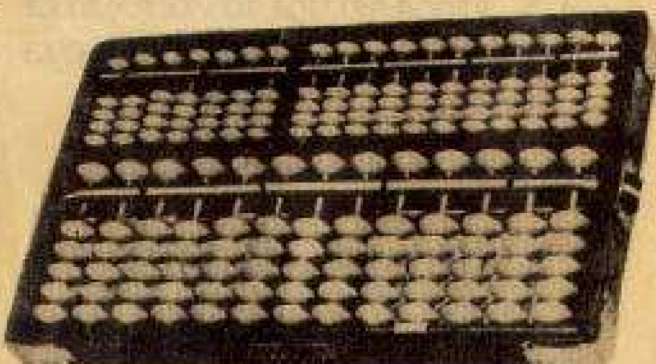
AUGUSTINE LEE

HOW TO LEARN LEE'S ABACUS

With Illustrations & Examples

By

Inventor: Lee Kai-chen



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CHAPTER 1.

INTRODUCTION

1. Merits of the Lee's Improved Abacus

The Lee's IMPROVED abacus may well claim to be the best of all kinds of existing arithmometer, in that:

(1) It makes speedy calculation—The findings of the international contests held in New York and Tokyo respectively in the year 1946 as well as the several contests held in Taipei in 1953 and 1956 have established the fact that even the electric calculator is no match to the abacus in solving arithmatical problems, let alone other calculating devices.

(2) It is error-proof—The very simple design of consistently assigning the value 5 to each bead counter on the upper deck and the value 1 to each bead counter on the lower deck, and the mechanical procedures called for in carrying out the operations, preclude any error derivable from either human or machine failures.

(3) It is easy for the beginner to learn—The notation system that each rod stands for a definite digit and each bead counter, a specific number, leaves no doubt whatsoever even in the mind of the very beginner. Especially when one is to carry out the operations for multiplication and division, one can master the methods employed on the Lee's IMPROVED abacus in a blink of an eye provided that one has an understanding on the methods employed in ordinary arithmatic, and do a much faster and better job than to write down all the steps on paper, which often becomes very tiresome, or to do it on the old-styled abacus, which involves many operations that deny direct reasoning.

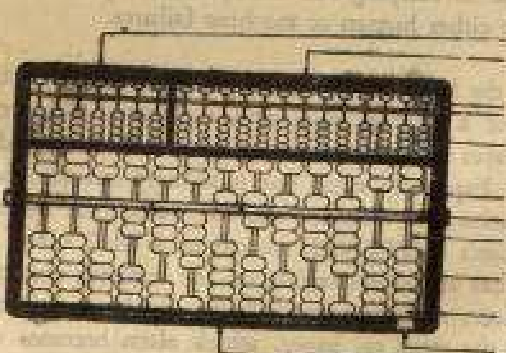
(4) It is handily carriable—The whole thing weighs only 1½ pound, and can be handled as easily as an ordinary office file-holder.

(5) It is easy to operate on—Unlike operating a calculating machine, which is necessarily heavy, one can lay the abacus on ones lap and complete the desired calculations without using a single sheet of paper or making a single stroke by the pencil, which are indispensable in the ordinary arithmatic practices

(6) It has beautiful outlook and durable construction—The bead counters are eye pleasing by themselves. They literally look as cute as so many gems. The beautifully painted wooden frame is extraordinarily durable. The Lee's abacus is, therefore, a valuable instrument and a treasurable ornament all in one.

(7) It is inexpensive—In spite of its multifarious merits, the Lee's abacus is obtainable at so very low a price that even a schooling teenager can afford to get one with his or her own pocket money.

2. Nomenclature



3. Notation system

Turn the place setting vernier to have the rod dot fall between any two rods. This done, the first rod to the left of the dot is ready for registering all numbers from 0 to 9, the second rod to its

left is to be used for registering all digits of the order of tens, the third rod to its left is to be used for registering all digits of the order of hundreds, etc. The rods to the right of the dot are to be used to register digits of the order of tenths, hundredths, thousandths, etc.

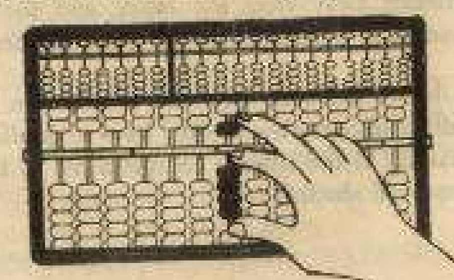
To register any given number on the abacus, remember: Each bead counter of the lower deck has the value of 1, while each bead counter of the upper deck has the value of 5. The number registered on the abacus in the above illustration should therefore read 12,345.6789.

The same notation system applies to both the left and the right A-O fields as well.

4. The way to move the bead counters

To use the Lee's improved abacus properly, it is essential to aim at speed and accuracy. Remarkable skill can be attained if the user is able to achieve coordination of his fingers, and his hand and brain. Accuracy and speed largely depend on skilful manipulation of the bead counters by the fingers.

To move the bead counters of the P-O field, we generally use the thumb, index finger or middle finger of the right hand. The three fingers are used in the following way:

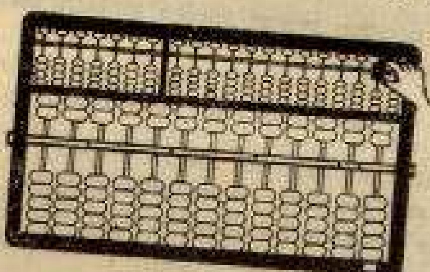


(1) The thumb controls the movement of the lower bead counters upwards to the beam.

(2) The index finger controls the movement of the lower bead counters downwards away from the beam.

(3) The middle finger controls the movement of the upper bead counters downwards to the beam, or away from the beam.

To move the bead counters of the left A-O field or the right A-O field, we generally use the thumb or index finger of the right hand. The two fingers are used in the following way:



(1) The thumb controls the movement of the lower bead counters upwards to the beam.

(2) The index finger controls the movement of the lower bead counters downwards away from the beam, and the upper bead counters downwards to the beam, or away from the beam.

Thus the thumb and each finger have their own special work to do. Naturally a learner may, at first, have to sacrifice speed for the sake of accuracy as the latter is indispensable in the operation of the Lee's improved abacus.

ADDITION

To make an addition on an abacus is simply to register the numbers to be added one after another in accordance with the notation system explained above, following a left-to-right direction as when one writes down the Arabic figures. So long as all digits are correctly placed, the final reading should be the sum in question. For all cases, the techniques involved in making additions are as follows:

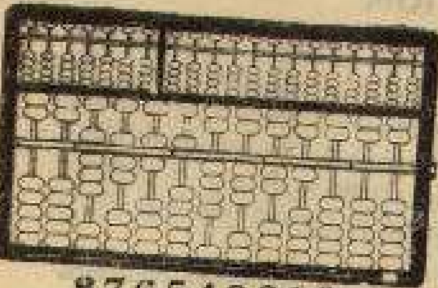
1. SIMPLE ADDING OF BEAD COUNTERS—This technique is used when, for instance, 6 is added to 2. In doing this, all one need to do is to move down one bead counter of the upper deck close to the beam and move another one bead counter of the lower deck up to complete the operation of adding 6, which, together with the two bead counters in the lower deck moved up to give the member 2 before carrying out the operation for addition, automatically shows the resultant sum 8.

Addition	Original number	Formula	Operations Involved	
			Bead counter of the lower deck	Bead counter of the upper deck
1	0.1.2.3.5.6.7.8.	1 = +1	+ 1	
2	0.1.2.5.6.7.	2 = +2	+ 2	
3	0.1.5.6.	3 = +3	+ 3	
4	0.5.	4 = +4	+ 4	
5	0.1.2.3.4.	5 = +5		+ 1
6	0.1.2.3.	6 = +6	+ 1	+ 1
7	0.1.2.	7 = +7	+ 2	+ 1
8	0.1.	8 = +8	+ 3	+ 1
9	0.	9 = +9	+ 4	+ 1

Remarks: "+" represents the operation of moving the bead counter(s) *close to the beam.*

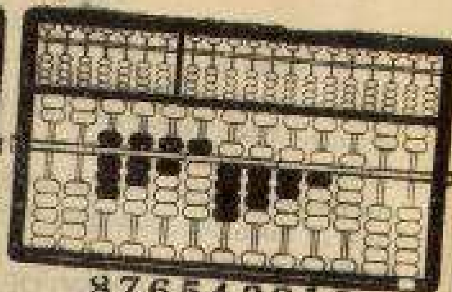
Example: 876,543,210+123,456,789=999,999,999

(1)



876543210

(2)



```
   876,543,210
+) 123,456,789
   999,999,999
```

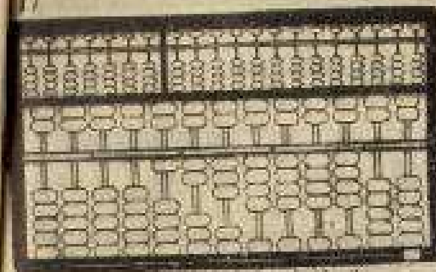
2. COMBINED ADDING-UP AND TAKING-OFF—This technique is called for when the original number registered on a rod is smaller than 5, but will become greater than 5 after the addition is made. The operation required is to move one bead counter of the upper deck down to the beam and one or more bead counters of the lower deck off the pack. For example, when 4 is to be added to 3, the operation required is to move one bead counter of the upper deck down to the beam and one bead counter off the original 3, leaving only two bead counters in the pack of the lower deck, which, together with the one bead counter of the upper deck, gives the answer 7.

