\mu}_{\mathcal{SP}}(B')$. Based on Definitions 13 and 14, the intrinsic properties of a belief base are focused on the subjective opinions of the formulae and the internal structure of each individual belief base. Since $B_1 \equiv_s B_1'$; therefore, we can conclude that $\zeta(B) = \zeta(B')$ and $\vartheta(B) = \vartheta(B')$. Furthermore, the disagreement property considers the subjective opinions of each formula in B and the subjective opinions of its image in B'. Based on Definition 5 and its consequence, $\delta(B) = \delta(B')$. Now, given that $fm^{\mu}_{\mathcal{SP}}(B) = fm^{\mu}_{\mathcal{SP}}(B')$; therefore, if $B_1 \prec_{fm} B_2$ then $B_1' \prec_{fm} B_2'$. ■

Theorem 1 shows that fitness measure-based ordering is syntax-independent.

It can be inferred that B_j is more convergent than B_i within the context of $\mathcal{SP}$ when $B_i \prec_{fm} B_j$. We can now finally define the choice function based on the fitness measure-based ordering.

Definition 19 *Let $\mathcal{SP} = \{B_1, ..., B_n\}$ be a subjective belief profile. A fitness measure-based choice function under the integrity constraint μ is defined as follows:*
$B_i \in g^{fm}_{\mu}$ *iff* $\forall B_j \in \mathcal{SP}$ *such that* $B_i^* \wedge B_j^* \wedge \mu \not\equiv \top$, *and* $B_j \prec_{fm} B_i$.

According to this choice function, from the set of mutually inconsistent belief bases, the ones that are divergent within the context of $\mathcal{SP}$ are selected. The fitness measure-based choice function g^{fm}_{μ} should be checked to see wether it satisfies $ec1 - ec3$. Based on Definition 19, $ec1$ is satisfied. $ec2$ is also satisfied from the definition of the choice function and the fitness measure-based ordering. $ec3$ is also satisfied according to the following corollary.

Corollary 1 *g^{fm}_{μ} satisfies condition $ec3$ of the Subjective belief game model choice function, that is, it is syntax-independent.*

The corollary is a straight derivation from Theorem 1. It can be seen that g_{μ}^{fm} satisfies $ec1 - ec3$ and is therefore a valid choice function.

4.2. Enhancement Functions

The subjective opinions attached to each belief base formulae provide a suitable basis for developing attractive enhancement functions. Here, we first introduce an enhancement function that deletes the weakest inconsistent formula, and also provides the possibility for the information source to offer justifications in the form of additional formulae. Second, an enhancement function is developed, which does not require any deletions. It only manipulates the subjective opinions of the belief bases.

Definition 20 *Let $\mathcal{SP}$ be a subjective belief profile and $(\phi, \chi) \in \cup(\mathcal{SP})$ be a subjective belief formula forming a singleton belief base $C_{\phi} = \{(\phi, \chi)\}$. (ϕ, χ) is considered to be one of the weakest inconsistent formula of $\mathcal{SP}$ if and only if:*

$(wif1)$ (ϕ, χ) is in conflict in $\cup(\mathcal{SP})$,

$(wif2)$ $\forall(\phi_i, \chi_i) \in \cup(\mathcal{SP})$ such that $\phi \wedge \phi_i \wedge \mu \not\equiv \top$ then $C_{\phi_i} \prec_{fm} C_{\phi}$.

Definition 21 *Let $\mathcal{SP} = \{B_1, .., B_n\}$ be a subjective belief profile, $\mathcal{SP}'$ be a subset of $\mathcal{SP}$, $B \in \mathcal{SP}'$ and $C = \{(\phi, \chi) \in B | (\phi, \chi)$ be one of the weakest inconsistent formula in $\cup(\mathcal{SP})\}$. The weakest inconsistent-based (WI) enhancement function for B is defined as:*

$\blacktriangledown^{WI}_{\mathcal{SP},\mathcal{SP}'} = B \setminus C$ *, if* $C \neq \varnothing$.

For each of the selected belief bases through the choice function, the WI enhancement function deletes those formulae which are among the weakest inconsistent formulae in $\cup\mathcal{SP}$, else the belief base remains intact. We now check that $\blacktriangledown^{WI}$ is a enhancement function. It is clear that $ee1$ is satisfied. $ee2$ is also satisfied since the information source is bound to respect this condition if any justifications are provided. Based on the definition of the weakest inconsistent formula (Definition 20), $ee3$ is satisfied because no weak subjective formula can be found for elimination under such a condition, and therefore, $B = \blacktriangledown^{WI}(B)$.

Let us now define a more flexible enhancement function that increases negotiability between the information sources.

