The Design of a Kind of Chirp-like Mother Wavelet by Neural Network

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Abstract Methods are presented for adaptive design of mother wavelets for the representation and processing of the chirp-like signal using neural networks. A new kind of network structure is proposed and its training algorithm is explored. The concept "wavelet atoms" is introduced to construct a new mother wavelet. Numerical simulations show that our approach is always effective to find a "satisfactory" mother wavelet for applications. In fact, our methods can also be used in the processing of other kind of signals.

1. Introduction

The linear frequency modulated (LFM) techniques are widely used in many occasions, such as sonar, audio and acoustics, etc., so the processing of the chirp-like signal is often needed to be considered. Since chirp-like signals are of a time-varying nature in time-frequency domain, the time-frequency methods such as short-time Fourier transform (STFT) and Wigner-Ville distribution (WVD), etc., are often used to analyze and process them. Recently, the wavelet transform has become a well-known time-frequency method as useful tool for various signals processing applications. The CWT and WS are best suited to signal analysis and representation [1], [2] because they have many good mathematical properties [4], [5].

Mann and Haykin [3] have proposed a new kind of transform named "chirplet transform", which is a generalization of the wavelet transform and useful for processing the chirp-like signal, but this transform may not remain the good mathematical properties of CWT due to its complexity. The choice of mother wavelet of CWT is very flexible, so it is very necessary and important to choose a better one to be suitable for the processing of the chirp-like signal.

Although there is now some literature dealing with the signal analysis and representation using CWT and WS [1], [2], how to choose a better mother wavelet for that is still seldom discussed. In this paper, a kind of the structure of neural network and a method to design a kind of chirp-like mother wavelets adaptively for the signal analysis and representation by neural network are proposed. Our approach is similar to [2] but differs in many important concepts, such as the "wavelet atoms", the criterion, and network structure, etc. The numerical examples show that our work is very effective and useful.

2. Wavelet Transform

The CWT is defined as [5]:

$$CWT_f^{\psi}(a,b) = \left\langle f, \psi^{a,b} \right\rangle = |a|^{-1/2} \int f(t)\overline{\psi}\left(\frac{t-b}{a}\right)dt \qquad (1)$$

where a and b are called scale and time parameters, respectively, and

$$\psi^{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right) \qquad (2)$$

where $\psi(t)$ is called mother wavelet and $\psi^{a,b}(t)$ is called wavelet.

For a function $\psi(t)$ to be a mother wavelet, it must be admissible, i.e., the following condition must be hold:

$$C_\psi \equiv \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \qquad (3)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$.

Due to the heavy redundancy, the CWT can be sampled in the time-scale plane (a, b) with the dyadic grid to form WS [5]. $\psi(t)$ should have properties such as regularity and vanishing moments [4], etc. We often expect $\psi(t)$ have good time-frequency localization and a signal $f(t)$ can be approximated by a few wavelet coefficients, i.e.,

$$\hat{f}(t) = \sum_{k=1}^{M} A_k \psi_k(t) = \sum_{k=1}^{M} A_k \psi\left(\frac{t-b_k}{a_k}\right) \qquad (4)$$

Since the wavelet transform is a linear transform, the limited linear combination of wavelets can also be treated as a mother wavelet, and it is easy to

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prove that this wavelet can maintain many properties of the original wavelets, such as regularity and vanishing moments.

In the following, we will use the concept of "wavelet atoms", which is a little similar to "time-frequency atoms" in [7], and construct a new structure of neural network for the adaptive design of the chirp-like mother wavelet.

3. The Design of Mother Wavelet By Neural Network

It is known that a Gaussian weighted LFM function can be treated as an admissible wavelet [1], which is similar to "Morlet wavelet". We choose a set of Gaussian weighted LFM functions as "wavelet atoms", then we construct mother wavelet as:

$$\psi(t) = \sum_{p=1}^{N} B_p h_p(t) \qquad (5)$$

where the Gaussian weighted LFM functions

$$h_p(t) = g\left(\frac{t-u_p}{s_p}\right)\exp\left(j2\pi f_p t + j2\pi l_p t^2\right) \qquad (6)$$

and where

$$g(t) = \pi^{-1/4}\exp\left(-t^2/2\right) \qquad (7)$$

As we known, the Gaussian function $g(t)$ is not localized in either time or frequency, but it does posses the best joint time-frequency resolution in the sense of having the minimum time-bandwidth product. In our approach, B_p, s_p, u_p, f_p and l_p are parameters that determine different mother wavelet $\psi(t)$ and the shape of resolution cell of $\psi(t)$. Because of the FM ingredient of $\psi(t)$, the shape of resolution cell of $\psi(t)$ varies more flexible and is especially suited to the LFM signal. It is easy to show that the family of $h_p(t)$ is extremely redundant [7], and $\psi_k(t)$ can not constitute an orthogonal basis.

