Signals and Systems I

Topic 2

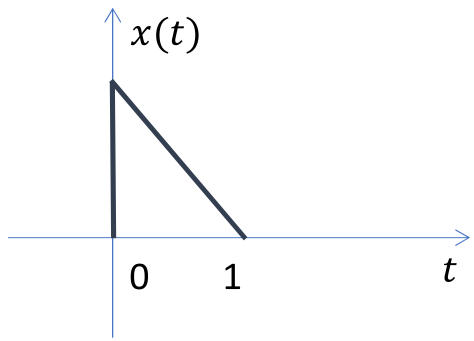
Today:

Useful Signal Operations

- Time Shift
- Amplitude Scaling
- Time Scaling
- Time Reversal
- Combined Operation

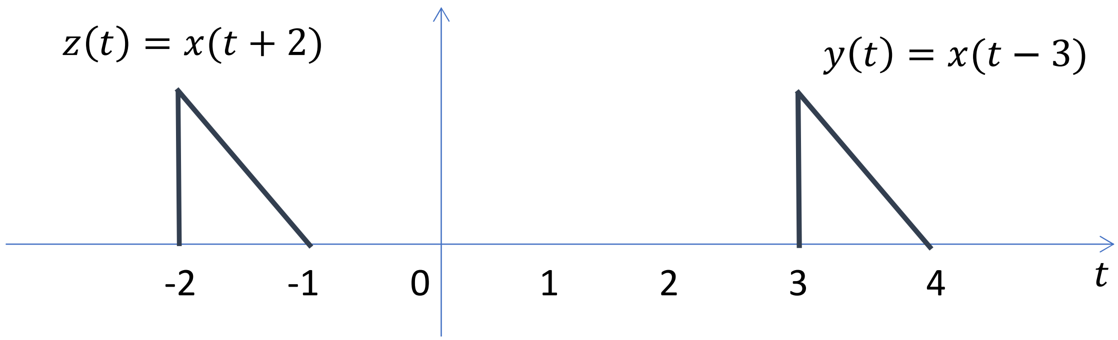
One more signal classitication: Odd and Even Signals

Time Shift:



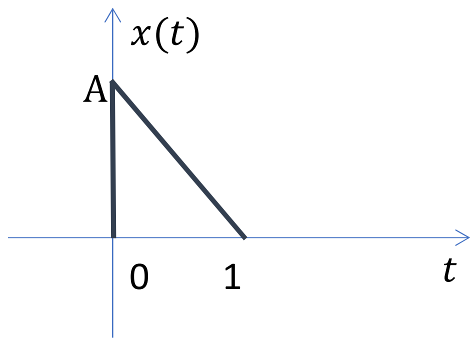
$x(t-T) \Rightarrow$ Shift to Right if $T>0$ (Delayed, After, Forward)

$x(t-T) \Rightarrow$ Shift to Left if $T<0$ (Advanced, Before, Backward)



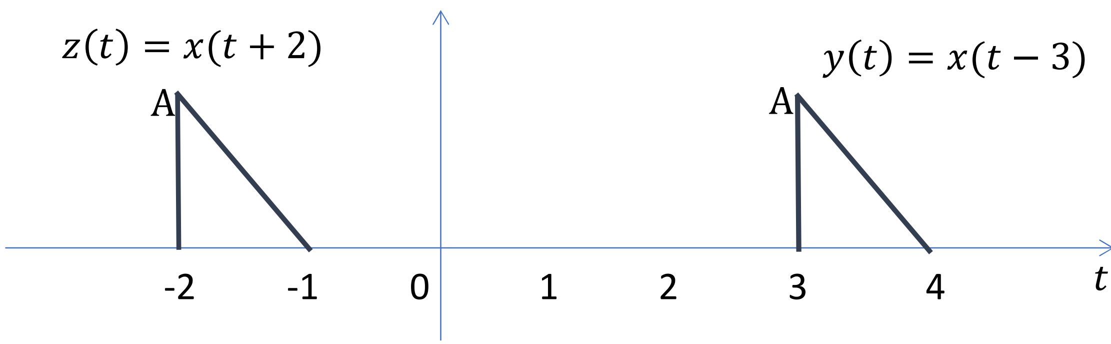
Example: For $y_2(t) = x(t+10)$ find the values for $y_2(-10), y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$

Time Shift:



$x(t-T) \Rightarrow$ Shift to Right if $T > 0$ (Delayed, After, Forward)

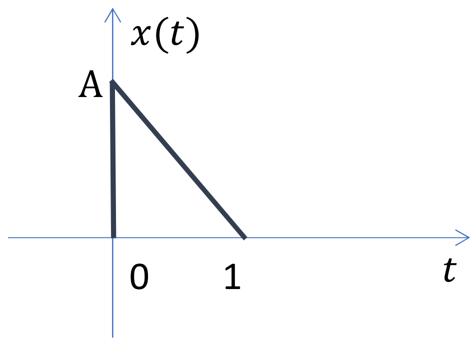
$x(t-T) \Rightarrow$ Shift to Left if $T < 0$ (Advanced, Before, Backward)



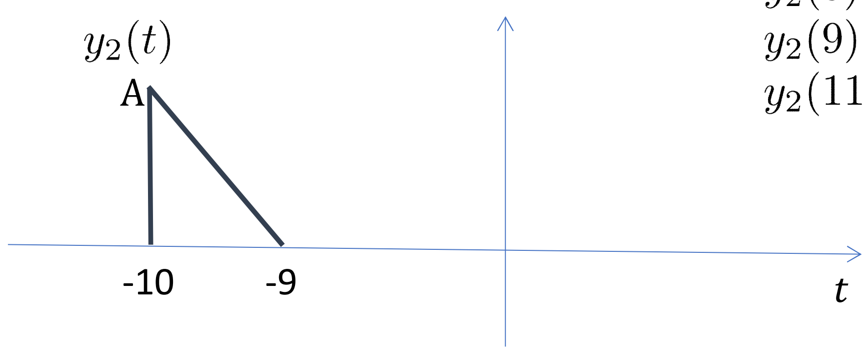
Note: get used to labeling any Transformed (operated) signal with a new name. For example here $y(t)$ and $z(t)$

Example: For $y_2(t) = x(t+10)$ find the values for $y_2(-10), y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$

Time Shift:

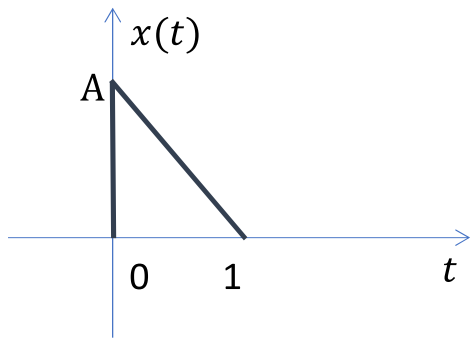


Example: For $y_2(t) = x(t{+}10)$ find the values for $y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$



$$y_2(-10) = x(-10+10) = x(0)$$
$$y_2(0) = x(0+10) = x(10)$$
$$y_2(5) = x(5+10) = x(15)$$
$$y_2(9) = x(9+10) = x(19)$$
$$y_2(11) = x(11+10) = x(21)$$

