The *Voltage Distribution Diagram*

A Simplified Analysis Method for Op-Amp Circuits

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Abstract

The *voltage distribution diagram* (VDD) is an easy-to-use tool for designing an inverting or non-inverting op-amp circuit with some arbitrary (linear) transfer function. It makes it easy to investigate the inverting and non-inverting circuit configurations to determine which will work best in a given application.

The same diagram may be used to illustrate the effect of various op-amp limitations such as finite open-loop voltage gain, offset voltage and slew-rate limiting.

It's useful in the design of positive feedback circuits as well. The voltage distribution diagram may be used to design as inverting or non-inverting Schmitt trigger with arbitrary thresholds and output bounds.

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1 Introduction

This paper describes a method of graphical analysis for op-amp circuits. The same technique, with minor modifications, applies both to linear amplifiers with negative feedback and Schmitt trigger circuits with positive feedback. The method is useful both as a practical tool in circuit design and an aid to understanding circuit operation.

Origins

A graphical method similar to the one presented here for inverting amplifier circuit design, is given in [5]. The application to linear amplifiers is mentioned briefly in reference [4]. An unrelated graphical method of Schmitt trigger design is given in [6]. The technique is not presented at all in the standard textbooks in op-amp circuit design, such as [1],[2] or [3]. This article extends and unifies previous work by showing how it may be applied to a variety of op-amp circuits.

Since there appears to be no generally accepted name for this technique, I shall refer to it as the *voltage distribution diagram*[1].

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[1] The application to linear amplifiers has been referred to elsewhere as a *teeter-totter* diagram. Franco [4] calls it a *mechanical analogy.* As far as I am aware there is no name for the technique when applied to Schmitt trigger circuits.

2 Negative Feedback: Linear Amplifiers

2.1 The Inverting Amplifier

The standard op-amp inverting amplifier circuit is shown in figure 1.

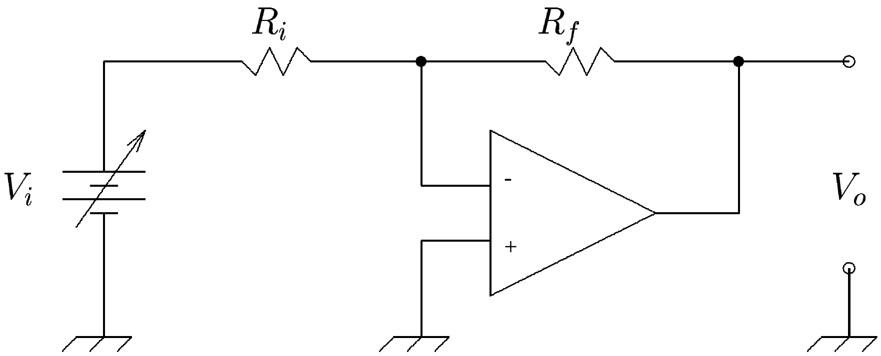

Figure 1: Inverting Op-Amp Circuit

The voltage-distribution-diagram for the inverting amplifier circuit of figure 1 is shown in figure 2.

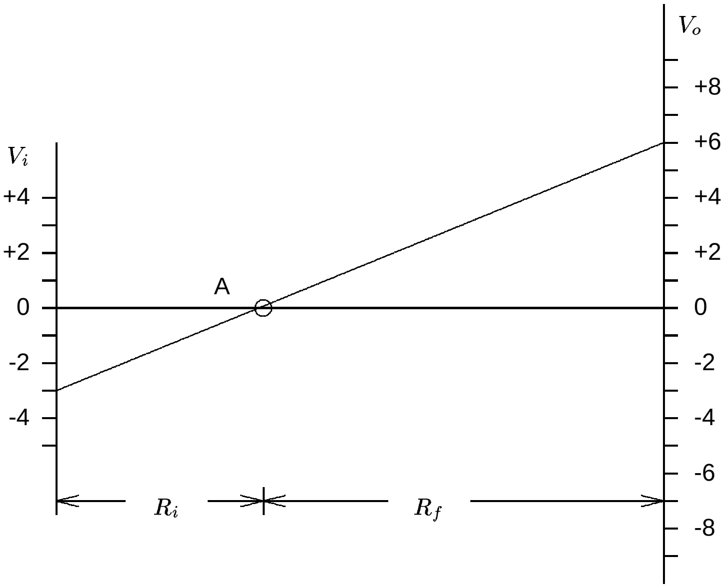

Figure 2: Inverting Op-Amp, Voltage Distribution Diagram

This diagram may be considered as a plot of the voltage distribution across the two resistors R_i and R_f for some value of input voltage V_i. In the case shown in figure 2, the input voltage is -3 volts. The circuit has a closed-loop voltage gain of -2, so the output voltage is $+6$ volts. Point A corresponds the virtual earth point.

When the input voltage reverses polarity, the voltage-distribution-diagram becomes as in figure 3. The distribution swings over the range between the dashed and solid lines on the diagram. The diagram behaves in the same fashion as a playground teeter-totter.

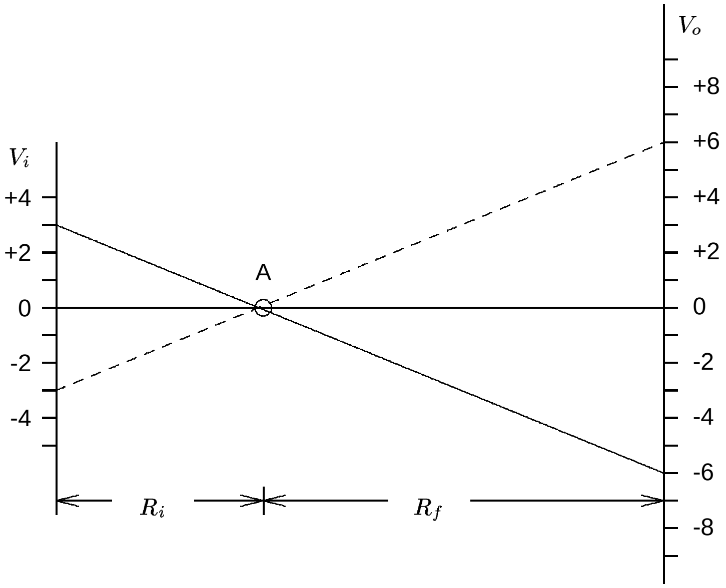

Figure 3: Reversing the Input Voltage

Some points are immediately obvious from this diagram:

Voltage Gain Ignoring polarity for a moment: because the two triangles are similar,

$$\frac{V_i}{R_i} = \frac{V_o}{R_f}$$

and

$$\frac{V_o}{V_i} = \frac{R_f}{R_i}$$

that is, the closed-loop voltage gain of the circuit is proportional to the ratio of the feedback to input resistance. In other words, as point A moves to the left on the diagram, the ratio of V_o/V_i increases and the gain increases.