Definition 22 *Let $\Re_B = (b_{\Re_B}, d_{\Re_B}, u_{\Re_B})$ be the general opinion about the trustworthiness of information source B, $\chi_p^B = (b_{\chi_p}^B, d_{\chi_p}^B, u_{\chi_p}^B)$ be B's opinion about some proposition p. Discounting of χ_p^B by $\Re_B$, denoted $\chi_p^B \otimes \Re_B$, is defined as follows:*

$$b_p^{\otimes} = b_{\Re_B} \times b_{\chi_p}^B \tag{12}$$

$$d_p^{\otimes} = b_{\Re_B} \times d_{\chi_p}^B \tag{13}$$

$$u_p^{\otimes} = d_{\Re_B} + u_{\Re_B} + b_{\Re_B} u_{\chi_p}^B \tag{14}$$

As an example, suppose that an agent provides (ϕ, χ) as a piece of information, where $\chi = (0, 1, 0)$. This means that the agent does not believe in the information contained in ϕ. Now, further suppose that our perception of the trustworthiness of the agent is represented by $(0, 0, 1)$ that conveys that we are unaware of the credibility of the agent. Given the information provided by the agent, we are able to discount it with the trustworthiness of the agent, and see how useful the provided information is. So, with the above definition, the discounted information will result in $(b_p^{\otimes}, d_p^{\otimes}, u_p^{\otimes}) = (0, 0, 1)$. This means that since we really do not have any idea how trustworthy the agent is, we cannot make any judgements about the utility of the information that it provides. However, if we had complete trust in the agent $(1, 0, 0)$, the result of the discount would have been $(0, 1, 0)$, which is equivalent to the information provided by the agent.

Definition 23 *Let $\mathcal{SP} = \{B_1, .., B_n\}$ be a subjective belief profile, χ represent all possible subjective opinions, $\Re_j$ be the trustworthiness of the information source j, and $\mathcal{SP}'$ be a subset of $\mathcal{SP}$. Let $B \in \mathcal{SP}'$, $(\phi_i, \chi_i) \in B$ and $C = \{(\phi', \chi'), ..., (\phi^{(n)}, \chi^{(n)})\}$ be the set of all images of (ϕ_i, χ_i) in other belief bases in $\mathcal{SP}$. The conformance subjective belief for (ϕ_i, χ_i) is a function $\wp : \mathcal{F}_B \to \chi$ defined as:*

$$\wp_{\phi_i}^{B} = \oplus_{(\phi_j, \chi_j) \in C} \left[\otimes (\Re_j, \chi_j) \right]. \tag{15}$$

The conformance subjective belief for $(\phi_i, \chi_i) \in B$, denoted $\wp_{\phi_i}^{B}$, represents the general consensus about a given formulae. It is produced by the application of the consensus operator on the set of images of ϕ_i in peer belief bases discounted by the trustworthiness of the information sources. For instance, in a case where we have two equally reliable information sources, one providing us with $(\phi_1, \chi_1), \chi_1 = (0, 1, 0)$ and the other $(\phi_1, \chi_2), \chi_2 = (1, 0, 0)$, we can use the consensus operator to compile the given information into one piece of consolidated information. In this case, this would be $\chi_1 \oplus \chi_2 = (0, 0, 1)$.

Definition 24 *Let $\mathcal{SP} = \{B_1, .., B_n\}$ be a subjective belief profile, $\mathcal{SP}'$ be a subset of $\mathcal{SP}$, $B \in \mathcal{SP}'$ and $C = \{(\phi, \chi) \in B | (\phi, \chi)$ be one of the weakest inconsistent formula in $\cup(\mathcal{SP})\}$. The conformance belief-based (CB) enhancement function for B is defined as:*

$$\blacktriangledown_{\mathcal{SP}, \mathcal{SP}'}^{CB} = \{(\phi_1, \chi_1'), ..., (\phi_n, \chi_n')\}$$

where χ_i' is a half-open interval between the original subjective belief χ_i and $\wp_{\phi_i}^{B}$.

In the CB-based enhancement function, each belief base in $\mathcal{SP}'$ is required to update the subjective belief values of the formulae in its belief base. The effect of the update on the subjective beliefs should remain in a half-open interval recommended by the enhancement function for each individual formula. This way the information sources have more freedom in updating and improving the contents of their belief base. If all belief bases conform to the recommended subjective opinion $\wp_{\phi_i}^{B}$ consensus is reached and inconsistencies are removed. Furthermore, according to $ee2$, the information sources are able to add justifications as long as new inconsistencies do not arise. Similar to $\blacktriangledown^{WI}$, it can be shown that $\blacktriangledown^{CB}$ satisfies $ee1 - ee3$, and is therefore, a valid enhancement function.

5. Framework Instantiation and Properties

5.1. Instantiation

Different arrangements of the choice functions and the enhancement functions will produce dissimilar Subjective belief game models, and belief base merging strategies that can produce disparate final outcomes. In the models below, we show and discuss how the different choice and enhancement functions can be integrated and what results they will each produce.