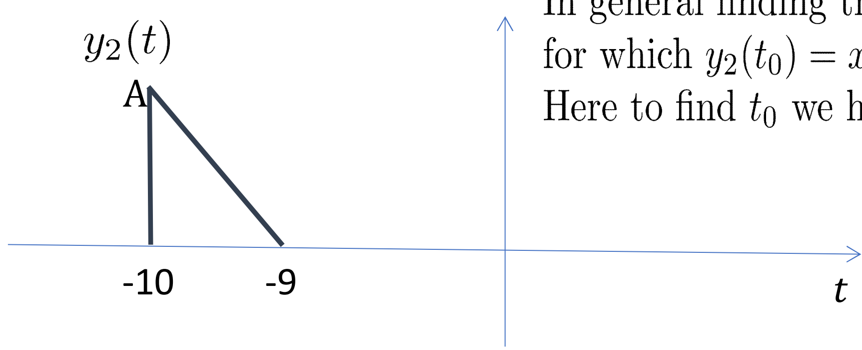
Time Shift:



Example: For $y_2(t) = x(t+10)$ find the values for $y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$

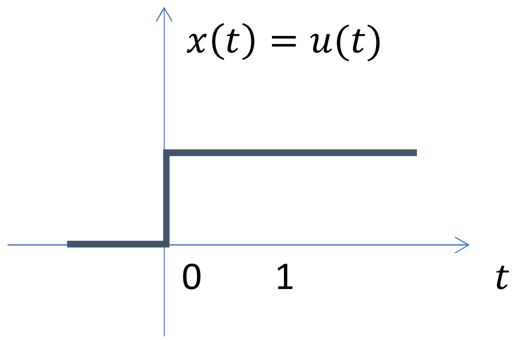
$y_2(-10) = x(-10+10) = x(\mathbf{0})$
$y_2(\mathbf{0}) = x(0+10) = x(10)$

In general finding the value of $y_2(0)$ and also value of t_0 for which $y_2(t_0) = x(0)$ are useful.
Here to find t_0 we have to have $t_0 + 10 = 0$ which means $t_0 = -10$



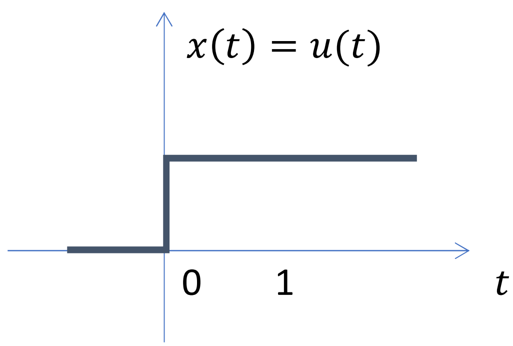
Time Shift:

Example: plot $y(t) = x(t-3)$



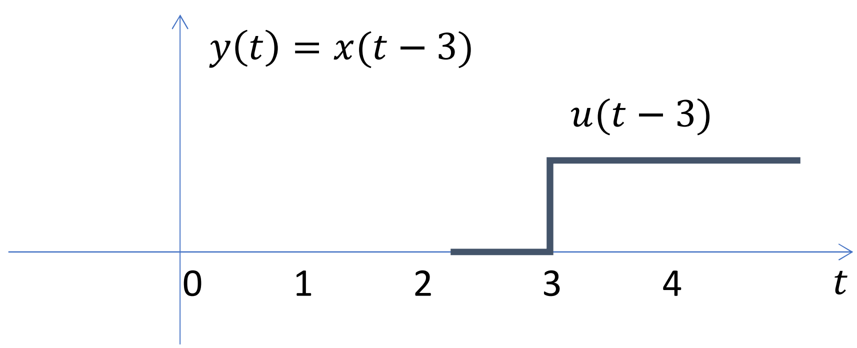
Time Shift:

Example: plot $y(t) = x(t-3)$



$y(3) = x(3-3) = x(\mathbf{0})$
$y(\mathbf{0}) = x(0-3) = x(-3)$

Finding the value of $y(0)$ and also value of t_0 for which $y(t_0) = x(0)$ are useful.
Here to find t_0 we have to have $t_0 - 3 = 0$ which means $t_0 = 3$



Time Shift:

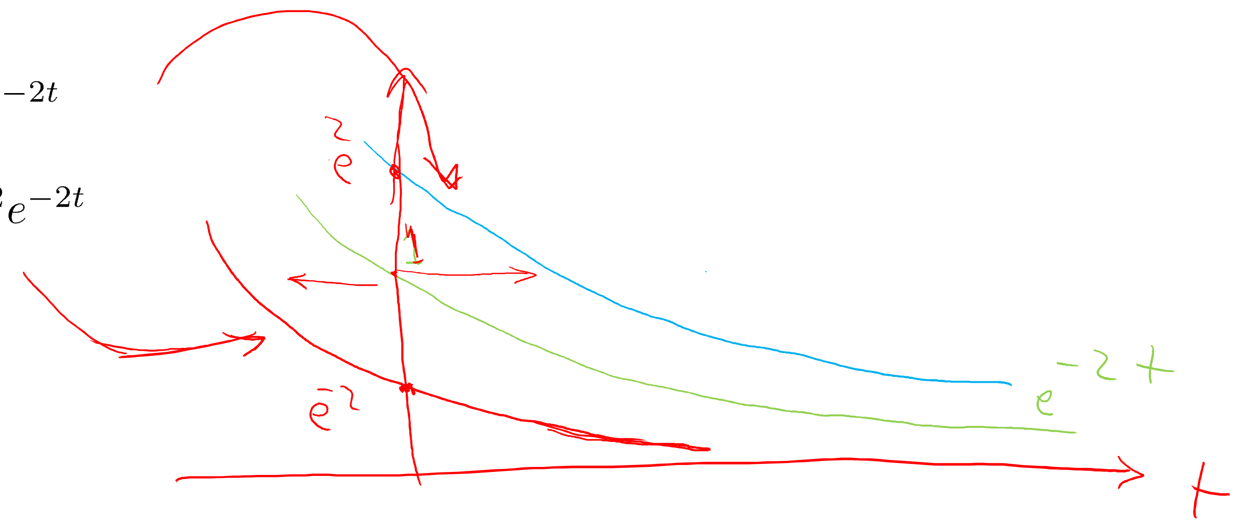
Example: If $x(t) = e^{-2t}$, then what are $y(t) = x(t-1)$ and $z(t) = x(t+1)$? Plot $y(t)$ and $z(t)$

Time Shift:

Example: If $x(t) = e^{-2t}$, then what are $y(t) = x(t-1)$ and $z(t) = x(t+1)$? Plot $y(t)$ and $z(t)$