Inverting Operation The direction of change of the output voltage is opposite to the input. As V_i moves up the page, V_o moves down.

Finite Gain The concept of the *virtual earth* point at A on the diagram is a very useful approximation idea for analyzing op-amp circuits. However, for the op-amp to develop a finite output voltage with a finite open-loop gain, there must actually be some non-zero signal at point A.

Exaggerating the effect (or for amplifiers where the open-loop gain cannot be neglected), the situation is as shown in figure 4. Finite open-loop voltage gain requires an error signal to appear at the input terminal and causes the closed-loop gain to decrease below $-R_f/R_i$. The effect is to move the pivot point from A to B by an amount which may be shown to be

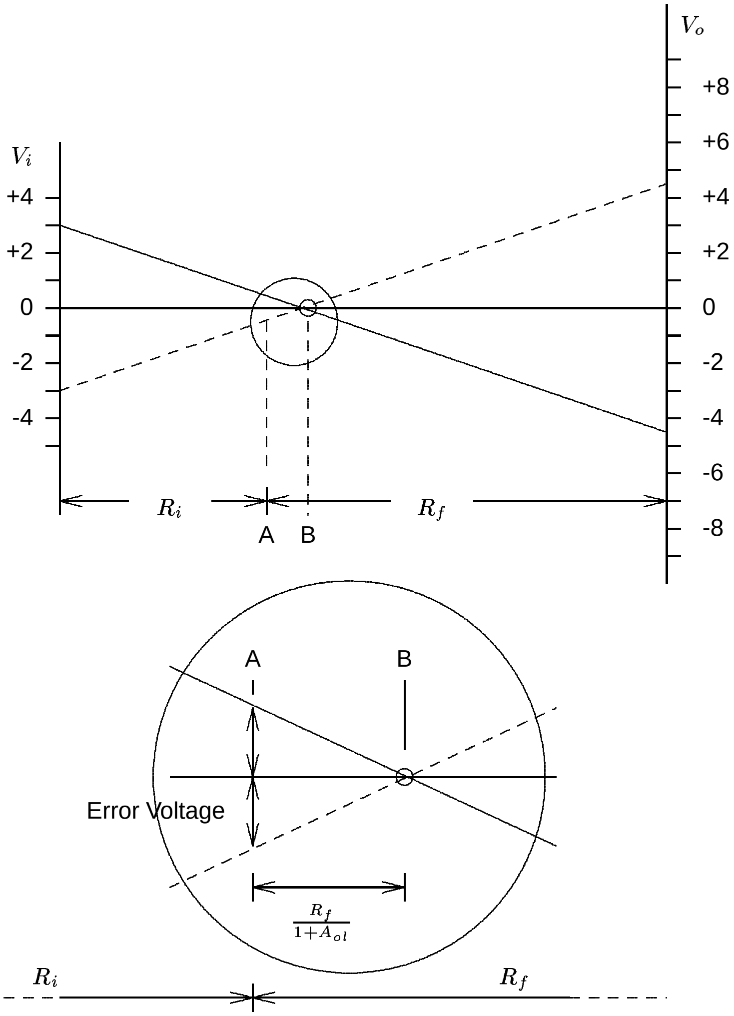

Figure 4: Finite Gain, Inverting Op-Amp

$$\frac{R_f}{1 + A_{ol}}$$

As the open-loop gain A_{ol} increases, this offset and the error signal both decrease in magnitude. The error voltage reverses sign with the input signal.

Offset Voltage Unlike error voltage, offset voltage does not change polarity with the input signal. The voltage distribution diagram is shown in figure 5. The input signal still swings symmetrically between ±3 volts but the output is offset in the positive direction by the input offset voltage.

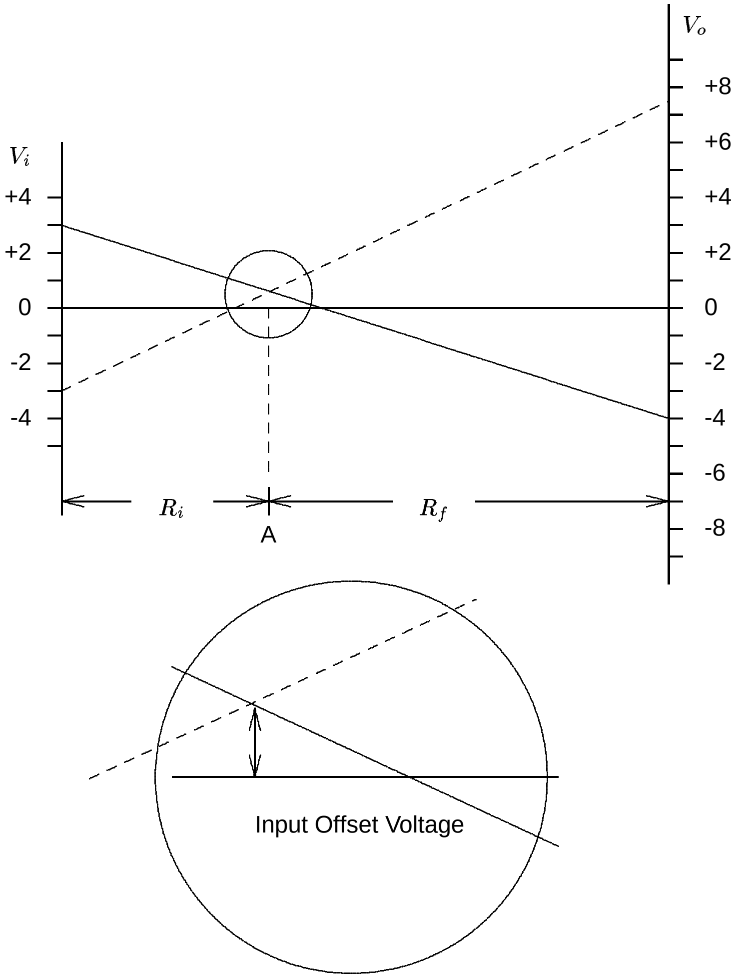

Figure 5: Offset Voltage, Inverting Op-Amp

The effect of offset is also demonstrated in the gain distribution diagram of figure 6. For an input signal of zero, the output signal should also be zero. Because of voltage offset, the output is non-zero and will increase as the closed-loop gain R_f/R_i increases (ie, point A moves to the left). The lesson for the circuit designer is that offset voltage is more likely to be a problem at large values of closed-loop gain.