Addition	Original number	Formula	Operations Involved	
			bead counter of the lower deck	bead counter of the upper deck
1	4.	1 = +5 − 4	− 4	+ 1
2	4, 3.	2 = +5 − 3	− 3	+ 1
3	4, 3, 2	3 = +5 − 2	− 2	+ 1
4	4, 3, 2, 1.	4 = +5 − 1	− 1	+ 1

Remarks: "—" represents the operation of moving the bead counter(s) *off the beam*.

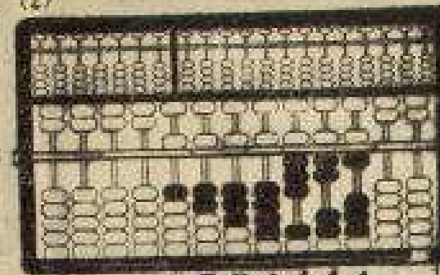
Example: 1,234,444+4,321,432=5,555,876

(1)



1,234,444.

(2)



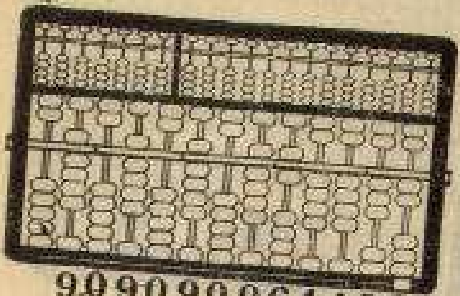
```
   1,234,444.
+) 4,321,432.
   5,555,876.
```

3. COMBINED TAKING-OFF AND PLACE ADVANCEMENT—This technique is called for when a sum greater than 10 occurs on a certain rod. The actual operation will then be to take off bead counters of the lower deck, or the upper deck, or both, and to add 1 on the rod to its left. The user may try to add 9 to 8 for practice.

Addition	Original number	Formula	Operations Involved		
			bead counter of the lower deck	bead counter of the upper deck	bead counter of the lower deck (on rod to the left)
1	9	1 = − 9 + 10	− 4	− 1	+ 1
2	8, 9	2 = − 8 + 10	− 3	− 1	+ 1
3	7, 8, 9.	3 = − 7 + 10	− 2	− 1	+ 1
4	6, 7, 8, 9.	4 = − 6 + 10	− 1	− 1	+ 1
5	5, 6, 7, 8, 9	5 = − 5 + 10		− 1	+ 1
6	4, 9	6 = − 4 + 10	− 4		+ 1
7	3, 4, 8, 9.	7 = − 3 + 10	− 3		+ 1
8	2, 3, 4, 7, 8, 9	8 = − 2 + 10	− 2		+ 1
9	1, 2, 3, 4, 6, 7, 8, 9	9 = − 1 + 10	− 1		+ 1

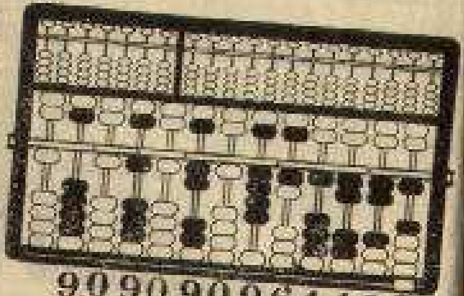
Example: 9,090,909,644.32+1,020,304,567.89=10,111,214,212.21

(1)



9,090,909,644,32

(2)



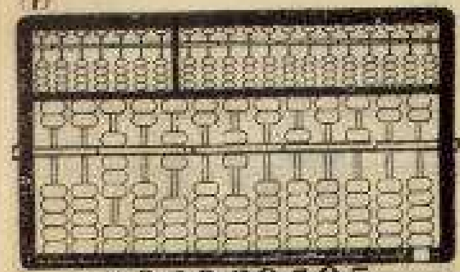
```
    9,090,909,644,32
+)  1,020,304,567,89
   10,111,214,212,21
```

4. COMBINED ADDING-UP, TAKING-OFF AND PLACE ADVANCEMENT—There are cases in which the actual operation required consists of adding-up in the lower deck, taking-off in the upper deck and again adding 1 on the rod for a higher order. This technique is called for when, for instance, 6 is to be added to 7.

Addition	Original Number	Formula	Operations Involved		
			Bead counter of the lower deck	Bead counter of the upper deck	Bead counter of the lower deck one rod to the left
6	5. 6. 7. 8.	6 = +1 −5 +10	+1	−1	+1
7	5. 6. 7.	7 = +2 −5 +10	+2	−1	+1
8	5. 6.	8 = +3 −5 +10	+3	−1	+1
9	5.	9 = +4 −5 +10	+4	−1	+1

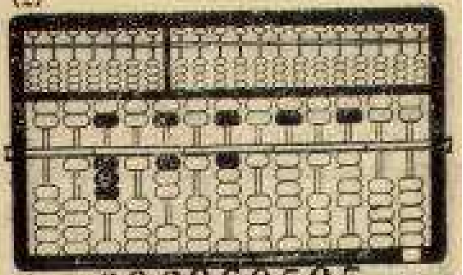
Example: 806,060,505 + 607,080,909 = 1,413,141,414

(1)



806,060,505.

(2)



```
    806,060,505.
+)  607,080,909.
  1,413,141,414.
```

It has been found to be a much simpler and faster process to make additions on an abacus than to do them on paper, especially when there are many numbers to be added together.

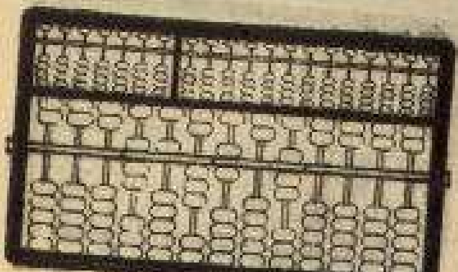
The actual procedures can be shown as follows:

```
    Original number
+)  1st addition
    1st sum
+)  2nd addition
    2nd sum
+)  3rd addition
    3rd sum
+)  4th addition
    ............
    ............
    ............

    (n − 1) th sum
+)  nth addition
    nth sum (Final result)
```

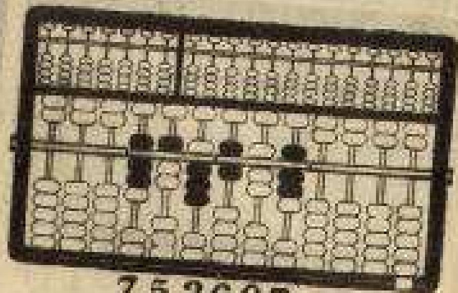
Example: 753,607+125,071+509,087+777,179=2,164,944

(1)



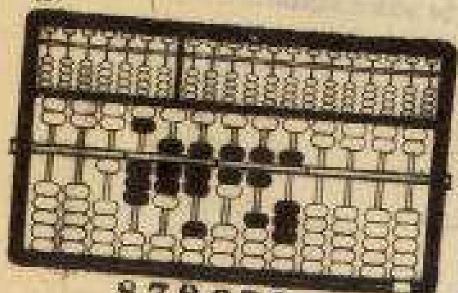
753607.

(2)



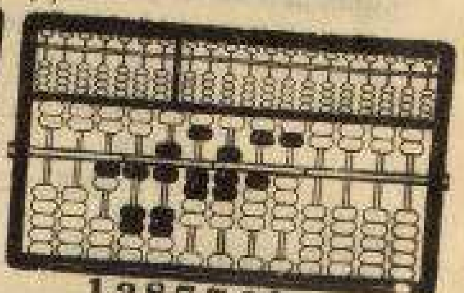
753607.
+) 125071.
878678.

(3)



878678.
+) 509087.
1,387765.

(4)



1387765.
+) 777179.
2,164944.

CHAPTER 3.

SUBTRACTION

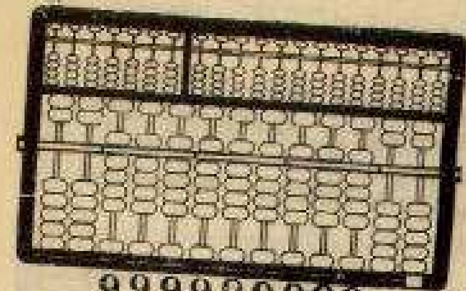
To subtract one number from another, first register the minuend on the abacus; then proceed to subtract place by place (rod by rod), starting from the left. The final reading on the abacus should give the answer required. For all cases, the techniques involved in making subtractions are as follows:

1. SIMPLE TAKING-OFF—This is achieved by simply taking off one or more bead counters from the pack in the lower deck, or upper deck, or sometimes, both. To subtract 7 from 9, for instance, all one has to do is to take off one bead counter in the upper deck and two in the lower deck from the original pack standing for 9, which is formed by one upper bead and four lower beads. The two bead counters left in the lower deck gives the remainder 2.

Sub-tractor	Original minuend	Formula	Operations Involved	
			Bead counter of the lower deck	Bead counter of the upper deck
1	1.2.3.4.6.7.8.9.	−1 = −1	−1	
2	2.3.4.7.8.9.	−2 = −2	−2	
3	3.4.8.9.	−3 = −3	−3	
4	4.9.	−4 = −4	−4	
5	5.6.7.8.9.	−5 = −5		−1
6	6.7.8.9.	−6 = −6	−1	−1
7	7.8.9.	−7 = −7	−2	−1
8	8.9.	−8 = −8	−3	−1
9	9.	−9 = −9	−4	−1

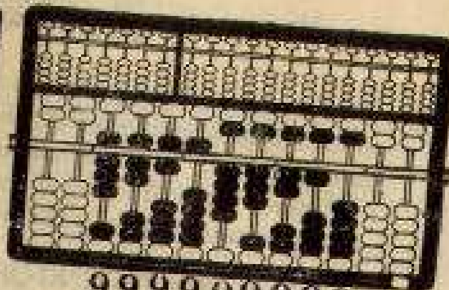
Example: 999,999,999−123,456,789=876,543,210

(1)



999,999,999.