- $\langle g^{fm,max}, \blacktriangledown^{WI}\rangle$: The choice function uses the maximum aggregation function. In each round of negotiation, those subjective belief bases that have the highest quantities of intrinsic and extrinsic properties assigned to their formulae are selected and the weakest inconsistent formulae are removed from $\cup\mathcal{SP}$.
- $\langle g^{fm,\Sigma}, \blacktriangledown^{WI}\rangle$: In this setting, those subjective belief bases that have the highest value of intrinsic and extrinsic properties assigned to their formulae are selected by the choice function that benefits from a sum aggregation function. The weakest inconsistent formulae are then removed.
- $\langle g^{fm,max}, \blacktriangledown^{CB}\rangle$: In this case, the selected belief bases (i.e. those belief bases that consist of formulae that have the highest quantity of intrinsic and extrinsic properties) through the choice function are required to update their formulae belief based on a recommended belief interval.
- $\langle g^{fm,\Sigma}, \blacktriangledown^{CB}\rangle$: In each round of negotiation, the choice function selects those subjective belief bases that have formulae which possess the highest value of intrinsic and extrinsic properties. Here, the enhancement process is similar to $\langle g^{fm,max}, \blacktriangledown^{CB}\rangle$.

Example 1 explains each of these cases in detail. By comparing these four possible settings, it can be seen that those compositions that benefit from conformance belief-based enhancement function have the lowest *information loss* with regards to each individual belief base. This is because in the enhancement process, conformance belief-based enhancement function does not directly delete any formulae, and only requires the information sources to amend their subjective opinions for each formula. Although this is a positive feature in many cases and provides the possibility for formal negotiation between the information sources by belief adjustment, it is not so desirable in situations where fast consensus achievement and simple computation is sought. For such conditions, the employment of the weakest inconsistent-based enhancement function yields faster results; however, the accuracy of its results is arguable.

5.2. Logical Properties

Merging operators derived from the Subjective belief game model possess several interesting logical properties. Here, we describe these properties and show how they are satisfied by the variants of the Subjective belief game model.

Definition 25 *Let Δ be a subjective merging operator, and $\mathcal{SP}, \mathcal{SP}_1$, and $\mathcal{SP}_2$ be subjective belief profiles, and μ be integrity constraints. A merging operator derived from the Subjective belief game model should satisfy the following logical properties:*

(lp1) $\Delta_\mu(\mathcal{SP}) \wedge \mu$ *is consistent,*

(lp2) Let $\mathcal{SP} = \{B_1, ..., B_n\}$. *If* $B_1 \cup ... \cup B_n$ *is consistent, then* $[\Delta_\mu(\mathcal{SP})]^* \equiv [B_1 \cup ... \cup B_n]^*$ *and* $\forall \phi$, *if* $\exists i$ *such that* $(\phi, \chi) \in B_i$ *then there exists* χ' *such that* $(b_\chi \leq b_{\chi'}$ *or* $d_\chi \leq d_{\chi'})$ *and* $(\phi, \chi') \in \Delta_\mu(\mathcal{SP})$,

(lp3) if $\mathcal{SP}_1 \equiv_s \mathcal{SP}_2$ *then* $\Delta_\mu(\mathcal{SP}_1) \equiv_s \Delta_\mu(\mathcal{SP}_2)$,

(lp4) Let $\mathcal{SP} = \{B_1, ..., B_n\}$ *and* $\bigcup \mathcal{SP} = B_1 \cup ... \cup B_n$. *Let* $(\phi, \chi) \in \bigcup(\mathcal{SP})$. *If* (ϕ, χ) *is not in conflict in* $\bigcup \mathcal{SP}$, *then* $(\phi, \chi') \in \Delta_\mu(SP)$ *where* $(b_\chi \leq b_{\chi'}$ *or* $d_\chi \leq d_{\chi'})$.

The first property $(lp1)$ states that the final subjective belief base should be consistent with regards to the integrity constraints. $lp2$ asserts that when the original information bases are consistent and there is no conflict between them, the merging operator should reinstate all the original information along with a reinforced effect on their subjective opinions. $lp3$ is the principle of irrelevance of syntax. Finally, $lp4$ emphasizes that all non-conflicting information of the knowledge bases should be present after merge.

Theorem 2 *Let $\oplus$ be the consensus operator, and let $\mathcal{N} = \langle g^{d,f}, \blacktriangledown \rangle$, where $d = fm$, $f = max$ or $f = \sum$, and $\blacktriangledown = \blacktriangledown^{WI}$. Then the operator $\Delta_{\mathcal{N},\oplus}$ satisfies properties $(lp1) - (lp4)$.*

Proof: —

$(lp1)$ The first property is satisfied by definition (see Definition 10), since the outcome of the Subjective belief game process is $\bigwedge \mathcal{SP}^\mu_\mathcal{N} \wedge \mu$.

$(lp2)$ Based on Definition 10, if $\mathcal{SP}$ is consistent, then $\mathcal{SP}_\mathcal{N} = \mathcal{SP}$; therefore, $\Delta_{\mathcal{N},\oplus}(\mathcal{SP})$ is equal to the effect of the consensus operator on the subjective opinions of the conjunction of all belief bases. Furthermore, the consensus operator has a reinforcement effect on consistent subjective opinions. Hence, we have if $\exists i$ such that $(\phi, \chi) \in B_i$ then there exists χ' such that $(b_\chi \leq b_{\chi'}$ *or* $d_\chi \leq d_{\chi'})$ and $(\phi, \chi') \in \Delta_\mu(\mathcal{SP})$, and $lp2$ is satisfied.