Clipping and the Error Voltage When the output voltage reaches the positive or negative limit of the op-amp, the system of negative feedback no longer functions and the error voltage becomes significant. Figure 7

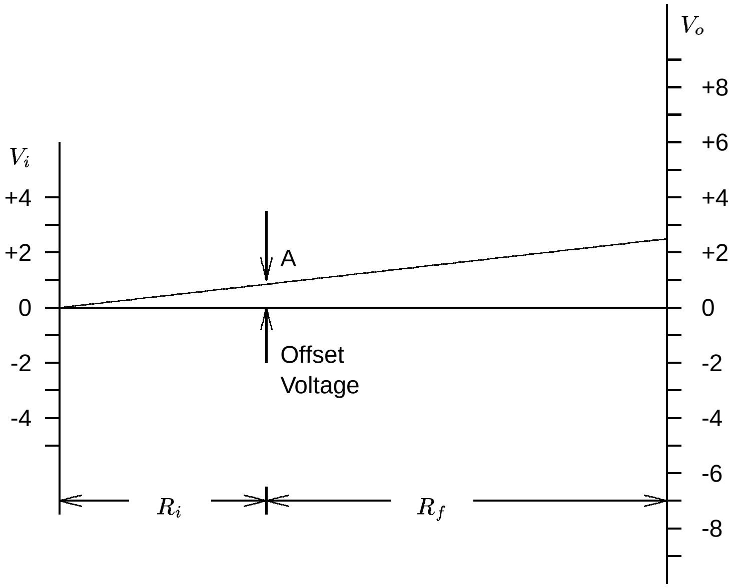

Figure 6: Offset Voltage, Zero Input

shows how clipping appears on the voltage distribution diagram. When clipping occurs, the pivot point of the line becomes the right-hand axis.

The closed-loop gain $-R_f/R_i$ is equal to 2 volts/volt and the limit of the op-amp output voltage V_o is ± 8 volts. In the case shown in figure 7 the input voltage is -6 volts, so the output limits at $+8$ volts. The resistors R_f and R_i function as a voltage divider and create a non-zero voltage at point A, the inverting input of the op-amp. When troubleshooting an inverting op-amp circuit, it is useful to check the voltage at the inverting terminal: if it is not equal to the voltage at the non-inverting terminal (often ground, or zero volts), then the op-amp is not operating as a negative feedback system.

Slew Rate Limiting A large voltage step of the input voltage V_i will cause slew rate limiting to occur, as the amplifier compensation capacitance is charged by a finite current. On the voltage distribution diagram, the input changes abruptly from one value to another. The output voltage ramps relatively slowly from its first to second value.

While the amplifier is slewing, the diagram shows that a large error voltage appears at the inverting input terminal of the op-amp during slew rate limiting. Consequently, a measurable transient voltage at the inverting input terminal indicates that the amplifier is slew rate limiting.

2.2 The Non-Inverting Amplifier

The non-inverting negative feedback amplifier configuration is shown in figure 8, and the corresponding voltage distribution diagram in figure 9.