(2)



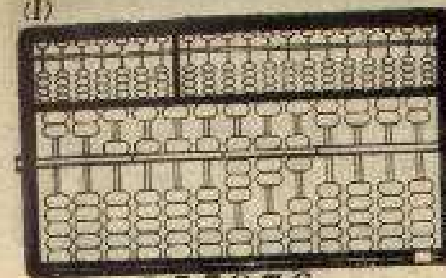
```
   999,999,999
-) 123,456,789
   876,543,210.
```

2. COMBINED ADDING-UP AND TAKING-OFF—This technique is called for when the number of bead counters in the lower deck is smaller than the subtractor, as in the case of 7-4. To carry out this operation, one has to add up 1 in the lower deck and take off 5 (one upper bead counter) in the upper deck, which means an actual reduction of 4, and gives the correct remainder 3.

Sub-tractor	Original minuend	Formula	Operations Involved	
			Bead counter of the lower deck	Bead counter of the upper deck
1	5.	−1 = +4 −5	+4	−1
2	5.6.	−2 = +3 −5	+3	−1
3	5.6.7.	−3 = +2 −5	+2	−1
4	5.6.7.8.	−4 = +1 −5	+1	−1

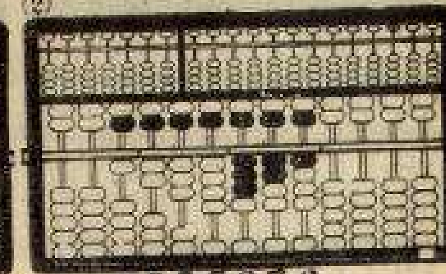
Example: 5,555,876—4,321,432=1,234,444

(1)



5,555,876.

(2)



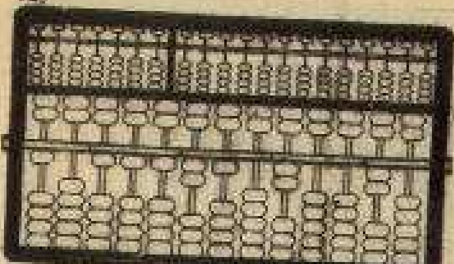
```
   5,555,876.
-) 4,321,432.
   1,234,444.
```

3. TAKING-OFF FROM A ROD OF HIGHER ORDER AND ADDING-UP—This is called for when the number on a certain rod is smaller than the supposed subtrahend as in the case of 13-4. To carry out this operation, one has to take off the one bead counter from the rod for the order of tens, move one bead counter up in the lower deck, and move down one bead counter in the upper deck, which means an actual reduction of 4, and gives the correct remainder 9.

Sub-tractor	Original minuend	Formula	Operations Involved		
			Bead counter of the lower deck [illegible]	Bead counter of the lower deck	Bead counter of the upper deck
1	0.	−1 = −10 + 9	−1	+4	+1
2	0.1.	−2 = −10 + 8	−1	+3	+1
3	0.1.2.	−3 = −10 + 7	−1	+2	+1
4	0.1.2.3.	−4 = −10 + 6	−1	+1	+1
5	0.1.2.3.4.	−5 = −10 + 5	−1		+1
6	0.5.	−6 = −10 + 4	−1	+4	
7	0.5.6.	−7 = −10 + 3	−1	+3	
8	0.1.2.5.6.7.	−8 = −10 + 2	−1	+2	
9	0.1.2.3.5.6.7.8.	−9 = −10 + 1	−1	+1	

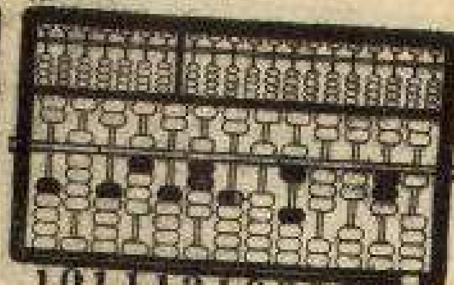
Example: 10,111,210,200.20—1,020,304,567.89=9,090,905,632.31

(1)



10,111,210,200,20

(2)



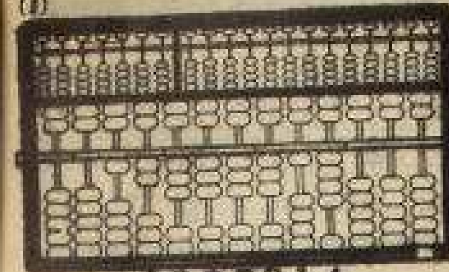
10,111,210,200,20
−) 1,020,304,567,89
9,090,905,632,31

4. COMBINED TAKING-OFF FROM A ROD OF HIGHER ORDER, ADDING-UP IN THE UPPER DECK AND TAKING-OFF IN THE LOWER DECK—This technique is called for when the number on a certain rod is smaller than the supposed subtrahend, but only in such cases as may be exemplified by 12-6. To carry out this subtraction, one takes off one bead counter from the rod for the order of tens, moves down one bead counter in the upper deck, and again takes off one bead counter in the lower deck, leaving the number 6 on the unit rod, which is the required answer. That this is correct can be confirmed by the fact that the algebraic sum resulted from the operation performed as above is 6, the subtractor.

Subtractor	Original minuend	Formula	Operations Involved		
			Bead counter of the lower deck one rod to the right	Bead counter of the lower deck	Bead counter of the upper deck
6	1, 2, 3, 4.	−6 = −10 + 5 − 1	−1	−1	+1
7	2, 3, 4.	−7 = −10 + 5 − 2	−1	−2	+1
8	3, 4.	−8 = −10 + 5 − 3	−1	−3	+1
9	4.	−9 = −10 + 5 − 4	−1	−4	+1

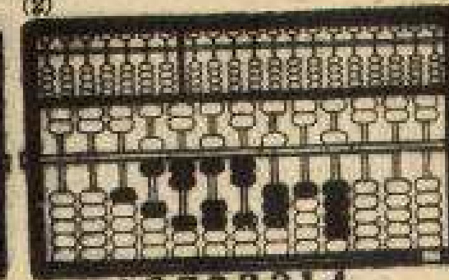
Example: 12,333,314−6,070,809=6,262,505

(1)



12,333,314.

(2)



12,333,314.
−) 6,070,809.
6,262,505.

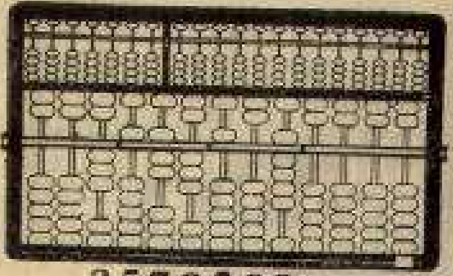
To carry out a series of subtractions, it is obviously easier to use an abacus than to make the calculations on paper.

The actual procedures can be shown as follows:

Original minuend
−) 1st subtractor
1st remainder
−) 2nd subtractor
2nd remainder
−) 3rd subtractor
3rd remainder
−) 4th subtractor
. . . .
. . . .
. . . .
(n − 1) th remainder
−) nth subtractor
nth remainder (Final result)

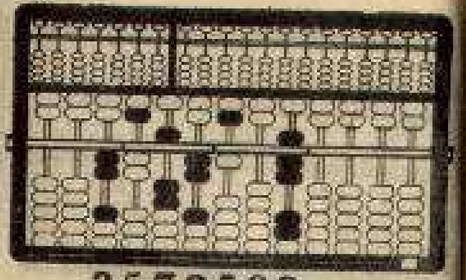
Example: 3,573,509−1,321,402−443,086−693,715=1,115,306

(1)



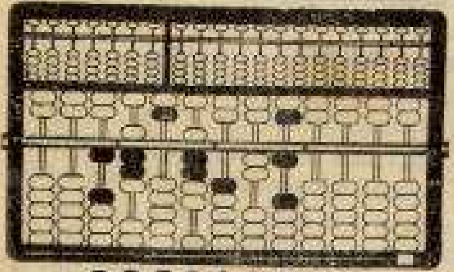
3,573,509.

(2)



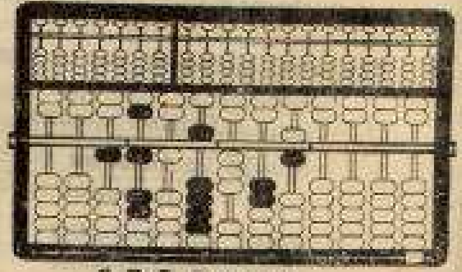
3,573,509.
−) 1,321,402.
2,252,107.

(3)



2,252,107.
−) 443,086.
1,809,021.

(4)



1,809,021.
−) 693,715.
1,115,306.

The user must have noticed that in carrying out additions and subtractions, one does not have to use the two auxiliary operation fields at all.

MULTIPLICATION

The method of multiplication on the Lee's abacus is similar to the method used in ordinary arithmatic, except for the fact that it can be carried out more rapidly, once the user gets used to it.

To carry out the operations for multiplication, one should follow carefully the steps prescribed below:

1. Register the multiplicant on the left A-O field.

2. Register the multiplier on the right A-O field.

3. Turn the place setting vernier of the P-O field so as to have the red dot fall between any two rods convenient for the operations. Move the indicator to a place immediately under the red dot.

4. Multiply the digits on the left A-O field, rod by rod, in a right-to-left order by the right-end digit on the right A-O field, and register the products one after another on the P-O field as in carrying out the operations for addition. Be careful, however, not to place the bead counters on wrong rods, and always remember to move one-rod to the left in registering the next product. The indicator should be consulted as a reliable guide for placement.

5. Use the second digit from right on the right A-O field as multiplier to operate on the digits registered on the left A-O field, rod by rod, in a right-to-left order, and register the products on the P-O field one after another as in carrying out operations for addition. To avoid misplacing bead counters, move the indicator one rod to the left before adding up the products.