Then equation (4) is used to approximate signal $f(t)$. This approximation can be expressed as the neural network of Fig. 1(a) and (b). The parameters A_k, a_k, b_k, B_p, s_p, u_p, f_p and l_p are chosen such that $\psi_k(t)$ is most similar to $f(t)$ on local basis. Here the least-mean-squares (LMS) energy is employed as a criterion. (In practice, we can choose other criterions according to different applications.) i.e.,

$$E = \frac{1}{2}\int\left\|\hat{f}(t) - f(t)\right\|^2 dt \approx \frac{1}{2}\sum_{t=1}^{T}\left\|\hat{f}(t) - f(t)\right\|^2 \qquad (8)$$

For simplicity, we assume $f(t)$ is a real signal, and adopt the real part of $h_p(t)$ as "wavelet atoms". We use a gradient descent method to train network and minimize E. We can stop the iterations when a sufficient small E is reached or $\psi_k(t)$ is enough similar to $f(t)$ on local basis. In most applications, we do not really need to get a global optimal solution but a satisfactory solution, which is proper for the application. In our method, to set initial value of parameters and take proper learning rate are important and we play some tricks on it.

4. Numerical Simulations

We consider two simple examples to demonstrate how to choose optimal wavelet by our neural network. The first signal $f(t)$ is a truncate LFM signal, shown as the solid line in Fig. 2. We take M=1, N=4, the initial wavelet $\psi(t)$ (normalized) is shown as the dashed line in Fig. 3, and the initial wavelet approximation $\hat{f}(t)$ is shown as the dashed line in Fig. 2.

Then we train the network and adjust parameters by our training algorithm. The dotted line in Fig. 2 shows the wavelet approximation $\hat{f}(t)$ after 235 iterations. The solid line in Fig. 3 shows the wavelet $\psi(t)$ (normalized) after 235 iterations.

The second signal is the sum of two truncate LFM waves occurred at different time, shown as the solid line in Fig. 4. We take M=2, N=4, the initial wavelet $\psi(t)$ (normalized) is shown as the dashed line in Fig. 5, and the initial wavelet approximation $\hat{f}(t)$ is shown as the dashed line in Fig. 4. The dotted line in Fig. 4 shows the wavelet approximation $\hat{f}(t)$ after 350 training iterations. The solid line in Fig. 5 shows the wavelet $\psi(t)$ (normalized) after 350 training iterations.

The simulation results show that the neural network and its training algorithm are always very effective for the chirp-like signal to converge to a good mother wavelet. The further numerical simulations show that the wavelets we have chosen are very good for the wavelet representation and analysis of the signals.

5. Conclusion

The processing of the chirp-like signal is often encountered and the wavelet transform can be used in this case. It is very important to find the optimal mother wavelet for the wavelet analysis and processing of the signal, but the problem is seldom considered. In this paper, the adaptive design of the mother wavelet for CWT and WS is discussed, then a kind of wavelet neural network and its training algorithm are developed. The mother wavelet are chosen from the linear combinations of the "wavelet atoms" adaptively by network. Our purpose is to find a wavelet that is suitable for the signal analysis or representation, that is, to find a "satisfactory" wavelet for applications but maybe not the "global optimal solution" for the neural network. The LMS energy is a common criterion but not necessary. One can use another criterion for the special application. The numerical simulations show that our method is always effective and especially suitable for the choice of wavelets for the representation and analysis of the chirp-like signal. One can use another criterion for the special application. The numerical simulations show that our method is always effective and especially suitable for the choice of wavelets for the representation and analysis of the chirp-like signal. The further research in radar signal detection [8] shows that this method is effective to choose a good mother wavelet for radar detection problem. In fact, our approach can also be used for other signals (This has also been proved by our work [9].)