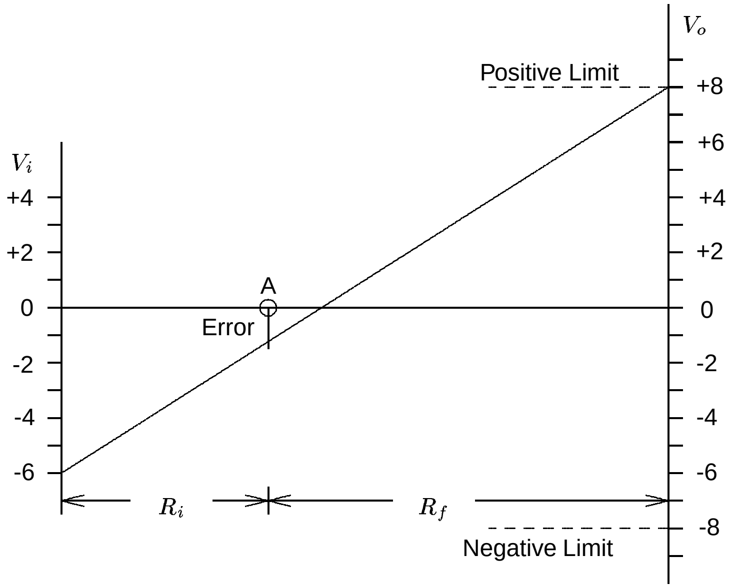

Figure 7: Positive Output Clipping

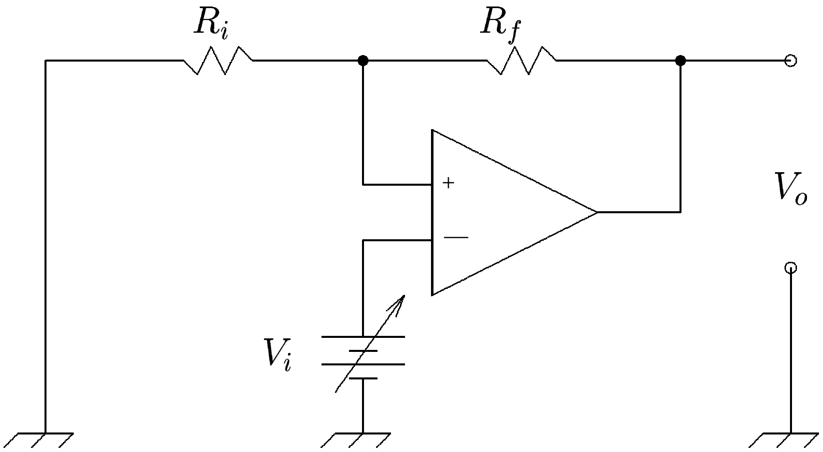

Figure 8: Non Inverting Op Amp Circuit

For the closed-loop gain, we have by similar triangles that

$$\frac{V_i}{R_i} = \frac{V_o}{R_i + R_f}$$

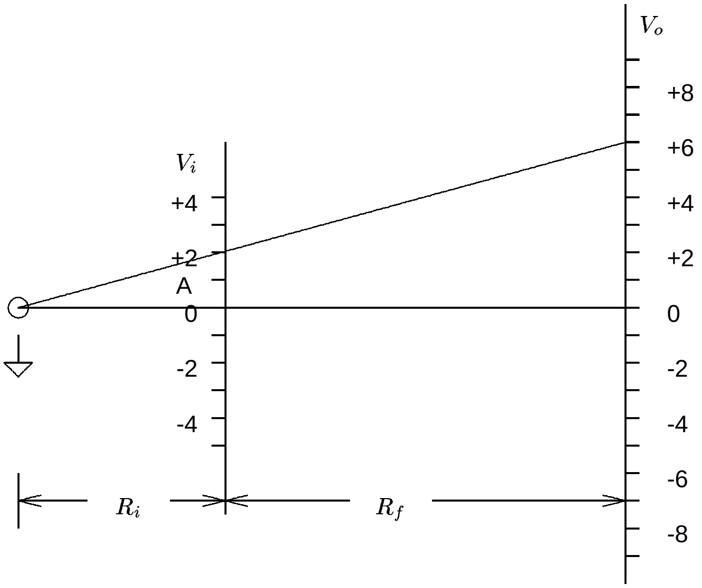

Figure 9: Voltage Distribution, Non Inverting Circuit

Rearranging,

$$\begin{aligned}\frac{V_o}{V_i} &= \frac{R_i + R_f}{R_i} \\ &= 1 + \frac{R_f}{R_i}\end{aligned}$$

which is the usual equation for the closed-loop gain, non-inverting op-amp configuration.

From the voltage distribution diagram of figure 9, it is evident that the output voltage changes in the same direction as the input, ie, the amplifier is non-inverting. As well, the output voltage can never be less than the input voltage, even when the feedback resistor is reduced to zero. The effect of finite gain open-loop voltage gain, offset voltage, output clipping and slew rate limiting may also be shown on the same diagram, as described in section 2.1 above. However, the real utility of the voltage distribution diagram is is designing the generalized inverting/non-inverting amplifier, shown in the next section.

2.3 General Inverting/Non-Inverting Amplifier

The inverting and non-inverting functions may be combined in the same circuit, as shown in figure 10.

This may be regarded as an inverting amplifier with input signal V_a and offset V_b, or as a non-inverting amplifier with offset V_a and input signal V_b. By superposition, the output voltage is equal to:

$$V_o = V_b \left(1 + \frac{R_f}{R_i}\right) - V_a \left(\frac{R_f}{R_i}\right)$$

The corresponding voltage distribution diagram is shown in figure 11.

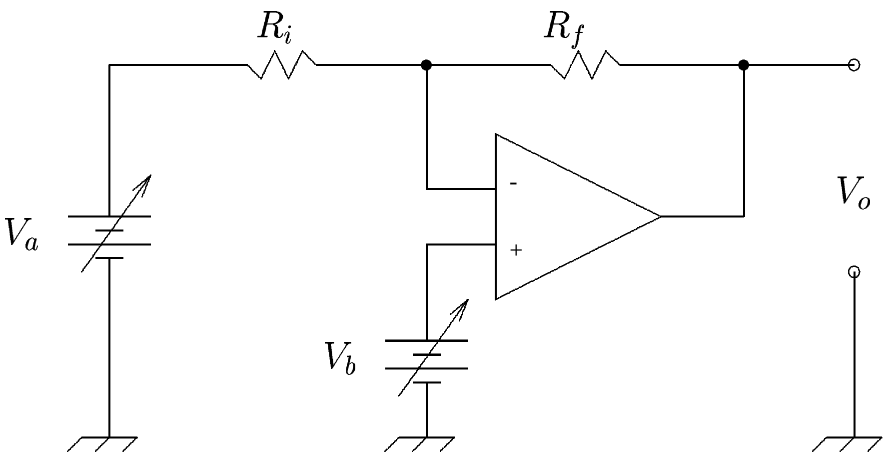

Figure 10: Generalized Inverting/Non-Inverting Amplifier

The equation for a straight line is

$$y = mx + b$$

where m is the Slope and b is the Y-Intercept. In the case of figure 11,

$$m = \frac{V_b - V_a}{R_i}, \quad b = V_a$$

Substituting these expressions for m and b in the general equation, we have

$$y = \left(\frac{V_b - V_a}{R_i}\right) x + V_a$$

One point on this line corresponds to $x = R_i + R_f, \quad y = V_o$. Substitute these values for x and y and simplify:

$$\begin{aligned} V_o &= \left(\frac{V_b - V_a}{R_i}\right)(R_i + R_f) + V_a \\ &= V_b\left(1 + \frac{R_f}{R_i}\right) - V_a\left(\frac{R_f}{R_i}\right) \end{aligned}$$

This is the same as the previous equation obtained by superposition, so the voltage distribution diagram corresponds to the behaviour of the circuit.

2.4 Application to Amplifier Design

When V_a is an input and V_b an offset voltage, we have a *generalized inverting amplifier*. When V_b is an input and V_a an offset voltage, we have a *generalized non-inverting amplifier*.