6. Use the third digit from right on the right A-O field as multiplier to operate on the digits registered on the left A-O field as before. Remember to move the indicator one rod to the left to avoid misplacing.

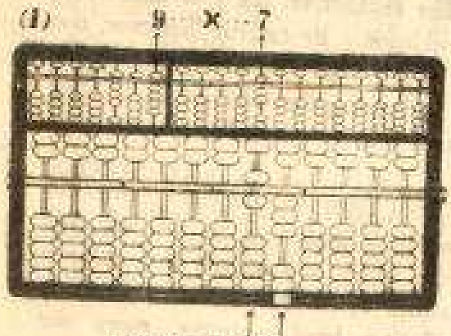
7. Continue the operations as above until every digit on the right A-O field has been used once.

8. The final reading on the P-O field gives the required product, and the place setting vernier now indicates the correct places for all digits.

Example 1. 859×7=6,013

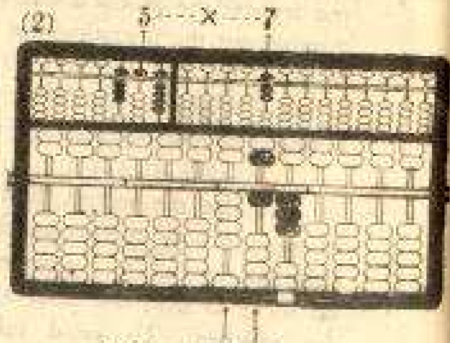
Multiplicant	859	……	Register on the left A-O field
Multiplier	×) 7	……	Register on the right A-O field
	·	←—	The position of the indicator
(9×7)	63	.. (1)	Add on the P-O field
(50×7)	35	.. (2)	"
(800×7)	+) 56	.. (3)	"
Product	6,013	……	The required product

(1)



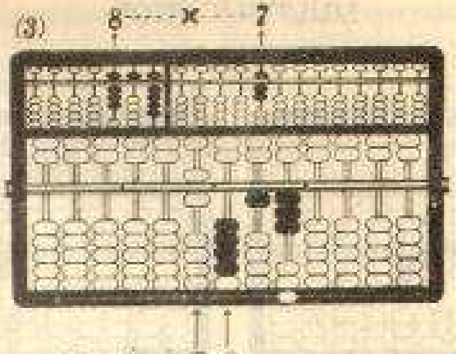
9×7=+63

(2)



5×7=+35

(3)

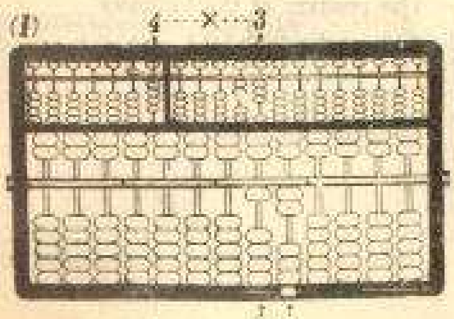


8×7=+56

Example 2. 54×23=1,242

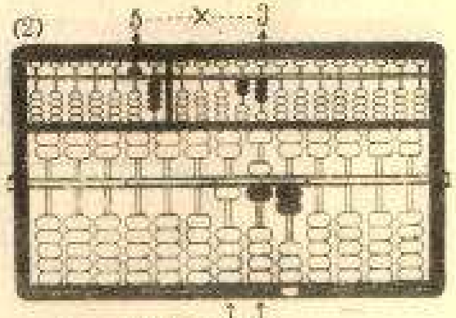
Multiplicant	54	……	Register on the left A-O field
Multiplier	×) 23	……	Register on the right A-O field
	·	←—	The position of the indicator
(4×3)	12	.. (1)	Add on the P-O field
(50×3)	15	.. (2)	"
	·	←—	The position of the indicator
(4×20)	8	.. (3)	Add on the P-O field
(50×20)	+) 10	.. (4)	"
Product	1,242	……	The required product

(1)



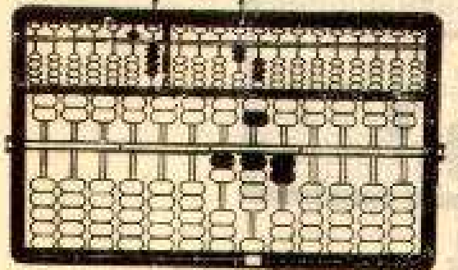
4×3=+12

(2)



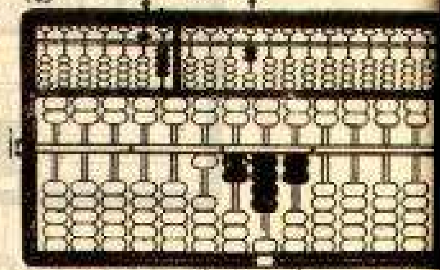
5×3=+15

(3) 4 × 2



4 × 2 = +8

(4) 5 × 2

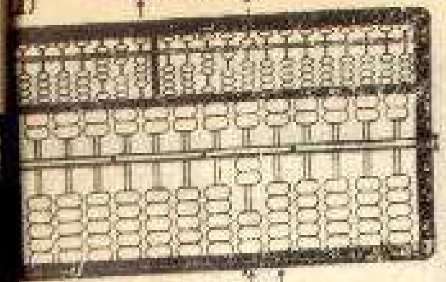


5 × 2 = +10

Example 3. 864×315=272,160

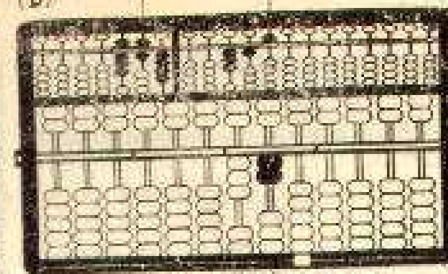
Multiplicant	864	......	Register on the left A-O field
Multiplier	×) 315	......	Register on the right A-O field
	•	←—	The position of the indicator
(4×5)	20	.. (1)	Add on the P-O field
(60×5)	30	.. (2)	"
(800×5)	40	.. (3)	"
	•	←—	The position of the indicator
(4×10)	4	.. (4)	Add on the P-O field
(60×10)	6	.. (5)	"
(800×10)	8	.. (6)	"
	•	←—	The position of the indicator
(4×300)	12	.. (7)	Add on the P-O field
(60×300)	18	.. (8)	"
(800×300	+) 24	.. (9)	"
Product	272,160	......	The required product

(1) 4 × 5



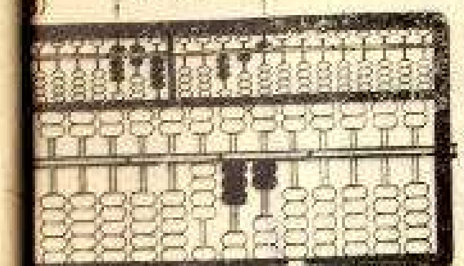
4 × 5 = +20

(2) 6 × 5



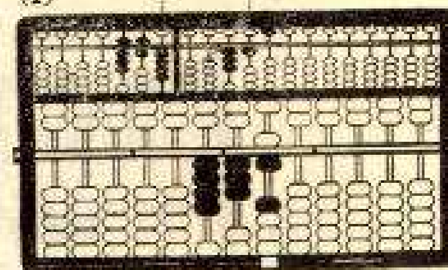
6 × 5 = +30

(3) 8 × 5



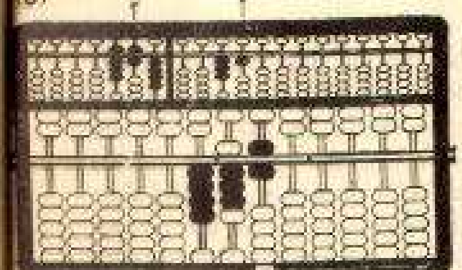
8 × 5 = +40

(4) 4 × 1



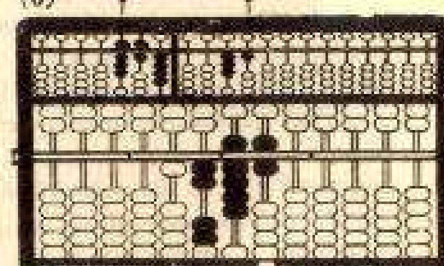
4 × 1 = +4

(5) 6 × 1



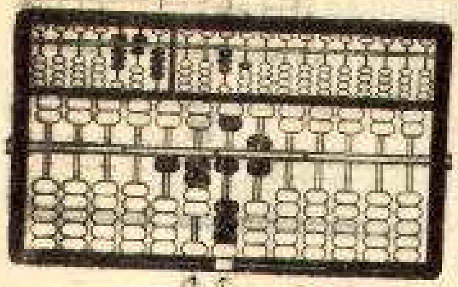
6 × 1 = +6

(6) 8 × 1



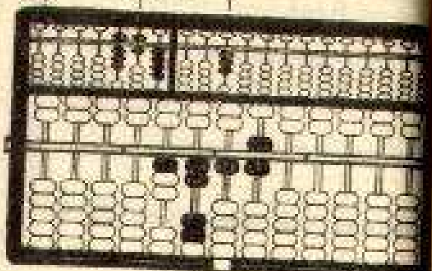
8 × 1 = +8

(7) 4 × 3



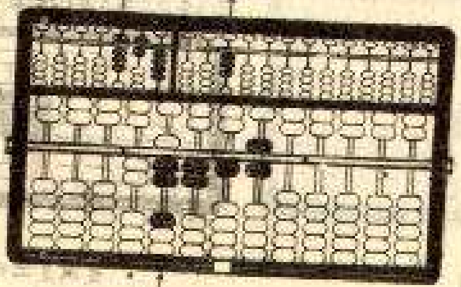
4×3=+12

(8) 6 × 3



6×3=+18

(9) 8 × 3



8×3=+24

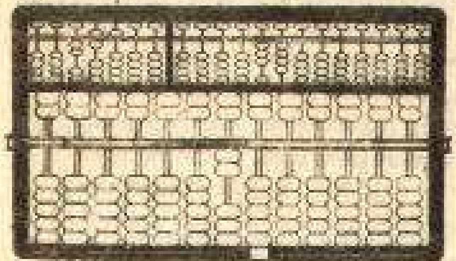
9. When decimal fractions occur in the multiplier, or the multiplicant, or both, whether they are of the improper or the proper kind, the procedures to follow are just the same as with integrals. However, special attention should be paid to the different handling of the indicator throughout the operations. If the right-end digit on the right A-O field is in the order of tenths, then set the indicator one-rod to the right of the red dot on the vernier, and move it one-rod each to the left as one goes on dealing with different digits on the right A-O field in the way prescribed above. If the right-end digit is in the order of hundredths, then set the