$(lp3)$ Let $\mathcal{SP}_1 \equiv_s \mathcal{SP}_2$ where $\mathcal{SP}_i = \{B^i_1, ..., B^i_n\}$. Suppose $\forall i$, $B^1_i \equiv B^2_i$. According to Corollary 1, the choice function g^{fm}_μ is syntax-independent, and also $\blacktriangledown^{WI}(B^j_i) \equiv_s \blacktriangledown^{WI}(B^k_i)$. The consensus operator is also clearly syntax-independent. Therefore, $\Delta_{\mathcal{N},\oplus}$ satisfies $lp3$.

$(lp4)$ Since the choice function selects the weakest inconsistent formulae, and consequently they are removed from the belief bases; hence, the non-conflicting formulae will certainly exist in the final outcome. The reinforcement effect is also satisfied by the consensus operator. It can be seen that $\Delta_{\mathcal{N},\oplus}$ satisfies $lp4$.

■

Theorem 3 *Let $\oplus$ be the consensus operator, and let $\mathcal{N} = \langle g^{d,f}, \blacktriangledown \rangle$, where $d = fm$, $f = max$ or $f = \sum$, and $\blacktriangledown = \blacktriangledown^{CB}$. Then the operator $\Delta_{\mathcal{N},\oplus}$ satisfies properties $(lp1) - (lp4)$.*

Proof: The proof of Theorem 3 is similar to Theorem 2. ■

6. Framework Application and Examples

Belief game models have mainly been applied to multiagent systems. In this section, we provide insight into the effectiveness of the Subjective belief game model for the definition of a formal collaborative conceptual modeling process. The Subjective belief game model can be useful for formalizing a negotiation process between the human participants of a design process in cases where discrepancies and conflicts arise.

6.1. Collaborative Conceptual Modeling

Collaborative conceptual modeling is the process through which multiple experts interact in order to develop a unique model (usually based on a given a meta-modeling formalism) for a given domain of discourse. In most cases in this process, each of the experts focuses on a special aspect of the problem domain and devises a separate model. The individually developed models should finally be integrated to form a unique model of the problem at hand. One of the greatest challenges in this process is the proper integration of the individual models. This is because many of the individual models have overlapping areas of concern. The specifications provided for the overlapping segments of each model may be inconsistent, and therefore, require meticulous methods for addressing their integration.

According to the definitions in [34], the inconsistencies between software models can be formally defined based on the overlap relationships between the interpretations assigned to domain models by the experts.

Definition 26 *The interpretation of a domain model DM represented through a set of interrelated model elements E is a pair (U,I) where U is the domain of interpretation of the model, and I is a total morphism that maps $\forall e_i \in E$ onto a relation R called the extension of e.*

An interpretation of a domain model relates each model element onto its corresponding concept in the domain of discourse; therefore, an interpretation conveys how an expert views the developed model with regards to the given domain.

Definition 27 *Let e_i and e_j be two model elements of domain models DM_i, and DM_j, and let $T_{i,A} = (U_{i,A}, I_{i,A})$, and $T_{i,B} = (U_{i,B}, I_{i,B})$ be interpretations of DM_i and DM_j assigned by experts A and B, respectively. The overlap relationships can be defined as follows:*

- *(no overlap) $I_{i,A}(e_i) \neq \varnothing,\ I_{j,B}(e_j) \neq \varnothing$, and $I_{i,A}(e_i) \cap I_{j,B}(e_j) = \varnothing$,*
- *(totally overlap) $I_{i,A}(e_i) \neq \varnothing,\ I_{j,B}(e_j) \neq \varnothing$, and $I_{i,A}(e_i) = I_{j,B}(e_j)$,*
- *(inclusively overlap) $I_{i,A}(e_i) \neq \varnothing,\ I_{j,B}(e_j) \neq \varnothing$, and $I_{i,A}(e_i) \subseteq I_{j,B}(e_j)$,*

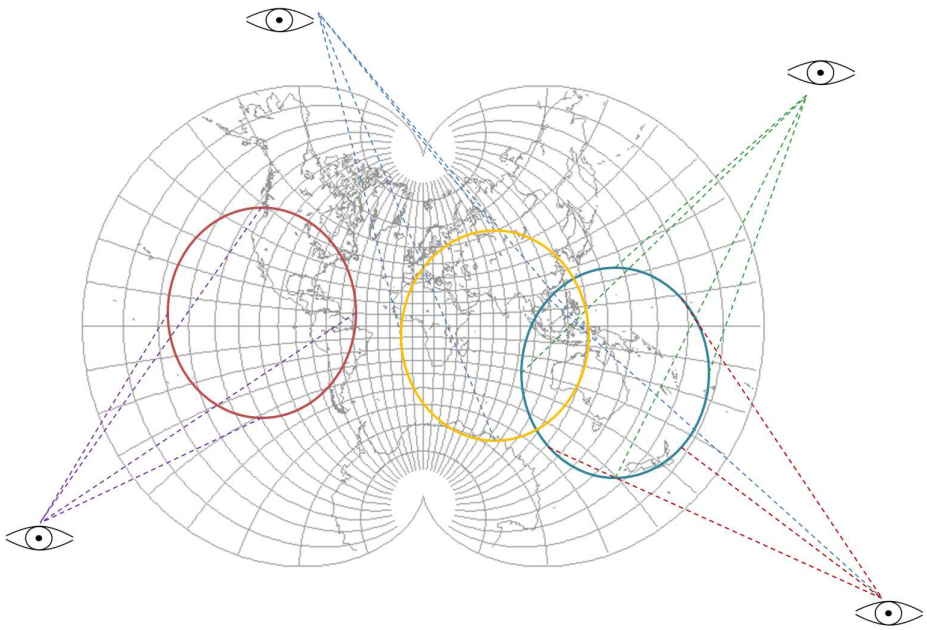

Figure 2. Different relationship types between the domain models of various experts.