Suppose that the design requirement is for an amplifier with an input voltage of ±1 volt input signal that corresponds to an output signal between 0 and +5 volts. The voltage distribution diagram for an inverting design

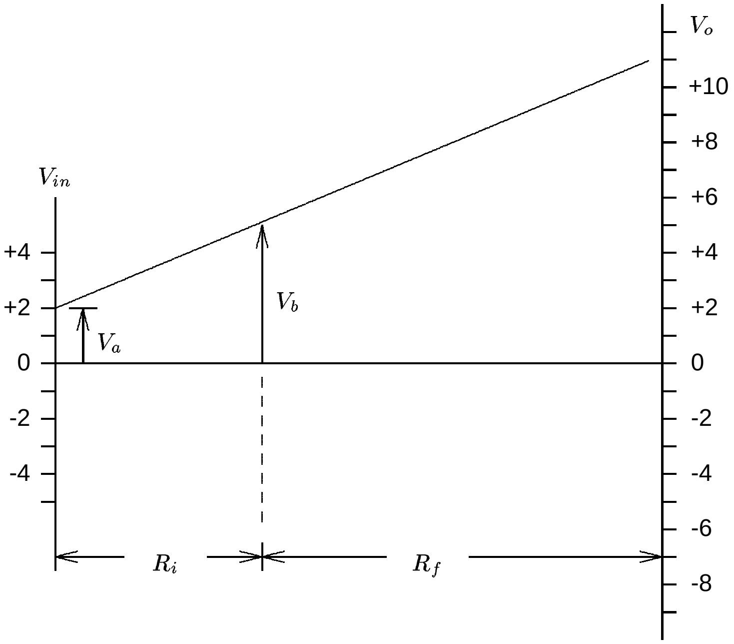

Figure 11: Voltage Distribution Diagram, Generalized Amplifier

is shown in figure 12. Two traces are drawn between the corresponding input and output voltages. The point of intersection defines both the offset voltage V_b and the ratio of $R_f : R_i$. Reading values[2] off figure 12, $R_f/R_i = 2.55$ and $V_b = +0.75$ volts. The values obtained by circuit analysis are $R_f/R_i = 2.5$ and $V_b = +0.714$ volts.

The voltage distribution diagram for a non-inverting amplifier that meets the same requirements is shown in figure 13. Reading values off figure 13, $R_f/R_i = 1.40$ and $V_a = -1.7$ volts. The values obtained analytically are $R_f/R_i = 1.5$ and $V_a = -1.66$ volts.

With the graphical images of figure 12 and figure 13 in mind, the circuit designer can easily explore how an inverting or non-inverting circuit design will implement some arbitrary transfer function.

3 Positive Feedback: The Schmitt Trigger

The voltage distribution diagram may be applied to positive feedback circuits. As in the case of linear amplifiers, there are two cases to consider: the inverting and non-inverting Schmitt Trigger.

3.1 Inverting Schmitt Trigger

The inverting Schmitt Trigger is shown in figure 14. It is identical to the non-inverting amplifier discussed previously except that the input terminals of the op-amp are exchanged.

The corresponding voltage distribution diagram is shown in figure 15. The amplifier receives positive feed-

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[2] We determined these values by scaling off the original xfig drawing.

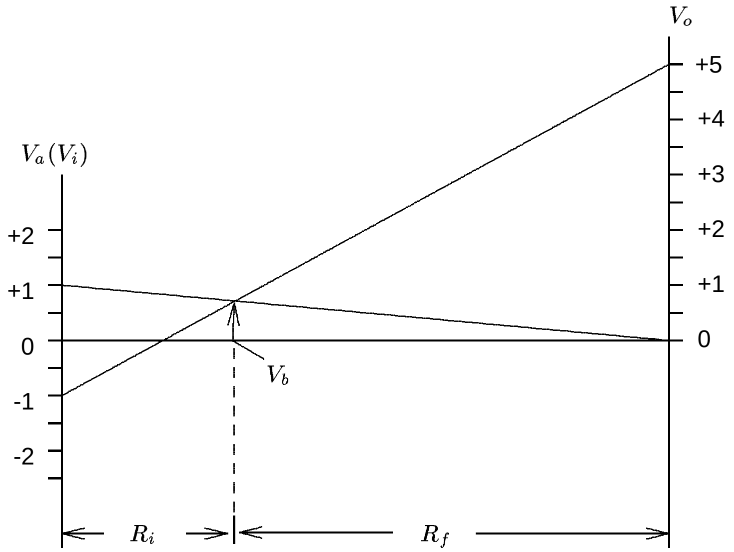

Figure 12: Voltage Distribution, Generalized Inverting Amplifier

back, so it will drive the output to one of its output limits. We refer to this configuration as the *inverting* Schmitt trigger because the output switches to its negative limit as the input voltage exceeds the positive input threshold, and switches to its positive limit as the input voltage exceeds (moving in a negative direction) the negative input threshold.

Let's assume that the output limits (bounds) are $V_{ub} = +3$ volts and $V_{lb} = -3$ volts. Assume that the input voltage is zero, and that the amplifier is currently at the upper bound. Resistors R_a and R_b are equal. Then the voltage at their junction, the non-inverting input terminal of the op-amp, will be at $+1.5$ volts. This is V_{ut}, the *upper threshold* of the Schmitt trigger, represented by the upper solid sloping line on figure 15.

The equation governing the generic behaviour of the op-amp is

$$V_o = A_{ol}(V^+ - V^-)$$

where A_{ol} is the open-loop gain of the op-amp, V^+ is the non-inverting input voltage (+1.5 volts), and V^- is the inverting input voltage (0 volts). In this circuit, with the output at its $+3$ volt positive limit, here is a net input voltage of 1.5 volts. This is multiplied by the large open-loop gain of the op-amp, driving the output up against its positive limit. This situation corresponds on the voltage distribution diagram of figure 15, the solid sloping line.

Now consider that the input voltage V_i at the inverting terminal increases positively to slightly more than the upper threshold V_{ut}. The net input to the op-amp will be negative $(V^+ - V_-)$, and this will drive the output of the amplifier negative. As the output moves negative, it increases the effective input voltage between the op-amp input