indicator two-rod to the right of the red dot on the vernier before going ahead with the operations. The user may rely on the same reasoning in setting the indicator when there should be multipliers having decimal fractions containing more than three decimal places. After all products have been properly registered on the P-O field, the red dot on the place setting vernier—now used as the decimal point—should be moved one-rod to the left, provided that the original multiplicant contains a decimal fraction of one decimal place. It should be moved two-rod to the left, provided that the multiplicant contains a decimal fraction of two decimal places. The user may apply the same reasoning to deal with multiplicants containing decimal fractions of more than three decimal places. The final reading on the P-O field as qualified by the place setting vernier will be the answer required.

Example 1. 26.5×3.4=90.1

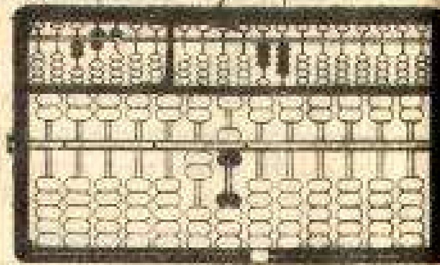
Multiplicant	26.5	......	Register on the left A-O field
Multiplier	×) 3.4	......	Register on the right A-O field
	•	⟵	The position of the indicator
(0.5×0.4)	2 0	.. (1)	Add on the P-O field
(6×0.4)	24	.. (2)	"
(20×0.4)	8	.. (3)	"
	•	⟵	The position of the indicator
(0.5×3)	15	.. (4)	Add on the P-O field
(6×3)	18	.. (5)	"
(20×3)	+) 6	.. (6)	"
Product	901.0	.. (7)	Move the red dot on the place setting vernier one-rod to the left
	90.10	......	The required product

(1) 5 × 4



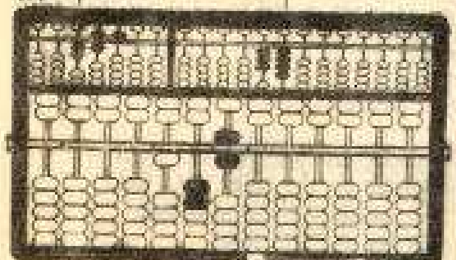
5×4=+20

(2) 6 × 4



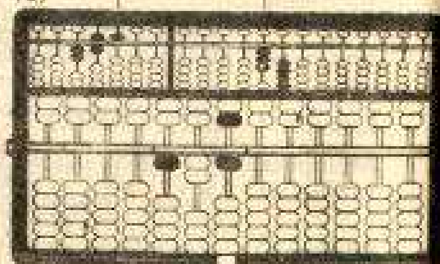
6×4=+24

(3) 2 × 4



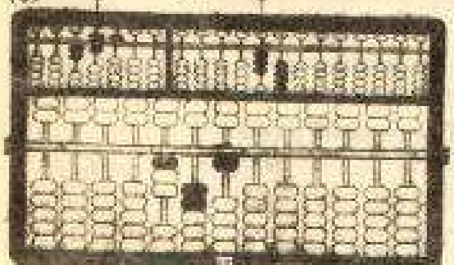
2×4=+8

(4) 5 × 3



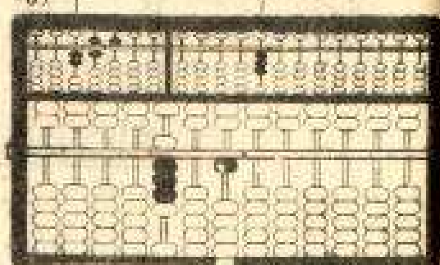
5×3=+15

(5) 6 × 3



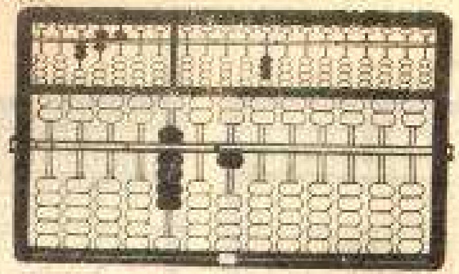
6×3=+18

(6) 2 × 3



2×3=+6

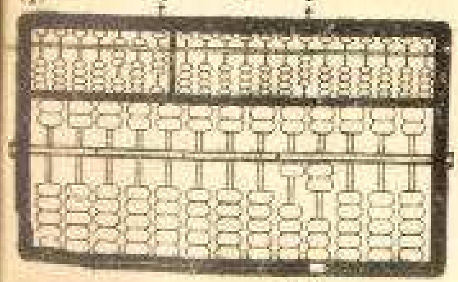
(7)



Example 2. 0.753×0.04=0.03012

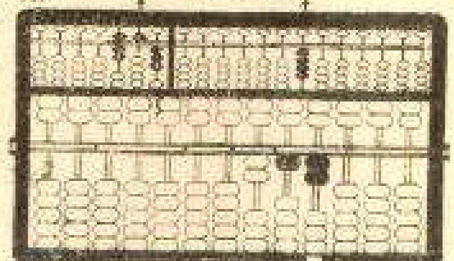
Multiplicant	0.753	……	Register on the left A-O field
Multiplier	×) 0.04	……	Register on the right A-O field
	·	←——	The position of the indicator
(0.003×0.04)	12	.. (1)	Add on the P-O field
(0.05×0.04)	2 0	.. (2)	"
(0.7×0.04)	+) 28	.. (3)	"
Product	0030.12		
	·←·	.. (4)	Move the red dot on the place setting vernier three rod to the left
	0.03012	……	The required product

(1) 3 × 4



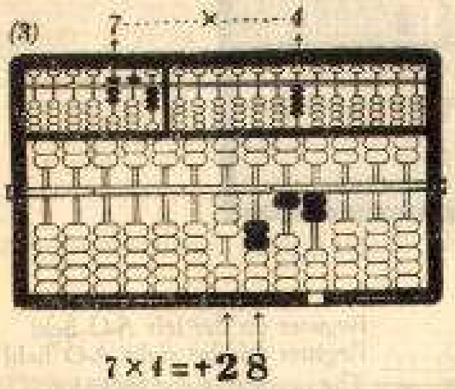
3×4=+12

(2) 5 × 4

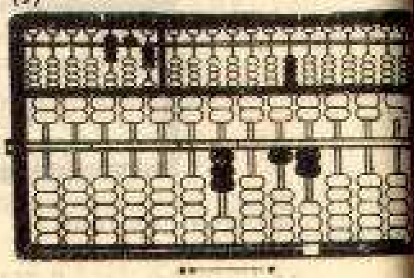


5×4=+20

(3)



(4)



DIVISION

The method of division on the Lee's abacus is also based on the same principle as that for finding quotients in ordinary arithmatic. However, the former has been found less strenuous than the latter. To carry out operations of division, one must be thoroughly familiar with the operations of subtraction which are fundamental to the work.

The following step are to be followed carefully:

1. Move the place setting vernier so as to have the red dot fall between any two rods on the P-O field, taking into consideration the number of digits in the dividend.

2. Place the dividend properly on the P-O field.

3. Place the divisor properly on the left A-O field.

4. When there is only one digit in the divisor, then set the indicator right under the rod representing the place of the highest order in the dividend. Should the digit registered on this rod be smaller than the divisor, then move the indicator one rod to the right to allow two digits to be operated upon. When the divisor contains two digits, set the indicator under the rod representing the place of the second highest order in the dividend. Should the number formed by the digits registered on the two left-end rods be smaller than the divisor, then move the indicator one more rod to the right to allow three digits to be operated upon. By the same reasoning the user will have no difficulty in placing the indicator correctly when the divisor contains more than three digits.

5. The place order the indicator shows on the P-O field is to be understood as the highest place order the quotient may have, i.e., when the indicator comes under the unit rod the quotient must be a number under 10; when the indicator comes under the rod representing the order of tens, the quotient then must be under 100; etc.

6. The quotient is to be found digit by digit. Whenever one digit of the quotient is found, it should be registered on the right A-O field in accordance with the place order suggested by the indicator on the P-O field, and the product obtained by multiplying this digit of the quotient to the divisor should be used as a subtrahend to be deduced from the dividend on the P-O field.

7. Special attention should be paid to the fact that the digit of the lowest order in the product falls exactly where the indicator is as the latter moves along throughout the process of the operations.

8. In subtracting the product of the quotient and the divisor from the dividend, the same procedures should be followed as in carrying out the operations for multiplication, except that all operations of addition should now be replaced by operations of subtraction.

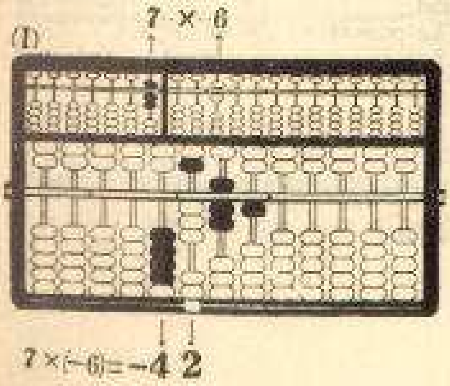
9. After the subtraction is made, one can then proceed to find the next digit in the quotient. The procedures to follow are exactly the same as in finding the first digit of the quotient.