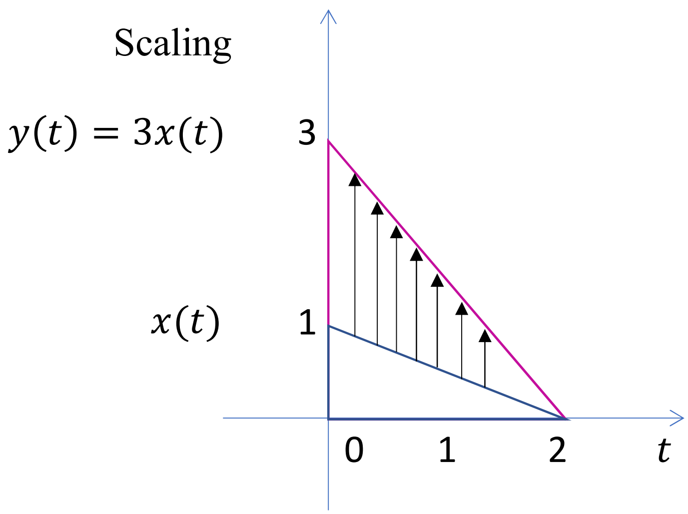
$$y(t) = x(t-1) = e^{-2((\mathbf{t-1}))} = e^{2}e^{-2t}$$

$$z(t) = x(t+1) = e^{-2((\mathbf{t+1}))} = e^{-2}e^{-2t}$$



Amplitude Scaling:

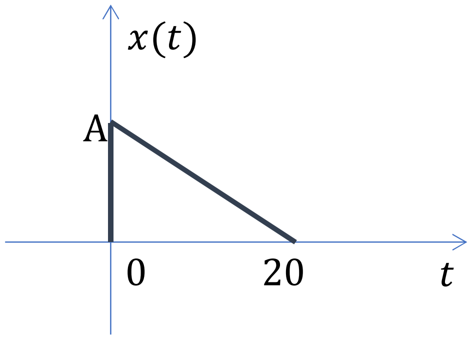
$$y(t) = Ax(t)$$



Time Reversal:

$y(t) = x(-t)$

Example: plot $y(t) = x(-t)$ first find $y(1)$, $y(2)$, $y(0)$, $y(-1)$, and $y(-2)$



Time Reversal:

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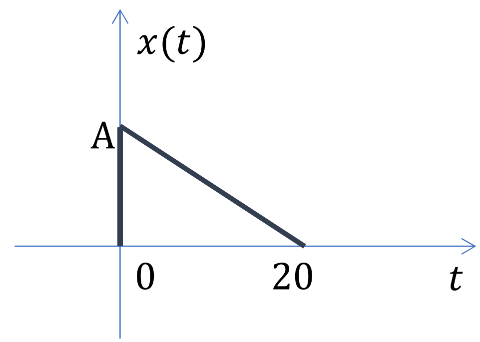
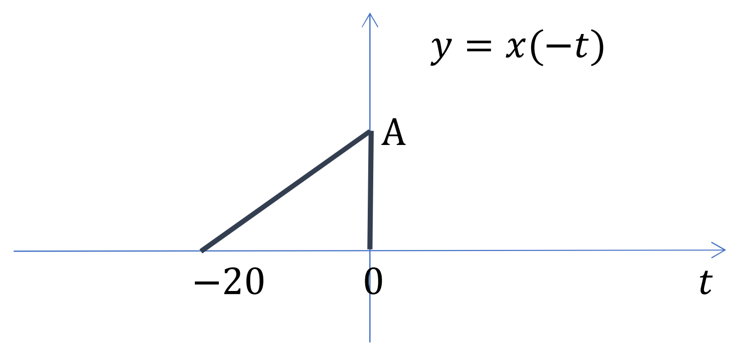


$$
\begin{aligned}
y(t) &= x(-t) \\
y(1) &= x(-1) \\
y(0) &= x(0) \\
y(-1) &= x(1) \\
y(-2) &= x(2)
\end{aligned}
$$

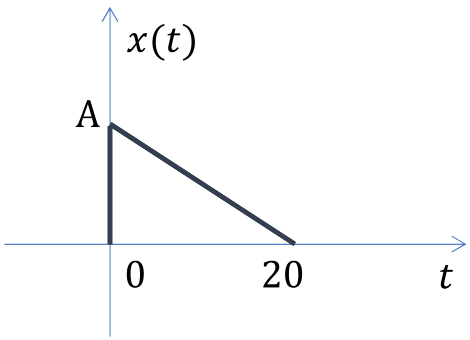


Time Scaling:

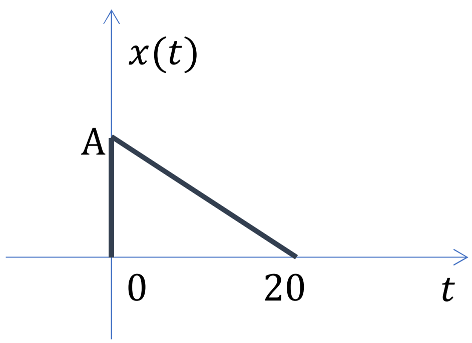
$$y(t) = x(\alpha t)$$

Note that Time Reversal is a special case of Time Scaling with $\alpha = 1$

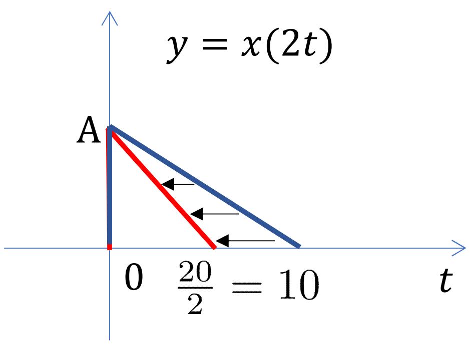
Example: Find $y(t) = x(2t)$ and $z(t) = x(\frac{t}{2})$.



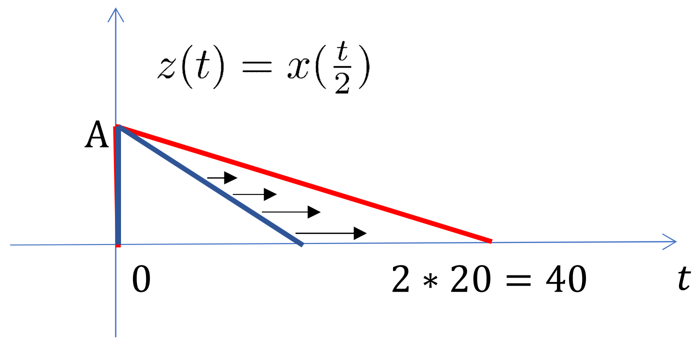
Time Scaling:



$y(t) = x(2t)$
$y(0) = x(0)$
$y(1) = x(2)$
$y(-1) = x(-2)$
$y(2) = x(4)$
$\vdots$
$y(10) = x(20)$
$y(11) = x(22)$