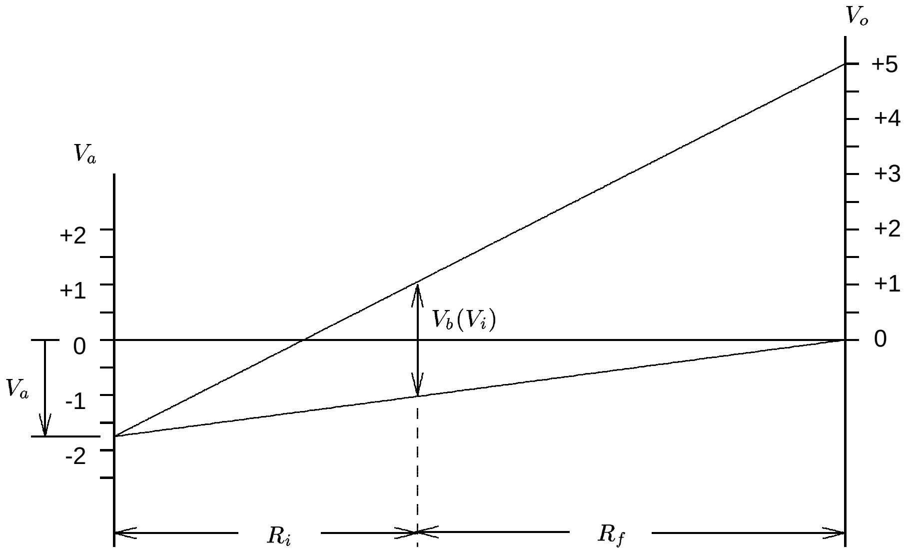

Figure 13: Voltage Distribution, Generalized Non-Inverting Amplifier

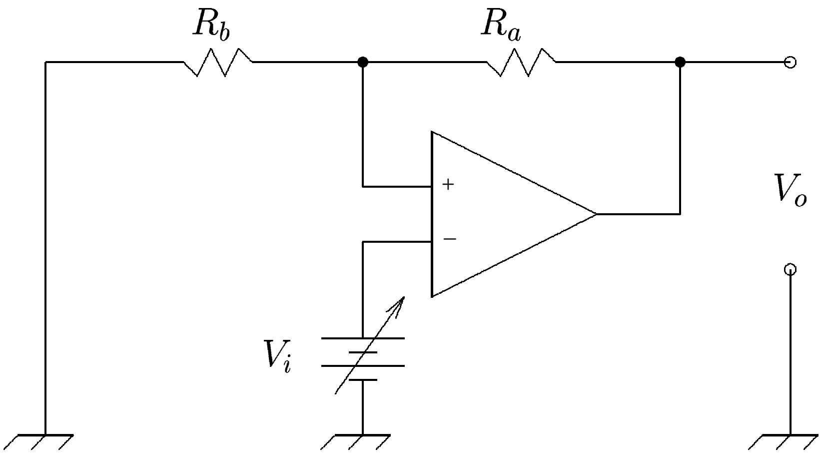

Figure 14: Inverting Schmitt Trigger

terminals, accelerating the movement of the output to its lower bound V_{lb}. On the voltage distribution diagram of figure 15, this new stable state corresponds to the dashed sloping line. In this state, the input must exceed the lower threshold $V_{lt} = -1.5$ volts to cause the Schmitt Trigger to switch back to its positive state.

As in the case of the linear amplifiers, a bias voltage source V_b may be used to modify the behaviour of the inverting Schmitt trigger, as shown in the design example of figures 17 and 18 in the section 3.2.

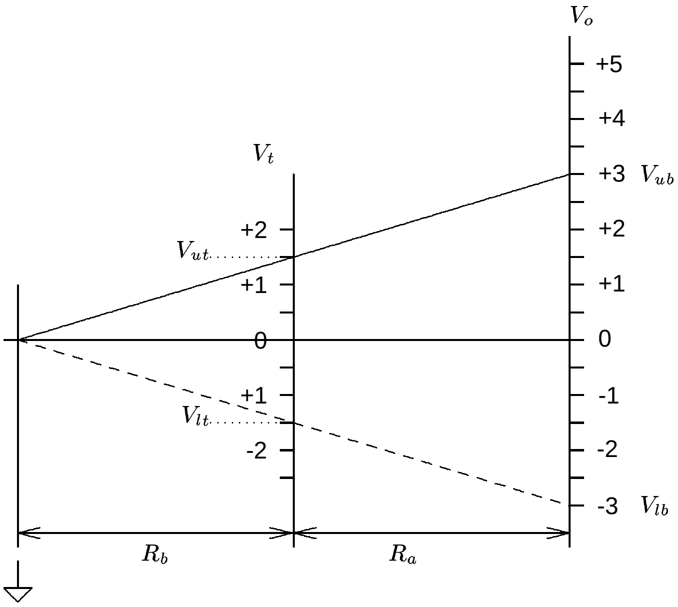

Figure 15: Voltage Distribution Diagram, Inverting Schmitt Trigger

3.2 Design Example

Suppose we wish to design an inverting Schmitt Trigger circuit where the output limits at +5 volts and 0 volts. The input thresholds are ±1 volt. This might correspond to an application where an AC coupled signal (one with zero DC component) is to be *squared up* for measurement by a microprocessor. The requirements are summarized in figure 16. Then the voltage distribution diagram must look like figure 17.

Parameter	Symbol	Value, volts
Input Upper Threshold	V_{ut}	$+1$
Input Lower Threshold	V_{lt}	-1
Output Upper Bound	V_{ub}	$+5$
Output Lower Bound	V_{lb}	0

Figure 16: Schmitt Trigger Design Requirements

According to the voltage distribution diagram of figure 17, the design requires that the bottom end of the voltage divider be biased to -1.75 volts. The ratio of $R_a : R_b$ is $10 : 7$. The schematic is shown in figure 18.

The voltage distribution diagram indicates immediately that a negative voltage is required for this circuit. If the circuit must operate from a single positive supply, the -1.75 volt bias signal may not be convenient to provide. Section 3.4 shows an alternative that uses a non-inverting Schmitt trigger circuit.