10. Repeat the procedures until the dividend on the P-O field is exhausted. The number represented by the bead counters on the right A-O field should be the quotient in question.

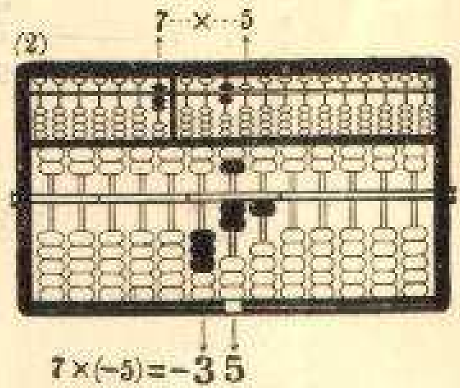
Example 1. 4,571÷7=653

....	Register the divisor on the left A-O field
653	Register the quotient on the right A-O field
→ 7) 4,571	Register the dividend on the P-O field
· ←———	The position of the indicator
—) 42	(1) Subtract from the P-O field
371	
· ←———	The position of the indicator
—) 35	(2) Subtract from the P-O field
21	
· ←———	The position of the indicator
—) 21	(3) Subtract from the P-O field
0	

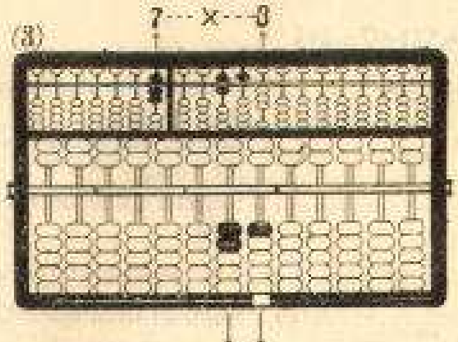
(1)



(2)



(3)

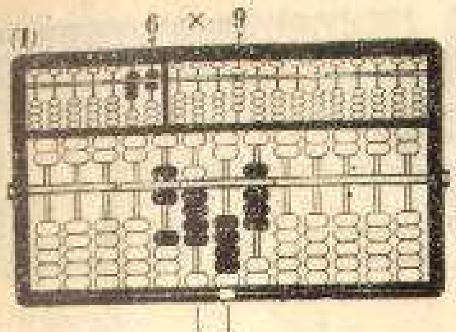


$7 \times (-8) = -21$

Example 2. 7,448 ÷ 76 = 98

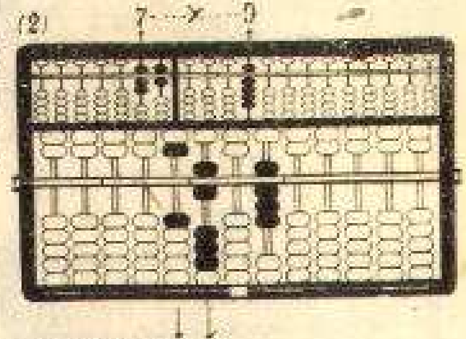
	....	Register the divisor on the left A-O field
98	....	Register the quotient on the right A-O field
→ 76) 7,448	....	Register the dividend on the P-O field
•	←———	The position of the indicator
54	.... (1)	Subtract from the P-O field
−) 63	.... (2)	"
608		
•	←———	The position of the indicator
48	.... (3)	Subtract from the P-O field
−) 56	.... (4)	"
0		

(1)



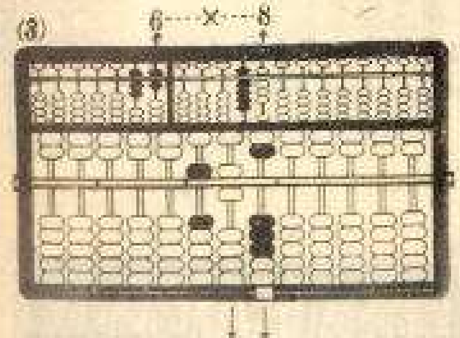
$6 \times (-9) = -54$

(2)



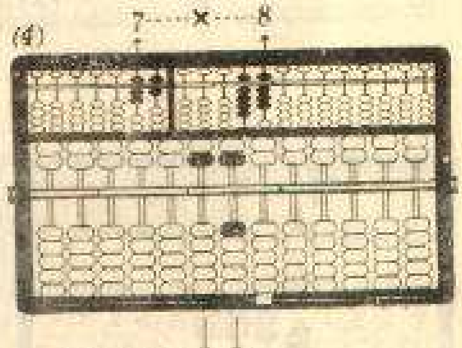
$7 \times (-9) = -63$

(3)



$6 \times (-8) = -48$

(4)

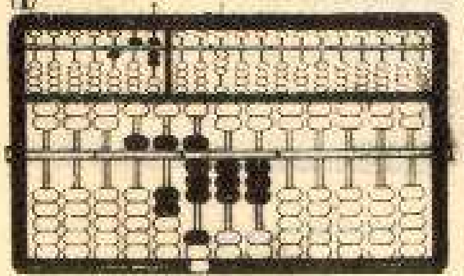


$7 \times (-8) = -56$

Example 3. 57,933÷157=369

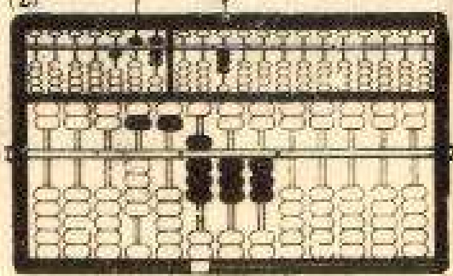
....	Register the divisor on the left A-O field
369	Register the quotient on the right A-O field
→ 157) 57,933	Register the dividend on the P-O field
· ←	The position of the indicator
21 .... (1)	Subtract from the P-O field
15 .... (2)	"
−) 3 .... (3)	"
10,833	
· ←	The position of the indicator
42 .... (4)	Subtract from the P-O field
30 .... (5)	"
−) 6 .... (6)	"
1,413	
· ←	The position of the indicator
63 .... (7)	Subtract from the P-O field
45 .... (8)	"
−) 9 .... (9)	"
0	

(1) 7 × 3



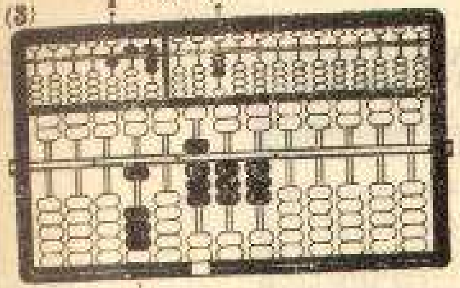
7×(−3)=−2 1

(2) 5 × 3



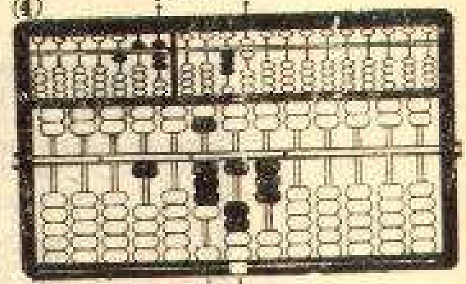
5×(−3)=−1 5

(3) 1 ··· × ··· 3



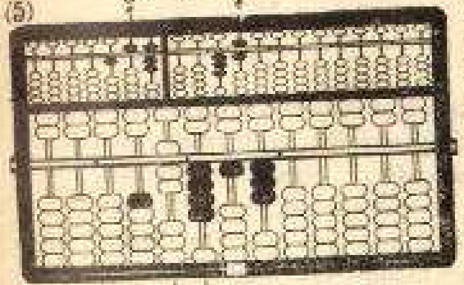
1×(−3)=−3

(4) 7 ··· × ··· 6



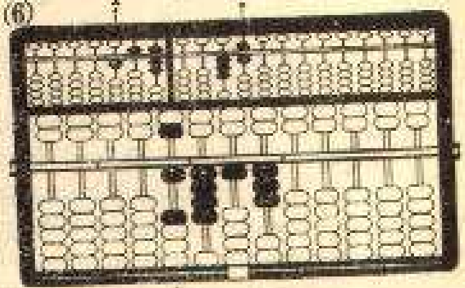
7×(−6)=−4 2

(5) 5 ··· × ··· 6



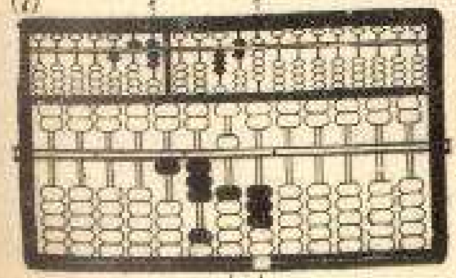
5×(−6)=−3 0

(6) 1 ··· × ··· 6



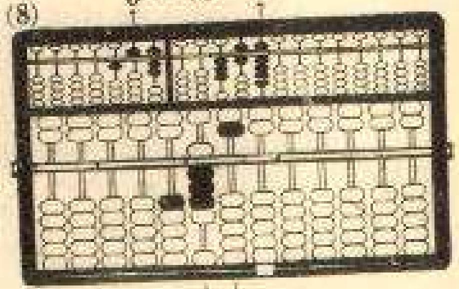
1×(−6)=−6

(7) 7 ··· × ··· 9



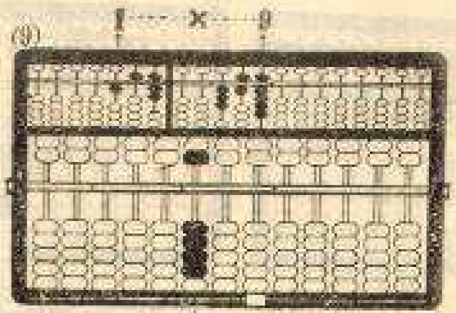
7×(−9)=−6 3

(8) 5 ··· × ··· 9



5×(−9)=−4 5

(9)



1×(−9)=−9

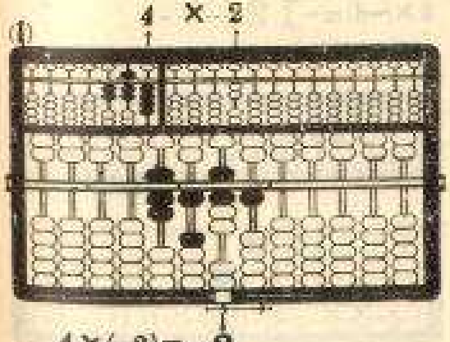
11. In handling problems in which the given divisor is a proper or improper decimal fraction, one has to transform the divisor into an integral before proceeding with the operations. To do this, one simply has to move the place setting vernier one-rod to the right if there be only one decimal place in the divisor, two-rod to the right if there be two decimal places in the divisor, three-rod to the right if there be three decimal places, etc, and do the same with the dividend, both of which are automatically achieved by moving the place setting vernier as many rods to the right in the P-O field as there are decimal place in the divisor.