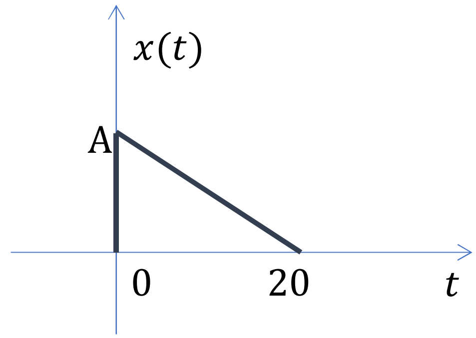


$z(t) = x(\frac{t}{2})$
$z(0) = x(0)$
$z(1) = x(\frac{1}{2})$
$z(-1) = x(\frac{-1}{2})$
$z(2) = x(\frac{2}{2})$
$\vdots$
$z(10) = x(\frac{10}{2})$
$z(11) = x(\frac{11}{2})$



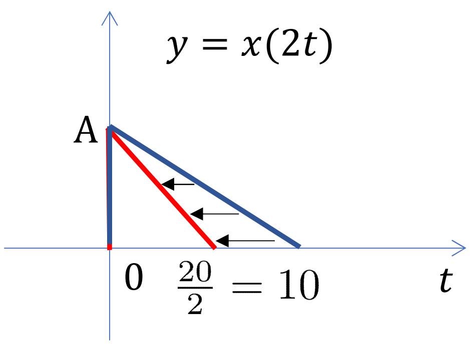
Generally, it is a good idea to always check for couple of points when per-forming time scale operation.

Time Scaling:

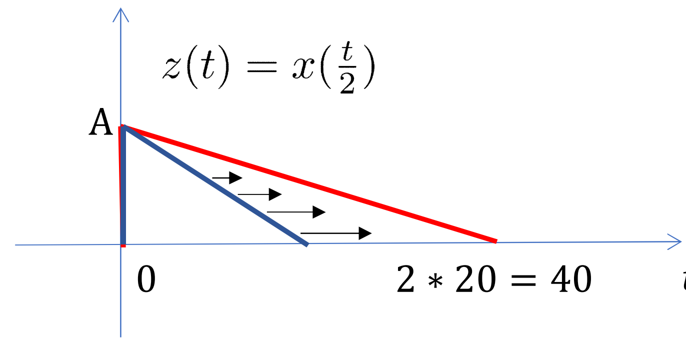


$$y(t) = x(\alpha t) \begin{cases} \text{Squeezing,} & \text{if} \quad \alpha > 1 \\ \text{Expanding,} & \text{if} \quad 0 < \alpha < 1 \end{cases}$$

$$\begin{aligned} y(t) &= x(2t) \\ y(0) &= x(0) \\ y(1) &= x(2) \\ y(-1) &= x(-2) \\ y(2) &= x(4) \\ &\vdots \\ y(10) &= x(20) \\ y(11) &= x(22) \end{aligned}$$



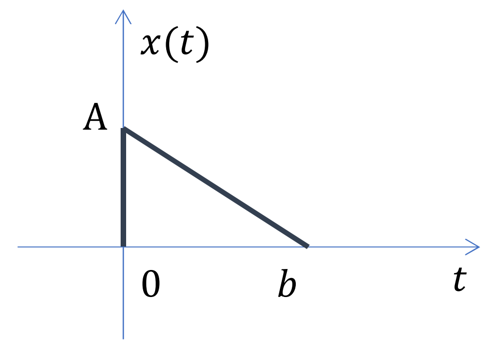
$$\begin{aligned} z(t) &= x(\tfrac{t}{2}) \\ z(0) &= x(0) \\ z(1) &= x(\tfrac{1}{2}) \\ z(-1) &= x(\tfrac{-1}{2}) \\ z(2) &= x(\tfrac{2}{2}) \\ &\vdots \\ z(10) &= x(\tfrac{10}{2}) \\ z(11) &= x(\tfrac{11}{2}) \end{aligned}$$



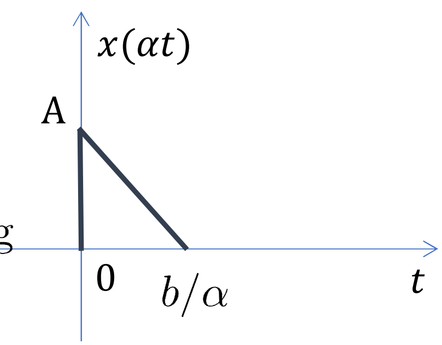
Generally, it is a good idea to check for couple of points when per-forming time scale operation.

Time Scaling:

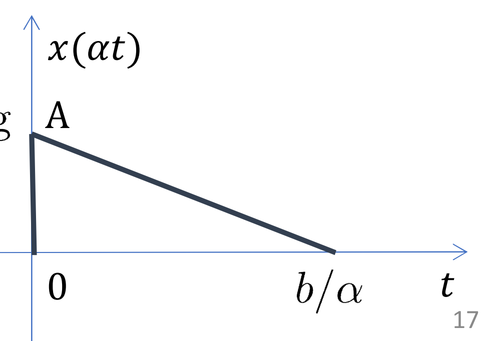
$$y(t) = x(\alpha t) \begin{cases} \text{Squeezing,} & \text{if} \quad |\ \alpha\ | > 1 \\ \text{Expanding,} & \text{if} \quad 0 < |\ \alpha\ | < 1 \end{cases}$$



$\alpha > 1$ signal squeezing

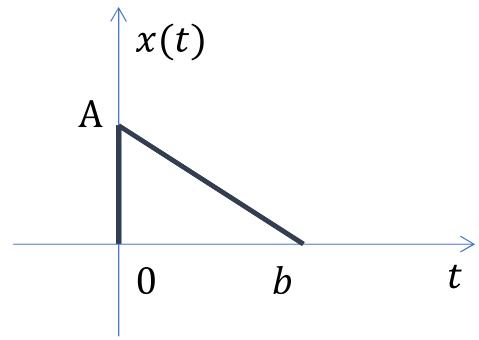


$0 < \alpha < 1$ signal expanding



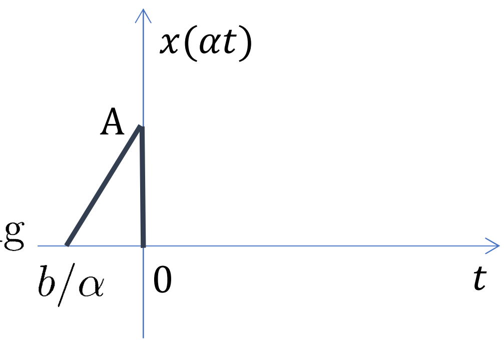
Time Scaling:

$$y(t) = x(\alpha t)\begin{cases}\text{Squeezing,} & \text{if} \quad |\alpha| > 1 \\ \text{Expanding,} & \text{if} \quad 0 < |\alpha| < 1\end{cases}$$



For $\alpha < 0$:

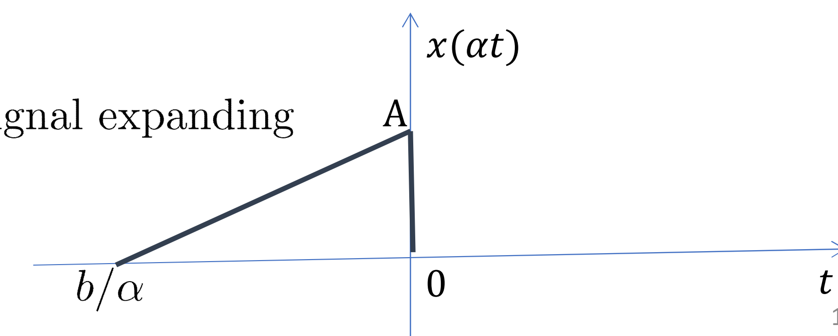
$\alpha < -1$ signal squeezing



In this case: $\alpha = -|\alpha|$

example: $-2 = -|-2|$

$-1 < \alpha < 0$ signal expanding

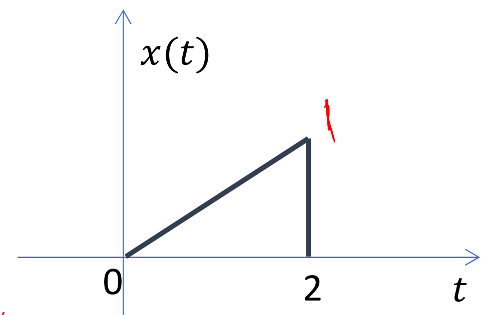


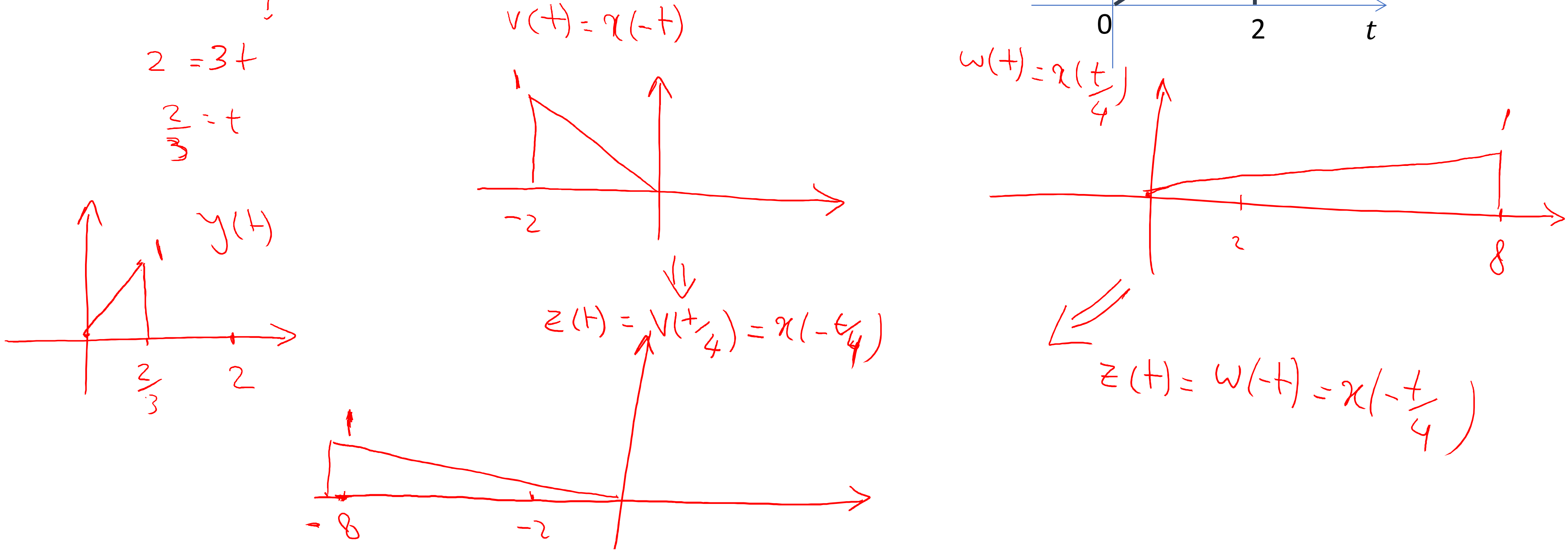
$y(t) = x(\alpha t) = x(-|\alpha| t)$

$x(-2t) = x(-(2t))$ additional flipping

Time Scaling:

Plot $y(t) = x(3t)$ and $z(t) = x(-t/4)$





Combined Operations:

$$z(t) = Ax(\alpha t - T)$$

We first plot $y(t) = x(\alpha t - T)$ then plot $z(t) = Ay(t)$

Two methods to plot $y(t)$

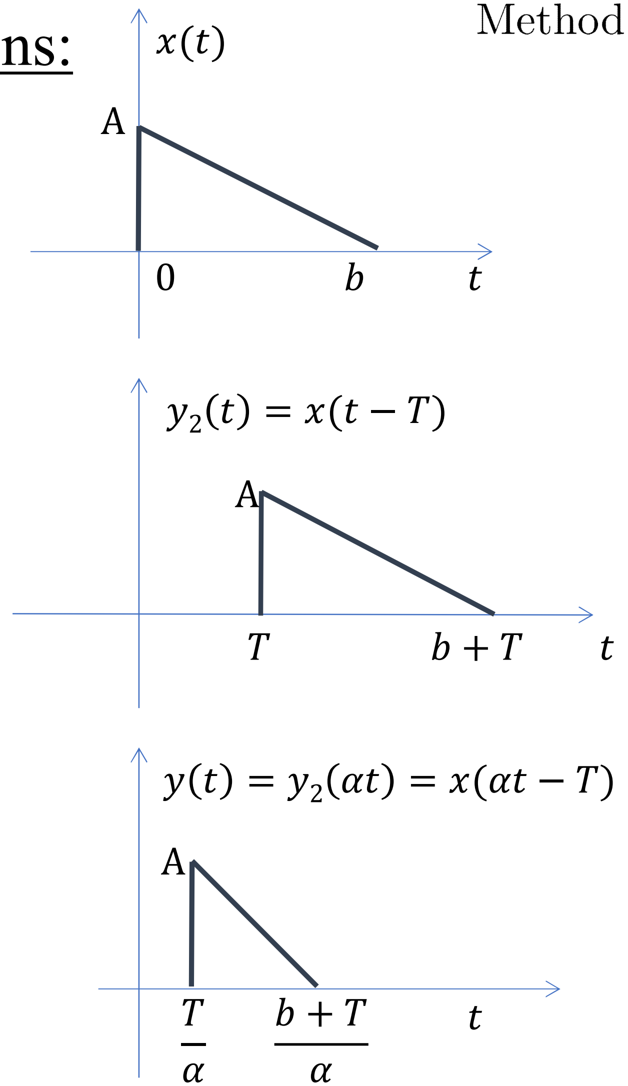
Method 1	Method 2
1- Shift by T	1- Time scale by α
$y_2(t) = x(t - T)$	$y_1(t) = x(\alpha t)$
2- Time scale by α	2- Shift by T/α
$y(t) = y_2(\alpha t) = x(\alpha t - T)$	$y(t) = y_1(t - T/\alpha) = x\left(\alpha(t - T/\alpha)\right) = x(\alpha t - T)$