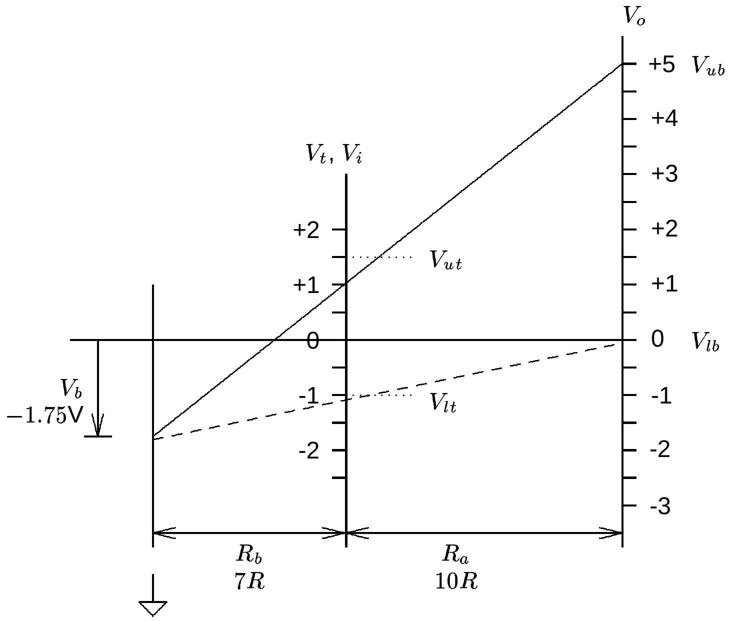

Figure 17: Inverting Schmitt Trigger, Example Circuit

3.3 Non-Inverting Schmitt Trigger

The non-inverting Schmitt Trigger is shown in figure 19. The input signal is moved to the ground end of the $R_a : R_b$ voltage divider. The corresponding voltage distribution diagram is shown in figure 20. In this case, we have assumed that the amplifier output limits are the V_{ub} (upper bound) at $+2.5$ volts and V_{lb} at -2.5 volts.

In this circuit configuration, the amplifier changes state whenever the voltage at the centre tap of the $R_a : R_b$ voltage divider moves through zero volts. From the straight line constructions on figure 20, we can determine that the upper threshold V_{ut} will be at $+1.5$ volts and the lower threshold V_{lt} at -1.5 volts.

Consider that the input voltage is positive and Schmitt is in its *output high* state, with the output voltage V_o equal to the *upper bound* value V_{ub}, $+2.5$ volts. This corresponds to the upper dashed line in figure 20.

Now suppose that the input voltage moves negative through zero, ending up in the position indicated by the upper solid line in figure 20. When the left end of this line passes through the lower threshold level V_{lt} of -1.5 volts, voltage at the op-amp non-inverting terminal passes through zero. The op-amp immediately switches into the *output low* state, with V_o equal to -2.5 volts. This new state corresponds to the lower solid line in figure 20, with the output voltage V_o equal to the *lower bound* value V_{lb}, -2.5 volts.

To revert back to the original state, output positive, the input voltage has to move positive to take the op-amp non-inverting terminal through zero volts, as indicated by the lower dashed line in figure 20. This requires that the input voltage move up through the upper threshold level V_{ut} of $+1.5$ volts.

The voltage distribution diagram is useful in visualizing the action of this circuit, but its main utility is in designing a circuit to some specific requirements, as described in section 3.4.

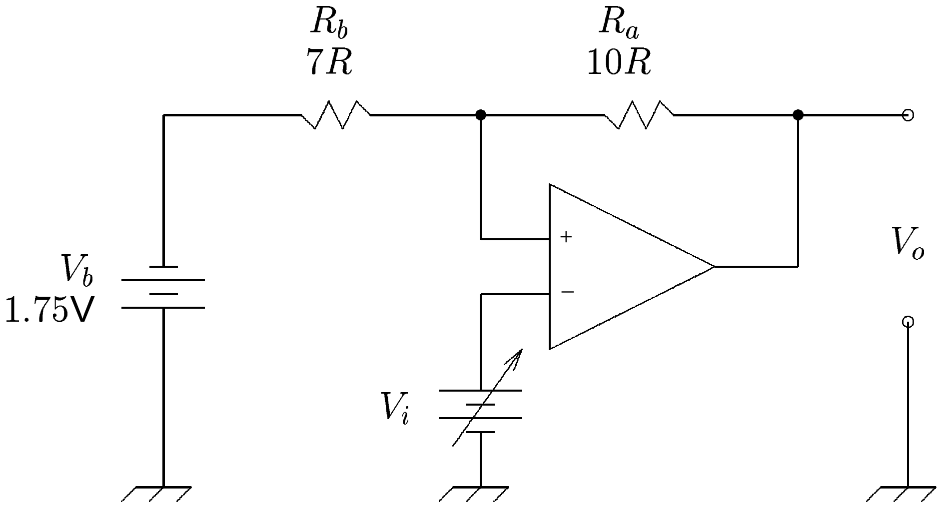

Figure 18: Inverting Schmitt Trigger, Example Design

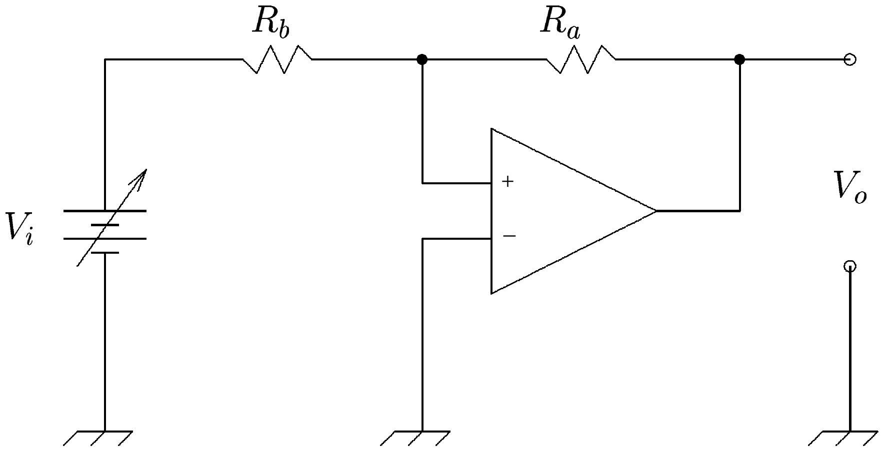

Figure 19: Non-Inverting Schmitt Trigger

3.4 Design Example

Now we will revisit the previous design, this time using the non-inverting Schmitt Trigger circuit. As previously summarized in figure 16, the output limits are at +5 volts and 0 volts, and the input thresholds are ±1 volt. The voltage distribution diagram for a non-inverting Schmitt trigger with these thresholds and output limits is shown in figure 21.

The diagram indicates that a bias voltage of $+0.75$ volts is required at the inverting terminal of the op-amp. The ratio of R_a:R_b is 2.53:1. The final circuit design is shown in figure 22.