- *(partially overlap)* $I_{i,A}(e_i) \neq \varnothing$, $I_{j,B}(e_j) \neq \varnothing$, *and* $I_{i,A}(e_i) \cap I_{j,B}(e_j) \neq \varnothing$, $I_{i,A}(e_i) - I_{j,B}(e_j) \neq \varnothing, I_{j,B}(e_j) - I_{i,A}(e_i) \neq \varnothing$.

Informally, the above definition can be expressed in terms of a venn diagram where each of the domain models reveals only a small fraction of the large target domain. Therefore, the domain models can have different relationships with each other, e.g., no overlap, total overlap, and partial overlap. Figure 2 shows how different perspectives over a given domain, the world in this figure, can result in different domain models based on the perception of the viewer. Based on these definitions, an inconsistency between domain models can be defined in terms of their overlaps.

Definition 28 *Let* $DM = \{DM_1, ..., DM_n\}$ *be a set of domain models, and* $O = \{(DM_i, DM_j) | DM_i \in DM,\ DM_j \in DM\}$ *be the overlap relationship between them. Furthermore, let* CR *be a set of consistency rules instructed by the meta-modeling language and domain restrictions.* DM *is said to be inconsistent if* CR *is not satisfied under merge strategy* Δ.

It is now possible to show that a collaborative conceptual modeling process can be supported by a Subjective belief game model. The domain models in a collaborative modeling process represent the subjective belief bases, and the experts symbolize the information sources in a Subjective belief game model.

Definition 29 *A collaborative modeling process is an extension to the Subjective belief game model,* $\mathcal{M} = \langle g, \blacktriangledown \rangle$, *where* g *is a choice function and* $\blacktriangledown$ *is an enhancement function.*

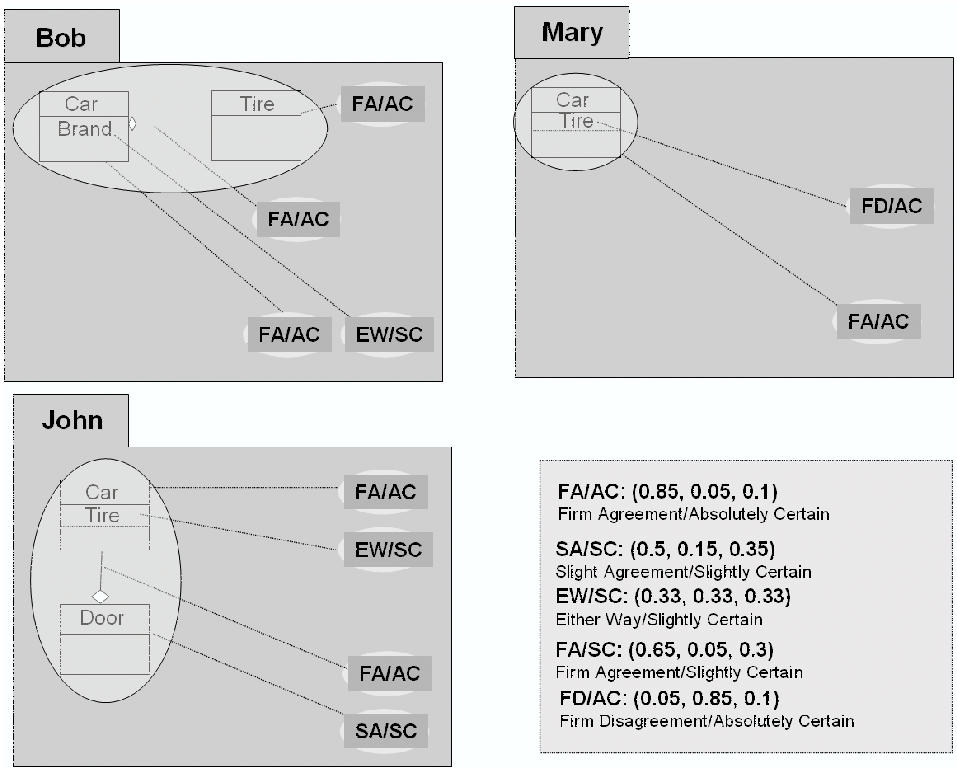

Figure 3. A small portion of the three models developed by three different experts.