12. After the decimal point in the divisor is removed, the operations for finding the quotient may then be carried out in the same way as prescribed above.

Example 1. 72.61÷2.74=26.5

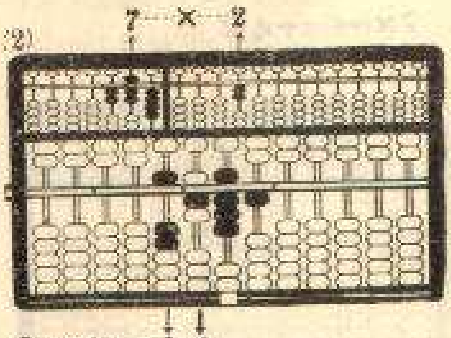
2.74.) 72.61 → →			Move the red dot on the place setting vernier two-rod to the right
....			Register the divisor on the left A-O field
26.5	....		Register the quotient on the right A-O field
→ 274) 7,261.	....		Register the dividend on the P-O field
•	←——		The position of the indicator
8	......	(1)	Subtract from the P-O field
14	......	(2)	"
−) 4	......	(3)	"
1,781.			
•	←——		The position of the indicator
24	......	(4)	Subtract from the P-O field
42	......	(5)	"
−) 12	......	(6)	"
137.0			
•	←——		The position of the indicator
20	......	(7)	Subtract from the P-O field
35	......	(8)	"
−) 10	......	(9)	"
0			

(1)

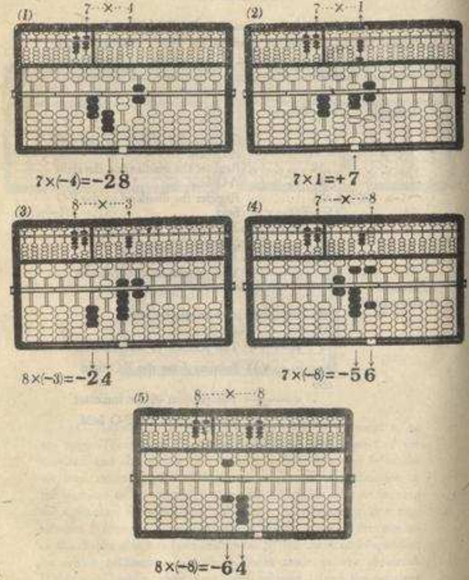


4×(−2)=−8

(2)



7×(−2)=−14



CHAPTER 6.

EXTRACTION OF SQUARE ROOTS

To extract square roots on the Lee's abacus is a much simple. job than to do it on paper. The steps to follow are set forth below:

1. Divide all digits of the square number into groups containing two digits each. In doing this, start from the decimal point toward the right for the decimal fraction part and toward the left for the integral part. Consider it a group by itself even when there is only one digit at the left-end. But, when there is only one digit left at the right-end of the decimal fraction part, "O" shall be supplemented to complete the two digit group.

2. For each group in the square number there will be one digit in the root to be found. There are, therefore, as many digits in the square root as there are groups in the square number. By examining the grouping of the square number, one can tell of what place order the first digit of the root will be.

3. Before starting to operate, the place setting vernier should be turned to a position most convenient for the calculation.

4. Register he square number on the P-O field in reference to the place setting vernier.

5. Move the indicator to mark out the first group from the left.

6. When the square number has only one group for the integral part, the root must be a number under 10. When there are two groups for the integral part of the square number, the root must be a number less than 100. When there are three groups

in the square number, there must also be three integral places in the root. The same reasoning holds good for all cases.

7. In making the calculation, first add one bead counter up in the left A-O field in accordance with the placement suggested by the indicator, and subtract 1 from the first group in the P-O field.

8. Secondly, add 2 to the original 1 on the left A-O field, and subtract the sum 3 from the first group on the P-O field; add 2 again to the original 3 on the left A-O field, and subtract the sum 5 from the first group on the P-O field; add 2 again to the original 5 on the left A-O field, and subtract the sum 7 from the first group on the P-O field;—repeat these operations until the remainder of the first group becomes smaller than the sum on the left A-O field.

9. Thirdly, add up one bead counter each to the first and the second digit from the left on the left A-O field (symbol 1.1 will be used hereafter), move the indicator two-rod to the right, and subtract the sum on the left A-O field from the number formed by the first and the second group of the square nunter. There may be cases when nothing is left on the first group. In such cases, just subtract the sum on the left A-O field from the number formed by the second group alone.

10. Fourthly, add 2 to the left A-O field, and subtract the sum from the P-O field; add 2 again to the left A-O field, and subtract the sum from the P-O field; and so on until the remainder of the second group on the P-O field becomes smaller than the sum on the left A-O field.

11. Repeat the operations described in Items 9 and 10 when necessary.

12. Fifthly, when all bead counters on the P-O field have been taken off the beam as a result of the operations, add 1 to the sum on the left A-O field and divide it up by 2. The quotient thus obtained is the root required.

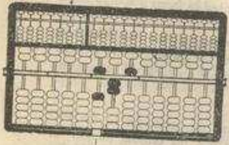
13. There may be cases when the remainder in the P-O field is inexhaustible. This means that the original number on which the operations of root extraction is made is not a perfect square number. Even so, the root obtained by carrying out the operations described in Item 12 gives a good approximation.

Example:

$$\sqrt{625} = 25$$

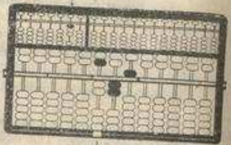
Figure No	"+" operations on the left A-O field					"−" operations on the P-O field			
				1st digit	2nd digit	First group		Second group	
1									
2				1			1		
3				2			3		
4				1	1			4	1
5					2			4	3
6					2			4	5
7					2			4	7
					2			4	9
			+	4	9	−	6	2	5
8	49 + 1 = 50								
9-10	50 ÷ 2 = 25								

(1) +1



−1

(2) +2

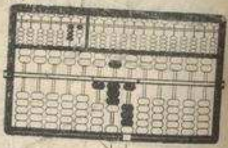


−3

(3) +1.1

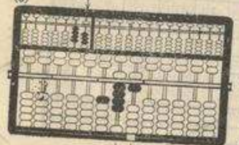
−41

(4) +2



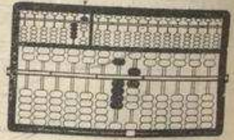
−43

(5) +2



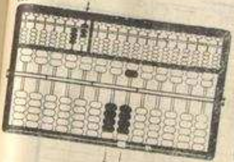
−45

(6) +2



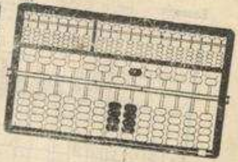
−47

(7) +2



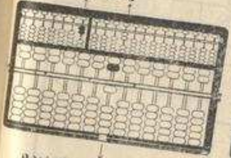
−49

(8)



$49+1=50$

(9) 2 ··· × ··· 2



$2\times(-2)=-4$

(10) 2 ··· × ··· 5

$2\times(-5)=-10$

14. In carrying out the operations described in Item 9 above, if the minuend in the P-O field is still smaller than the sum in the left A-O field after having gone through the procedures of adding 1.1 to the left A-O field and moving the indicator two-rod to the right on the P-O field, one has then to move the indicator two-rod more to the right, and add 1.01 instead of 1.1 to the left A-O field.

Example: $\sqrt{1.664.64} = 40.8$

Operation Procedures	"+" operations on the left A-O field			"−" operations on the P-O field					
	1st digit	2nd digit	3rd digit	First group		Second group		Third group	
1	1				1				
2	2				3				
3	2				5				
4	2				7				
5	1	0	1			0	8	0	1
6			2				3	0	3
7			2				8	0	5
8			2				8	0	7
9			2				8	0	9
10			2				8	1	[illegible]
11			2				8	1	3
12			2				8	1	5
	+ 8 1 . 5			−1, 6 6 4 . 6 4					
13	81.5 + 0.1 = 81.6								
14–15	81.6 ÷ 2 = 40.8								

15. After having followed the prescriptions in Item 14, the minuend in the P-O field may still be smaller than the sum in the left A-O field. In such cases, the indicator should be moved again two-rod to the right and, instead of 1.1 or 1.01, the number to be added to the left A-O field should then be 1.001. The same reasoning applies to all cases of similar nature.