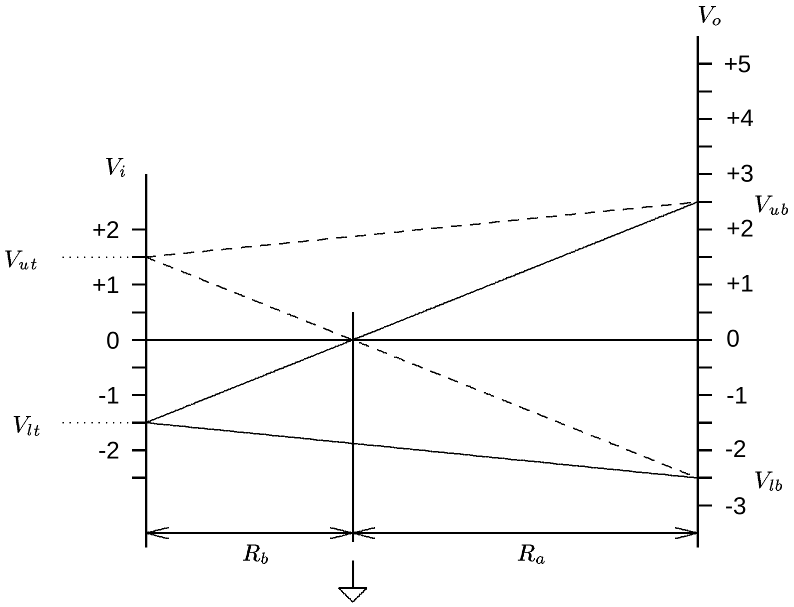

Figure 20: Voltage Distribution Diagram, Non-Inverting Schmitt Trigger

Comparing the circuit designs in figures 18 and 22, the non-inverting version in figure 22 is probably preferable[3]. In that design, the $+0.75$ volt bias voltage may be derived from the system positive supply voltage with another voltage divider.

4 Voltage Distribution Diagram and Transfer Function

A plot of the transfer function for an amplifier or Schmitt trigger circuit is another useful aid to design and understanding. For example, in teaching material about the Schmitt trigger, it is quite common to show the transfer function because it illustrates the concept of hysteresis.

As a design aid, however, the voltage distribution diagram is a more useful technique since values of offset voltage and resistor ratio can be read directly from the diagram.

5 Summary

Section 2.4 showed how the voltage-distribution-diagram technique can be used to design a specific amplifier circuit where some combination of input signal and offset must be combined to produce a specified range of output voltage.

Similarily, the two design examples of sections 3.2 and 3.4 show how voltage distribution diagrams may be used to explore alternative Schmitt trigger designs.

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[3] This assumes that the phase of the output with respect to the input is not a major issue and can be accounted for easily further in the signal chain. This is certainly the case if the signal is used by a microprocessor.

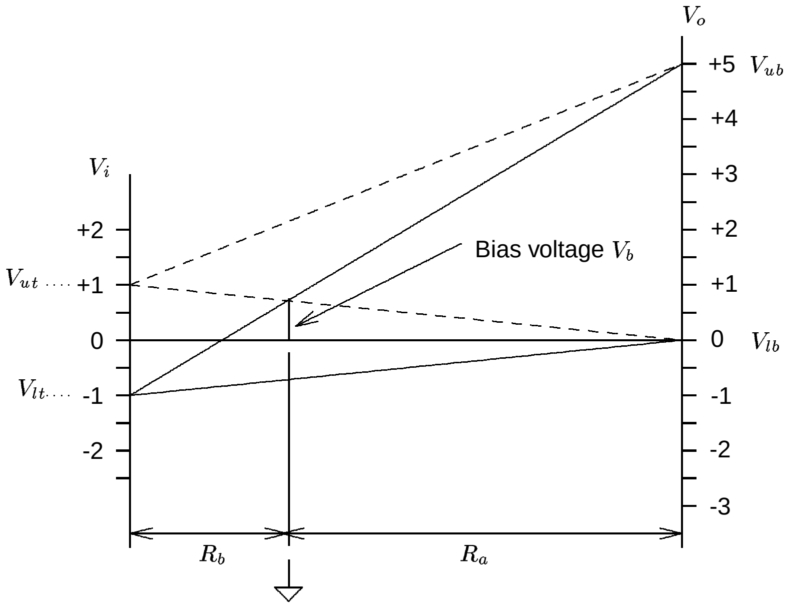

Figure 21: Non-Inverting Schmitt Trigger Design

Once a workable linear amplifier or Schmitt trigger circuit configuration is identified along with approximate circuit values, more accurate circuit values may be calculated by analytical methods.

6 Acknowledgements

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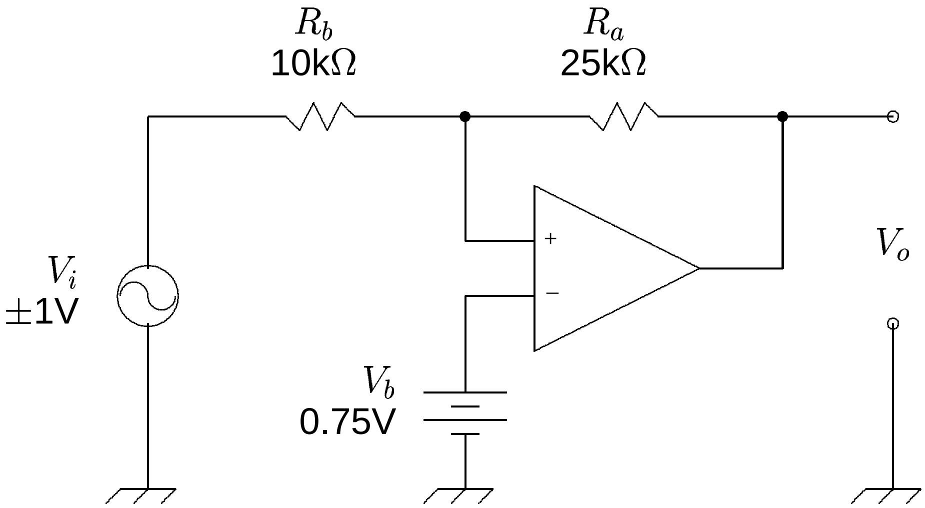

Figure 22: Non-Inverting Schmitt Trigger Design

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