The final unique model developed from a set of domain models $\mathcal{DM}$ developed by various experts under a set of consistency rules CR, denoted $\mathcal{M}_{\mathcal{CR}}(\mathcal{DM})$, is the domain model $\mathcal{DM}_{\mathcal{M}}^{\mathcal{CR}}$ defined as:

(cm1) $\mathcal{DM}_0 = \mathcal{DM}$,

(cm2) $\mathcal{DM}_{i+1} = \blacktriangledown_{\mathcal{DM}_i, g(\mathcal{DM}_i)}(\mathcal{DM}_i)$,

(cm3) $\mathcal{DM}_{\mathcal{M}}^{\mathcal{CR}}$ *is the first* $\mathcal{DM}_i$ *that is consistent with* CR.

Definition 29 shows that a collaborative conceptual modeling process can be conducted within the framework of the proposed Subjective belief game model. This is because we have been able to show that each of the domain models developed by the experts can be viewed as a Subjective belief base, and therefore, the collection of all of the domain models forms a Subjective belief profile. Now, since in Definitions 26 and 27 and also Figure 2, we have shown that domain models can have overlaps and hence may contain inconsistencies, the process of developing a unified representative domain model from all the individual domain models could be performed using the Subjective belief game model introduced in this paper.

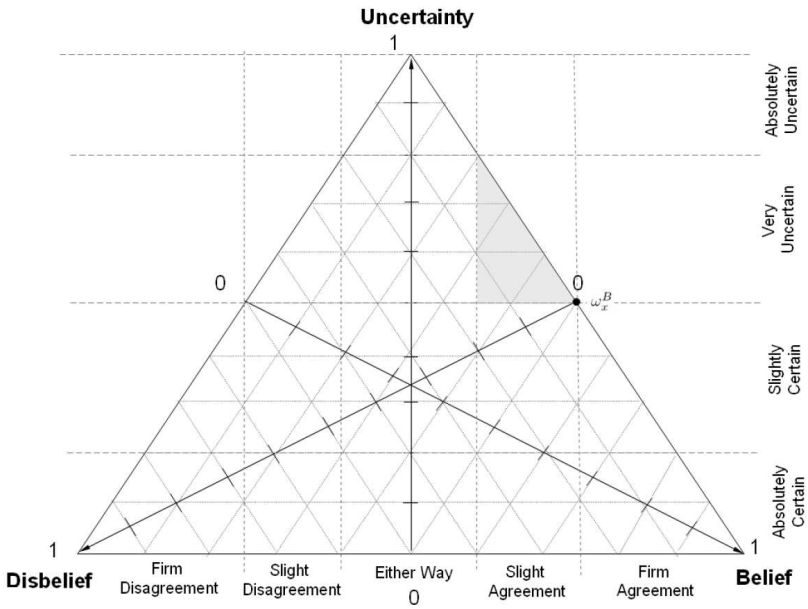

Figure 4. Developing easier to understand Subjective opinions from the opinion space.

6.2. *Illustrative Example*

In this section, we develop an example to show the behavior of two Subjective belief game model based merge methods. We show and compare the outcome of merge according to $\langle g^{fm,max}, \blacktriangledown^{WI}\rangle$ and $\langle g^{fm,max}, \blacktriangledown^{CB}\rangle$.

Example 1 *Three experts: Bob, John, and Mary are collaboratively designing the exterior and interior of a Car using the Unified Modeling Language [35]. Each of them designs a model from his/her own perspective and according to his/her area of expertise. We focus on a small segment of the models highlighted in Figure 3. The models are summarized as subjective propositions in a subjective belief profile* $\mathcal{SP} = \{\mathcal{M}_{Bob}, \mathcal{M}_{John}, \mathcal{M}_{Mary}\}$, *where:*

$\mathcal{M}_{Bob} = \{(p, [FA/AC]), (q, [EW/SC]), (r, [FA/AC]),$
$(t, [FA/AC]^c), (s, [FA/AC])\},$
$\mathcal{M}_{John} = \{(p, [FA/AC]), (t, [FA/AC]), (u, [SA/SC]),$
$(v, [FA/AC])\},$
$\mathcal{M}_{Mary} = \{(p, [FA/AC]), (t, [FD/AC])\},$

p*: Car is a class,* q*: Brand is an attribute of Car,* r*: Tire is a class,* s*: A Car can have multiple Tires,* t*: Tire is an attribute of Car,* u*: Door is a class,* v*: A Car can have multiple Doors, and* $(t, [FA/AC]^c)$ *means that* t *is believed to be true with the complement of*