Example: $\sqrt{1,004,004} = 1,002$

Operation Procedures	"+" operations on the left A-O field				"−" operations on the P-O field							
	1st digit	2nd digit	3rd digit	4th digit	First group		Second group		Third group		Fourth group	
1	1											
2	1	0	0	1		1						
3							0	0	2	0	0	1
				2					2	0	0	3
	+ 2, 0 0 3				− 1,0 0 4,0 0 4							
4	2,003 + 1 = 2,004											
5–6	2,004 ÷ 2 = 1,002											

CHAPTER 7.

EXTRACTION OF CUBIC ROOTS

The operations involved in finding cubic roots on the Lee's abacus are but those of additions and subtractions. To extract cubic roots, follow the steps set forth below:

1. Divide all digits of the cubic number into groups of three digits each. In doing this, start from the decimal point toward the right for the decimal fraction part and toward the left for the integral part. Consider it a group by itself even when there is only one, or two, digit left at the left-end. However, when there is only one, or two, digit left at the right-end of the decimal fraction part, "O" must be supplemented to make the group contain three digits.

2. For each group in the cubic number, there will be one digit in the root to be found. There are, therefore, as many digits in the cubic root as there are groups in the cubic number. By examining the grouping of the cubic number, one can tell of what place order the first digit of the root will be.

3. Before starting to operate, the place setting vernier should be turned to a position most convenient for the calculation.

4. Register the cubic number on the P-O field in reference to the place setting vernier.

5. Move the indicator to mark out the first group from the left.

6. When the cubic number has only one group for the integral part, the root must be a number less than 10. When there are two groups for the integral part of the cubic number, the

root must be a number under 100. When there are three groups for the integral part of the cubic number, the root must then be a number under 1,000. The same reasoning holds good for all cases.

7. In making the calculation, first add one bead counter up to the left A-O field in accordance with the placement suggested by the indicator (hereafter referred to as "L-number"), add one bead counter up to the right A-O field (hereafter referred to as "R-number"), and then substract 1 from the first group on the P-O field.

8. Secondly, add 1 to the L-number, add the resultant L-number to the R-number, add 2 to the L-number, add the resultant L-number (4) to the R-number, and subtract the resultant R-number (7) from the first group on the P-O field. Then, add 1 to the L-number, add the resultant L-number to the R-number, add 2 to the L-number, add the resultant L-number to the R-number, and subtract the resultant R-number from the first group on the P-O field. Repeact these operations until the remainder of the first group of the cubic number becomes smaller than the corresponding R-number.

9. Thirdly, add 1 to the L-number and add the resultant L-number to the R-number.

10. Fouthly, add up one bead counter each to the first and the second digit from the left of the L-number (symbol 1.1 will be used hereafter) add the resultant L-number to the R-number, lowering the place order of the former by "two" in this addition, move the indicator three-rod to the right, and then subtract the R-number from the number formed by the first and the second group of the cubic number.

11. Fifthly, add 1 to the second digit of the L-number, add the resultant L-number to the R-number, add 2 to the second digit of the L-number, add the resultant L-number to the R-number, subtract the resultant R-number from the second group of the cubic number on the P-O field. Repeat these operations until the remainder of the second group of the cubic number on the P-O field becomes smaller than the corresponding R-number.

12. Continue the operations described in Items 10 and 11 when necessary.

13. Sixthly, when all bead counters in the P-O field have been taken off the beam as a result of the operations, add 2 to the L-number and divide it up by 3. The quotient thus obtained will be the root required.

Example:

$$\sqrt[3]{12,167} = 23$$

Figure No.	"+" operations on the left A-O field (L-number)		"+" operations on the right A-O field (R-number)		"−" operations on the P-O field	
	1st digit	2nd digit	Order 1	Order 2	First group	Second group
1	1					
2	1		1			
3	1		2		1	
4	2		4			
5	1		5		7	
6	1	1		6 1		
7		1		6 2	1 2	6 1
8		2		6 4		
9		1		6 5	1 3	8 7
		2		6 7		
	+ 6	7			1 5	1 9
					−1 2,	1 6 7
10	67 + 2 = 69					
11-12	69 ÷ 3 = 23					

11. **Fifthly**, add 1 to the second digit of the L-number, add the resultant L-number to the R-number, add 2 to the second digit of the L-number, add the resultant L-number to the R-number, subtract the resultant R-number from the second group of the cubic number on the P-O field. Repeat these operations until the remainder of the second group of the cubic number on the P-O field becomes smaller than the corresponding R-number.

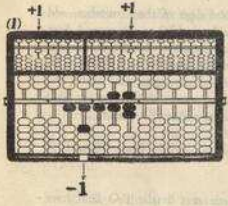
12. **Continue** the operations described in Items 10 and 11 when necessary.

13. **Sixthly**, when all bead counters in the P-O field have been taken off the beam as a result of the operations, add 2 to the L-number and divide it up by 3. The quotient thus obtained will be the root required.

Example:

$$\sqrt[3]{12,167} = 23$$

Figure No.	"+" operations on the left A-O field (L-number)			"+" operations on the right A-O field (R-number)		"−" operations on the P-O field	
		1st digit	2nd digit	Order 1	Order 2	First group	Second group
1		1					
2		1		1		1	
3		2		2			
4		1		4		7	
5		1		5			
6			1		6 1	1	2 6 1
7			1		6 2		
8			2		6 4	1	3 8 7
9			1		6 5		
			2		6 7	1	5 1 9
		+ 6	7			−1 2,	1 6 7
10	67 + 2 = 69						
11–12	69 ÷ 3 = 23						

(1)


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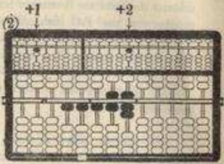

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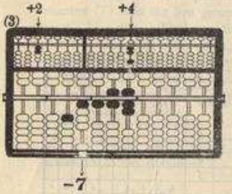

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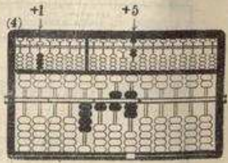

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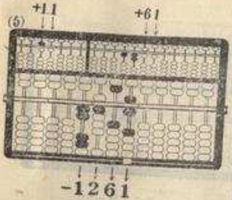

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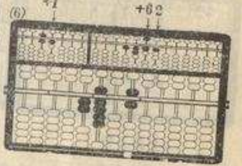

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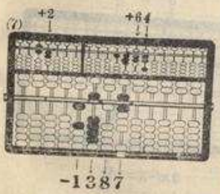

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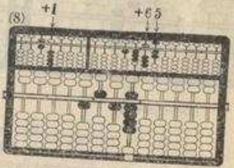

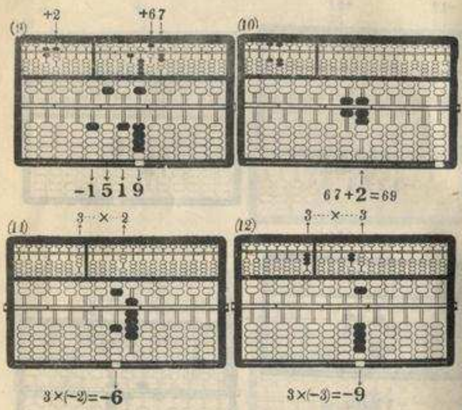

14. In carrying out the operations described in Item 10 above, if the minuend in the P-O field is still smaller than the R-number after having gone through the procedures of adding 1.1 to the L-number and moving the indicator three-rods to the right on the P-O field, one has then to move the indicator three-rod more to the right, add 1.01 instead of 1.1 to the L-number, and add the resultant L-number to the R-number, lowering the former's place order by "four" instead of "two" in this addition.

Example: $\sqrt[3]{8,615.125} = 20.5$

operation procedures	"+" operations on the left A-O field				"+" operations on the right A-O field			"−" operations on the P-O field		
		1st digit	2nd digit	3rd digit	Order 1	Order 2	Order 3	1st group	2nd group	3rd group
1		1								
2		1			1			1		
3		2			2					
4		1			8			2		
5		1	0	1	5					
6					0	6	0 1		1 2 0	6 0 1
7				1		6	0 2			
8				2		6	0 4		1 2 1	8 0 7
9				1		6	0 5			
10				2		6	0 7		1 2 3	0 1 9
11				1		6	0 8			
12				2		6	1 0			
13				1		6	1 1		1 2 4	2 3 7
	+6 1 . 3			2	1	6	1 3		1 2 5	6 6 1
								−8,6 1 5 . 1 2 5		
14	61.3 + 0.2 = 61.5									
15–16	61.5 ÷ 3 = 20.5									

15. After having followed the prescriptions in Item 14, the minuend in the P-O field may still be smaller than the R-number. In such cases, the indicator should be brought again three-rod more to the right, and what is supposed to be added to the L-number should be 1.001 instead of 1.1 or 1.01. When adding the resultant L-number to the R-number, the former's place order should then be lowered by "six". The same reasoning holds good for all cases of similar nature.

Example: $\sqrt[3]{8{,}036.054027} = 20.03$

Operation Procedures	"+" operations on the left A-O field: 1st digit	2nd digit	3rd digit	4th digit	"+" operations on the right A-O field: Order 1	Order 2	Order 3	Order 4	"−" operations on the P-O field: 1st group	2nd group	3rd group	4th group
1	1				1				1			
2	2				2							
3	2				4				7			
4	1				5							
5	1	0	0	1		0 0	6 0	0 1		1 2	0 0 6	0 0 1
6				1			6 0	0 2				
7				2			6 0	0 4		1 2	0 1 8	0 0 7
8				1			6 0	0 5				
9				2			6 0	0 7		1 2	0 3 0	0 1 9
	+60.07								−8,036.054027			
10	60.07 + 0.02 = 60.09											
11–12	60.09 ÷ 3 = 20.03											

16. There may be cases when the remainder in the P-O field is inexhaustible. This means that the original number on which the operations of root extraction is made is not a perfect cubic number. Even so, the root obtained by carrying out the operations described above makes a good approximation.