References

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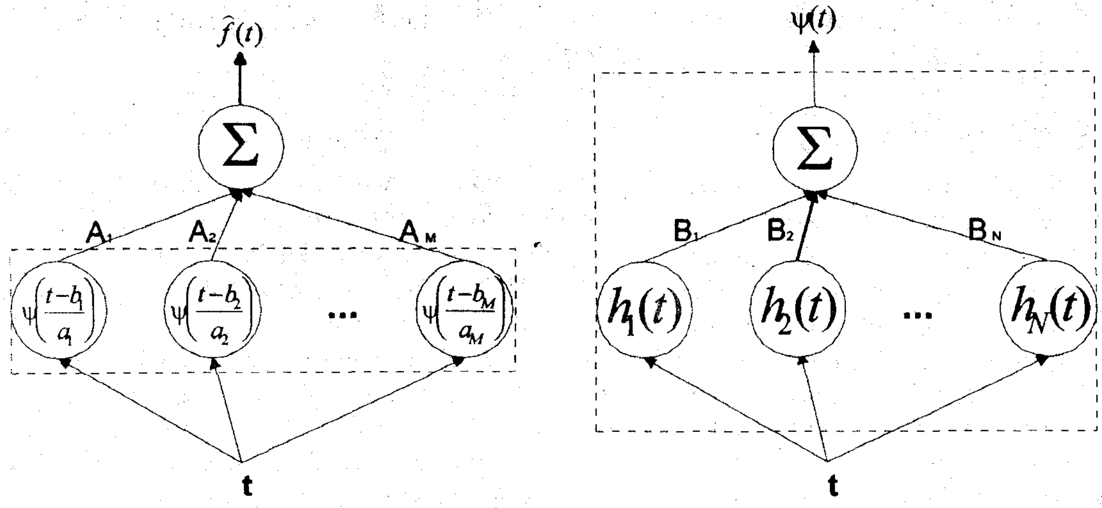

(a) (b)

Fig. 1

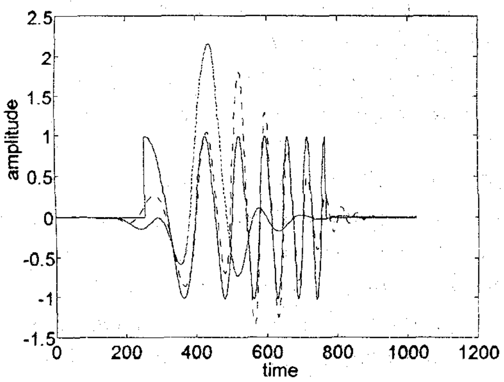

Fig. 2

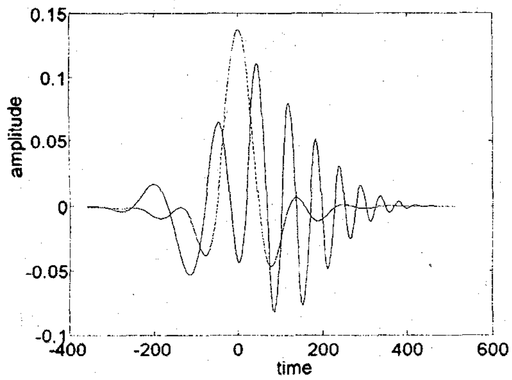

Fig. 3

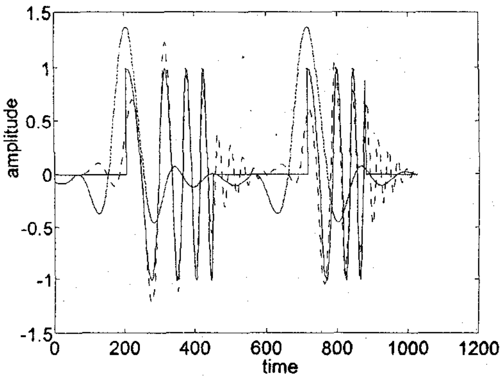

Fig. 4

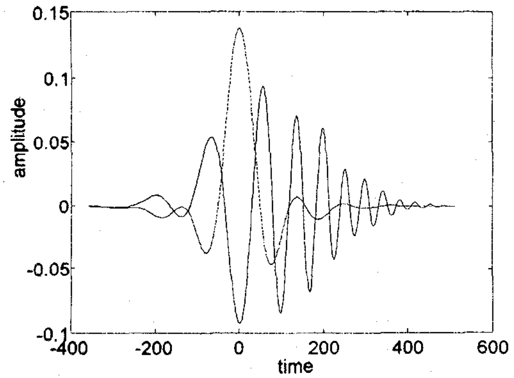

Fig. 5