$[FA/AC]$.
In order to show, how the Subjective values in the squared brackets have been developed from a Subjective opinion, Figure 4 has been shown. As it can be seen in the figure, the uncertainty axis and the probability axis have been divided into four and five sections, respectively. Each of these divisions represent a linguistic term. For example, the partitions on the uncertainty axis have developed the absolutely uncertain, very uncertain, slightly certain, and absolutely certain terms that can be used by the experts. On the other hand, the divisions on the probability axis has created five linguistic terms, namely firm disagreement, slight disagreement, either way, slight agreement, and firm agreement. If we consider the spaces that are created by the intersection of these two divisions, we will have various sub-spaces within the triangle that roughly represent certain linguistic expressions. For example, the gray area in the figure represents the kind of opinions by the experts where the expert is rather ignorant of the correctness of the proposition but thinks that the proposition is more likely to be correct; therefore, an opinion like $\omega_x^D = (0:3; 0:05; 0:65)$ *is located in the gray area and is interpreted as 'expert C is Very Uncertain about x; however, at the same time, prefers to Slightly Agree with it'. In this way the corresponding Subjective values such as [FA/AC], [SA/SC], and others can be developed.*
Without loss of accuracy, let us assume that all three experts are equally reliable ($\Re_i = (1, 0, 0)$*); and hence ignore its effect. In order to merge the developed models, a two step game should be performed. In the first step the weakest model should be selected as the loser of the game, and subsequently, in the second step the loser has to make some concessions through the enhancement function; therefore, we initially calculate the intrinsic and extrinsic properties of all models and compute the fitness measure, in order to find the weakest model through the fitness measure-based choice function. The values of the properties of the models in the first round of negotiation are shown in Table 1.*

Table 1. First Round Property Values for Each of the Models in Figure 3

	$\mathcal{M}_{Bob}$	$\mathcal{M}_{John}$	$\mathcal{M}_{Mary}$
Ambiguity	0.28	0.29	0.09
Indecisiveness	0.35	0.35	0.1
Disagreement	0.77	0.77	0.77
fm	0.88	0.89	0.78

For instance, to calculate fm *for* $\mathcal{M}_{Bob}$*, we can employ Definition 17. Hence, we would have:*

$$fm^{\mu}_{\mathcal{SP}}(\mathcal{M}_{Bob}) = \sqrt[2]{(0.28)^2 + 0.(35)^2 + (0.77)^2} = 0.88$$

Notice that $\Re$ *is not considered here since all participants are assumed to be equally reliable. Based on such calculations shown in Table 1, a fitness measure-based ordering can be inferred that is:* $\mathcal{M}_{Mary} \prec_{fm} \mathcal{M}_{Bob} \prec_{fm} \mathcal{M}_{John}$*. Furthermore, from Figure 3,* $\mathcal{M}_{John}$ *is inconsistent with both* $\mathcal{M}_{Mary}$ *and* $\mathcal{M}_{Bob}$ *on the same formulae; therefore,* $\mathcal{M}_{John}$ *is selected as the loser of this round by the fitness measure-based choice function (*$g^{fm,max}$*). The behavior of the enhancement functions in this step are different. The WI-enhancement function selects the weakest inconsistent formulae and removes them from the model. The weakest inconsistent formula in* $\mathcal{M}_{John}$ *is* t *which will be removed from*

the model. This removal will make all three models compatible and they can be merged easily.

In the CB-enhancement function, the function recommends suitable opinion values that can make the experts beliefs converge. The upper value for the intervals that are proposed by the CB-enhancement function ($\wp^{B}_{\phi_i}$) for all formula are: for $p = (0.89, 0.05, 0.06)$, for $t = (0.05, 0.89, 0.06)$, for $u = (0.5, 0.15, 0.35)$, and for $v = (0.85, 0.05, 0.1)$, which clearly have a reinforcement effect. It is up to John to decide about the final value that it wants to select from within this range (e.g. for t the range is from $(0.85, 0.05, 0.1)$ to $(0.05, 0.89, 0.06)$). Lets suppose that John selects the proposed values of the CB enhancement function, so inconsistencies are removed and integration can easily be performed (if he makes other decisions other rounds of negotiation are required):

$$\begin{aligned}\mathcal{M}_{\oplus^{\mathcal{WI}}} = \{&(p, [(0.91, 0.06, 0.03)]), (q, [(0.33, 0.33, 0.33)]),\\ &(r, [(0.85, 0.05, 0.1)]), (t, [(0.05, 0.89, 0.6)]),\\ &(u, [(0.5, 0.15, 0.35)]), (v, [(0.85, 0.05, 0.1)]),\\ &(s, [(0.85, 0.05, 0.1)])\},\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{\oplus^{\mathcal{CB}}} = \{&(p, [(0.92, 0.05, 0.03)]), (q, [(0.33, 0.33, 0.33)]),\\ &(r, [(0.85, 0.05, 0.1)]), (t, [(0.05, 0.92, 0.3)]),\\ &(u, [(0.5, 0.15, 0.35)]), (v, [(0.85, 0.05, 0.1)]),\\ &(s, [(0.85, 0.05, 0.1)])\}.\end{aligned}$$

Although t exists in the final belief base, the belief base is still consistent because t possess a definite disbelief mass, which is interpreted as $\neg t$.

As it can be seen, the WI-enhancement function is faster in making a final decision but has a high information loss; whereas, CB-enhancement function provides more flexibility for negotiation, but is obviously slower. The CB-enhancement function is most suitable for cases where a correct belief base is singled out due to the negligence or lack of knowledge of the other information sources. It allows the proper justification of the belief bases standpoints by having more patience in merge.

7. Related Work

The Belief Game Model [2] and the Belief Negotiation Model [3] are two important work in the field of belief merging. Although these models are structurally quite similar, they have some fundamental differences. One of their major differences is that BGM focuses on the logical content of the belief bases in making the choice for contraction, while BNM considers each source as a candidate. Therefore, BNM may weaken one of the two identical belief bases, and leave the other intact; whereas in BGM, these two belief bases are always dealt with similarly. This feature of BGM adds more anonymity to the process, since individual characteristics of the belief bases are not important. Furthermore, BNM provides the possibility to consider the history of the rounds of negotiation while making a decision in the choice function. For instance, it can make the condition that each belief base cannot be weakened two times in a row. However in BGM, the choice decisions

are more Markovian meaning that each round of negotiation is totally independent of the previous rounds.

More recently, a class of general belief merge operators called DA^2 have been proposed that encode many previous merging operators (both model-based and syntax-based) as its special cases [28]. The general DA^2 framework is defined using a distance function d and two aggregation function $\oplus$ and $\odot$. The major shortcoming of the mentioned models is that they function over pure propositional logic belief bases, which makes the definition of choice functions complicated. It is therefore important to consider the role of formulae priorities while merging belief bases.

Some researchers have defined formulae priorities through the employment of the necessity degree of each formula in possibility theory [36]. There are two approaches to possibilistic belief base merging. In the first approach, researchers believe that inconsistency is totally undesirable and should be removed after merging [8, 37]. The second approach claims that inconsistencies are unavoidable and they can exist after merging [38, 39, 9]. Qi et al. [4] propose a prioritized belief negotiation model that merges information sources represented by possibilistic belief bases. They introduce several negotiation and weakening functions that manipulate the necessity degrees of the formulae and employ the conjunctive operator (that can have a reinforcement effect) over the obtained consistent belief bases for combination.

The split-combination (S-C) merging operators [40] are another set of operators that effectively employ possibilistic information to merge belief bases. The basic idea of the S-C approach is that it initially splits the belief profile into two sub-profiles based on a splitting method ($\mathcal{B} = \langle \mathcal{C}, \mathcal{D} \rangle$). Then the belief bases in each belief profile are merged separately resulting in two belief bases $\mathcal{C}$ and $\mathcal{D}$. The final outcome of the S-C merging approach is $B_{S-C} = \mathcal{C} \cup \mathcal{D}$. The authors have also proposed two splitting methods, namely: *upper-free-degree based* splitting and the *free-formula based* splitting. In a different approach, Amgoud and Prade [41] propose an argumentation framework for merging conflicting prioritized belief bases. In this model, the preference orderings between the arguments makes it possible to distinguish different types of relations between the arguments. A set of acceptable arguments can be developed from the framework, which can be considered as the required results. Other merging models based on possibilistic theory can be found in the related literature [42, 43, 44, 45, 46].

8. Concluding Remarks

In this paper, we have proposed a Subjective belief game model which extends Konieczny's belief game model [2]. The proposed model functions over a subjective belief profile which consists of different belief bases annotated with subjective opinions. Our model is different from BGM in several aspects:

- First, our model considers extrinsic and intrinsic properties of the belief bases in order to make a final decision regarding the belief bases that are selected by the choice function; therefore, similar to BNM our model does not perform uniformly over content-wise identical belief bases.

- Second, Konieczny argues that social pressure is a permissible influence on the information sources [2]. This may lead to cases where a belief base is singled out and forced to conform with the general opinion although it possesses the correct information. To overcome this issue, our model employs enhancement functions instead of weakening functions that provide more space for negotiation by permitting the addition of justifications in the enhancement function (Definition 9). The flexible definition of the enhancement function provides more *room to maneuver* for those belief bases that have to concede in order for them to justify their stand point.

- Third, we consider the trustworthiness of the information sources expressing the content of a belief base as one of its extrinsic properties. The trustworthiness value is a static subjective belief about the trustworthiness of the information source. Our previous experiments show that in many cases the trustworthiness value that dynamically changes as a result of the rounds of negotiation is not very different from the initial trustworthiness value that was ascribed to a belief base [47]; therefore, trustworthiness is a static subjective opinion in our model.

It is important to notice that unlike our proposed model, the prioritized belief negotiation model [4] is also restricted with regards to the above-mentioned points 2 and 3. We have also introduced several choice and enhancement functions. The CB enhancement function demonstrates enduring behavior, while the WI enhancement function shows to be more impatient, removes inconsistencies as soon as they are encountered. This is because the CB function tolerates inconsistencies and requires partial conformance to its proposed recommendations, but the WI function immediately removes the weakest inconsistent formula from the belief base. Moreover, the CB function exhibits greater respect for *minimality of change* by tolerating inconsistencies to a great extent. It has also been formally shown that a collaborative modeling process can be supported by a Subjective belief game model if the inconsistent model elements are appropriately annotated with subjective opinions by the modeling experts.

Very few researchers have considered the use of game-theory concepts such as the Nash equilibrium to develop numerical models of belief revision [48, 13]. As future work, we are interested in looking at the possibility of developing a cooperative non-zero-sum game for merging multiple belief bases. For instance, we can look for a pareto optimal solution to a belief merging process where the result of the merge cannot be any better in any way other than disadvantaging one of the belief bases.

Notes

1. See http://en.wikipedia.org/wiki/Subjective_logic.

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