Combined Operations:

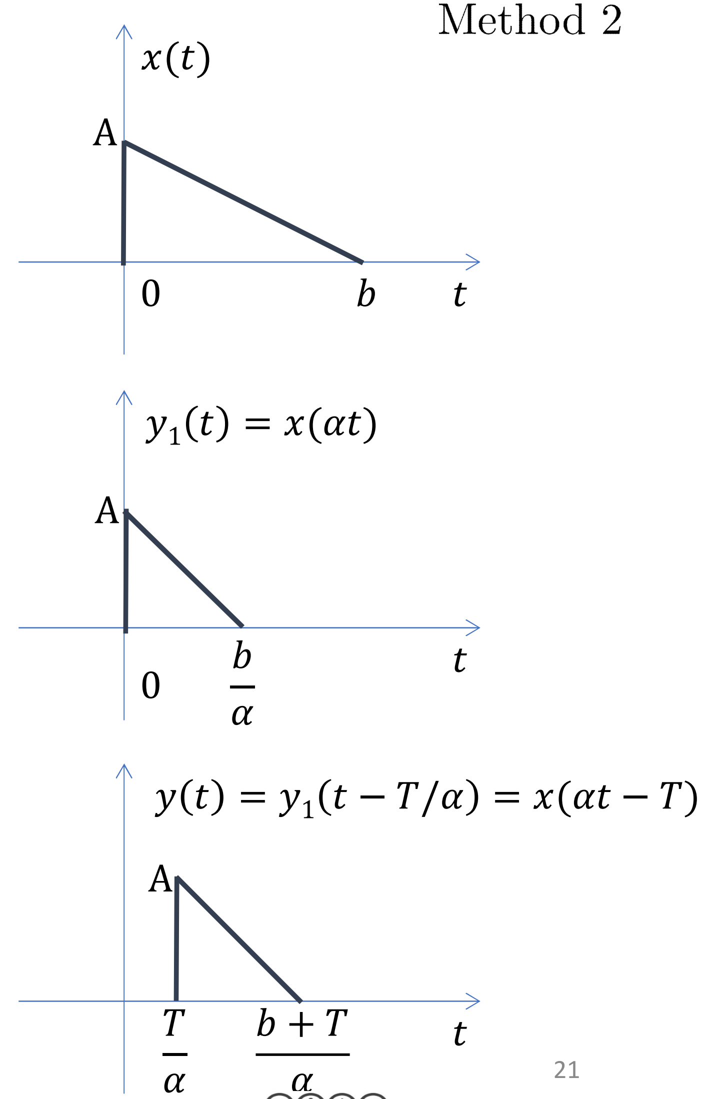
$y(t) = x(\alpha t - T)$

Example $\alpha > 1$, $T > 0$

Method 1



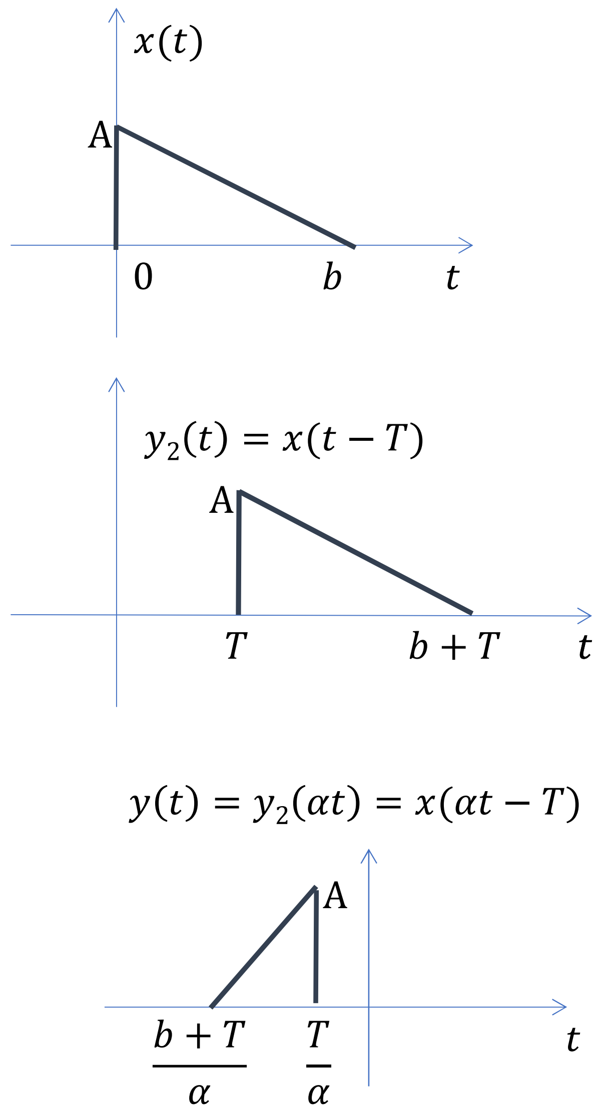
Method 2

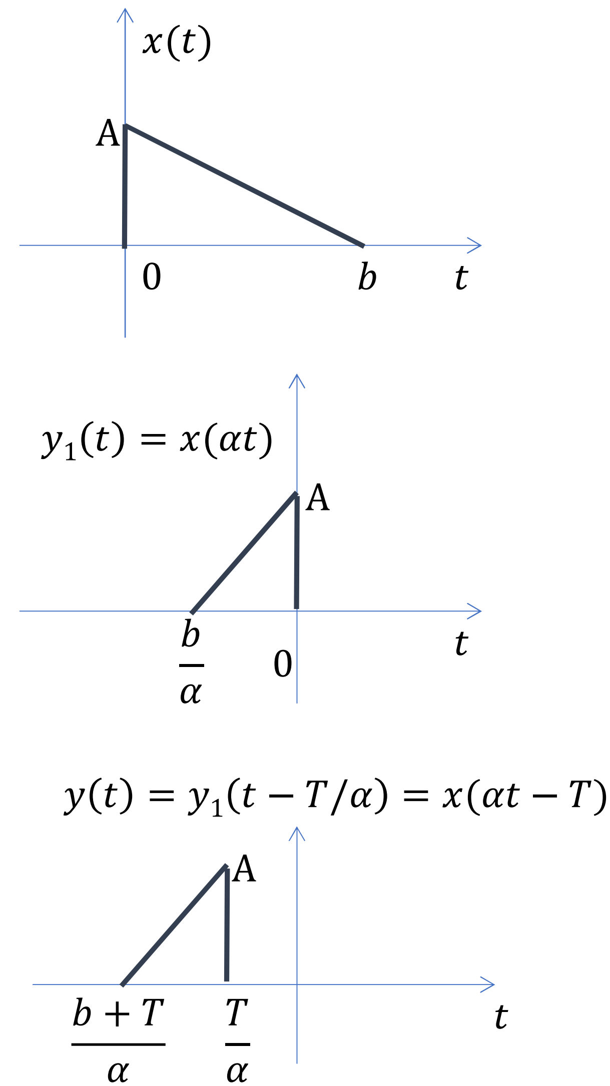


Combined Operations:

$$y(t) = x(\alpha t - T)$$

Example $\alpha < -1$, $T > 0$



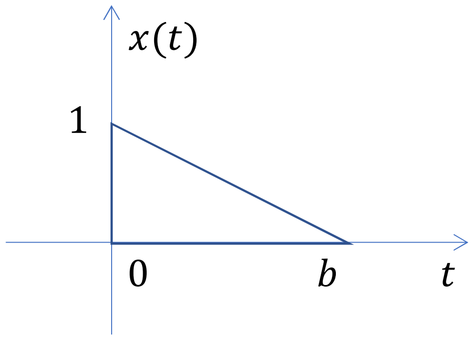


Combined Operations:

Easy steps for combined operations

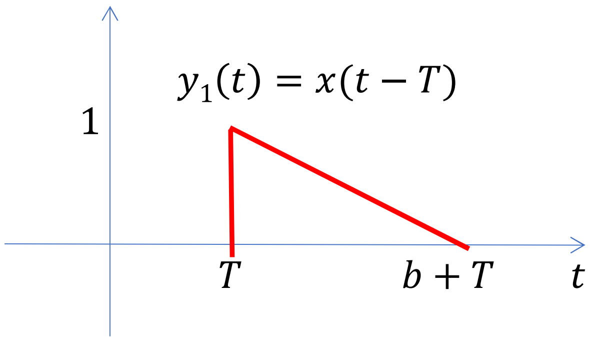
Given $x(t)$ plot $z(t) = Ax(\alpha t - T)$
1- Shift by T
2- Time scale by $|\alpha|$
3- If α is positive go to step 4. If α is negative, flip the signal (time reverse)
4- Scale the signal by A

Combined Operations:

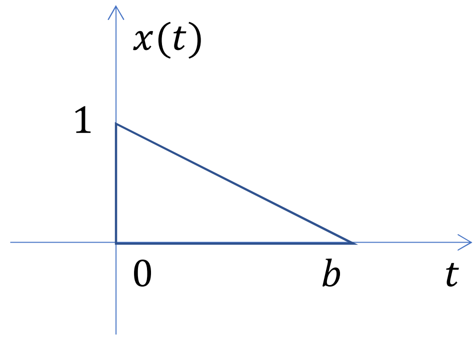


Find $y(t) = Ax(\alpha t - T)$

Step One

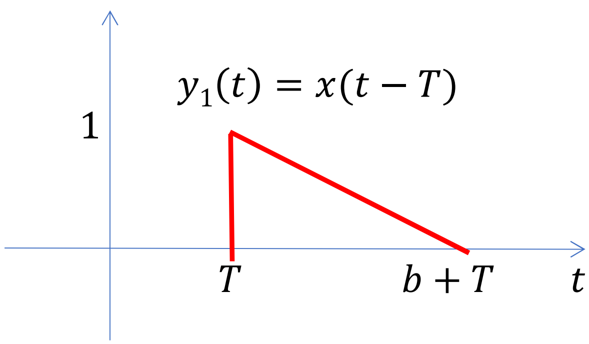


Combined Operations:

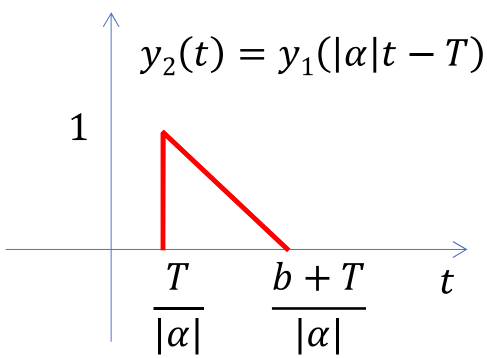


Find $y(t) = Ax(\alpha t - T)$

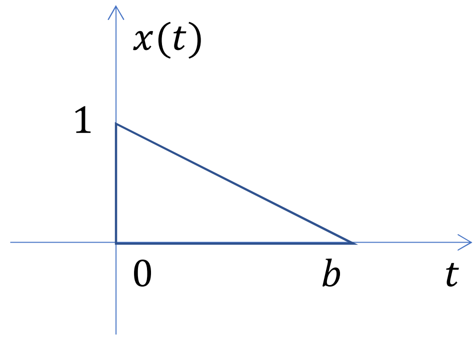
Step One



Step Two



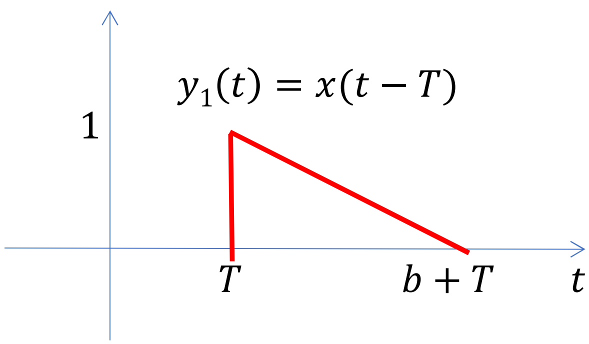
Combined Operations:



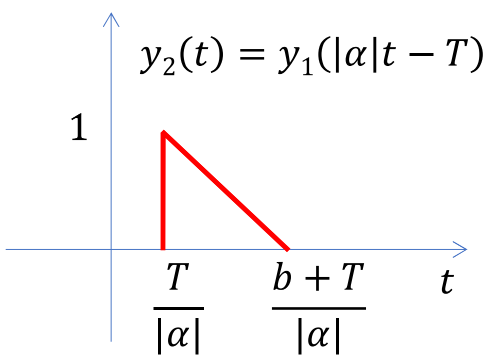
Find $y(t) = Ax(\alpha t - T)$

$\alpha < 0$

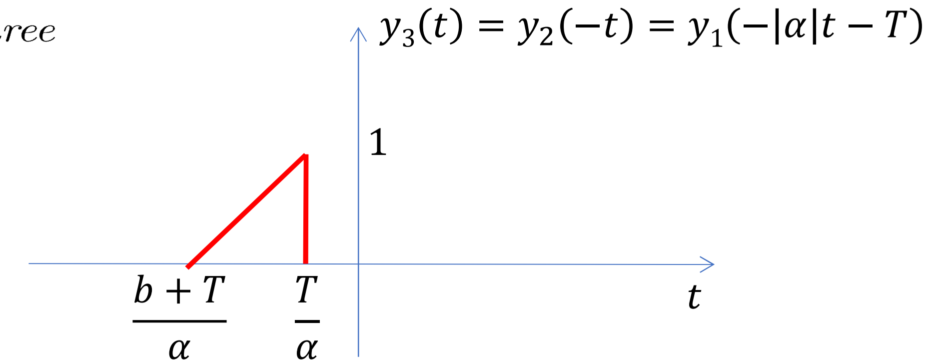
Step One



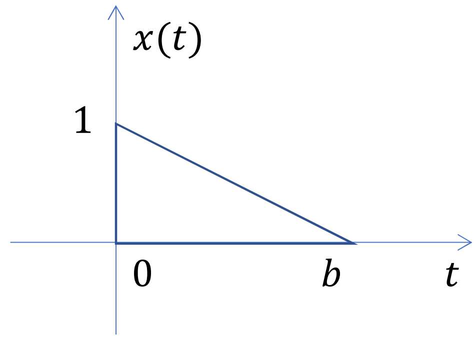
Step Two



Step Three



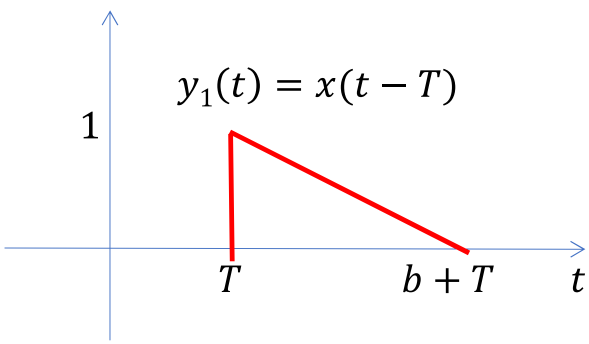
Combined Operations:



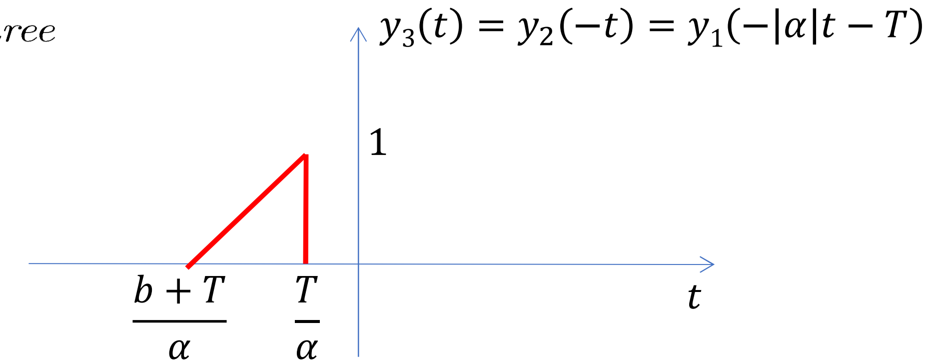
Find $y(t) = Ax(\alpha t - T)$

$\alpha < 0$

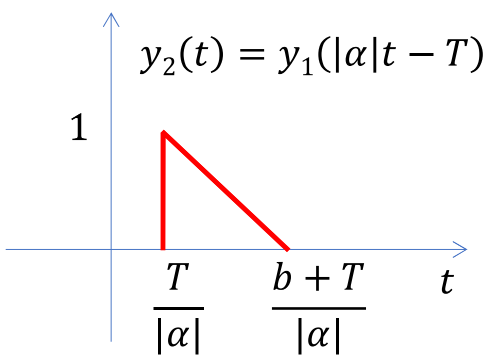
Step One



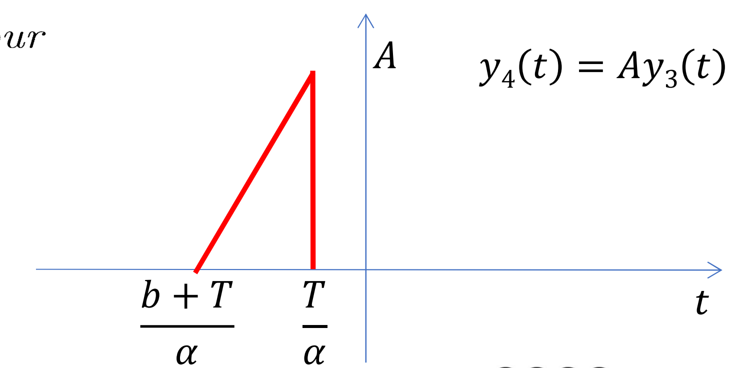
Step Three



Step Two

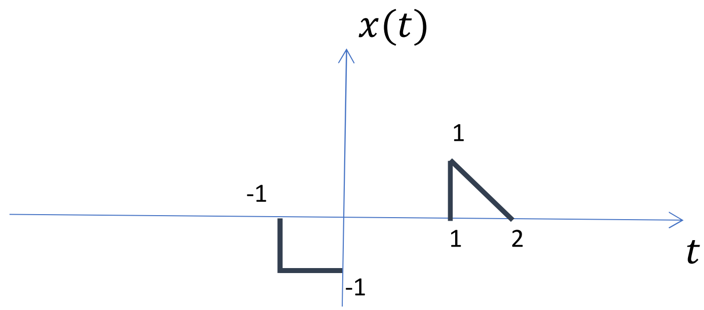


Step Four



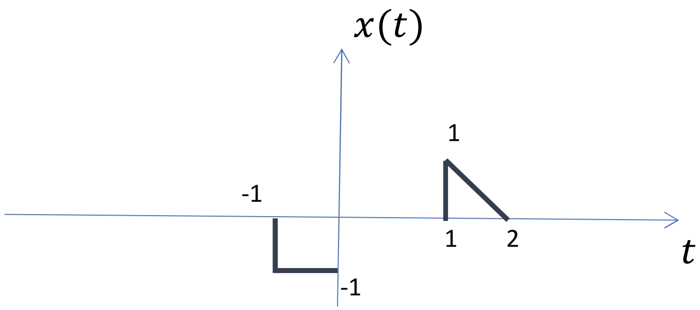
Combined Operations:

Example: $Plot\ x(3t),\ x(t+2),\ -4x(3t+2), and\ x(\frac{-t}{2}-3)$



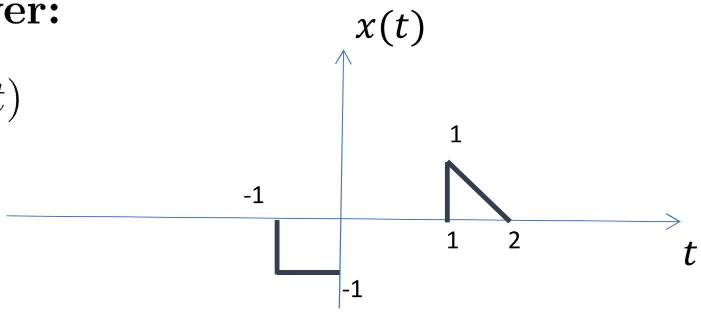
Combined Operations:

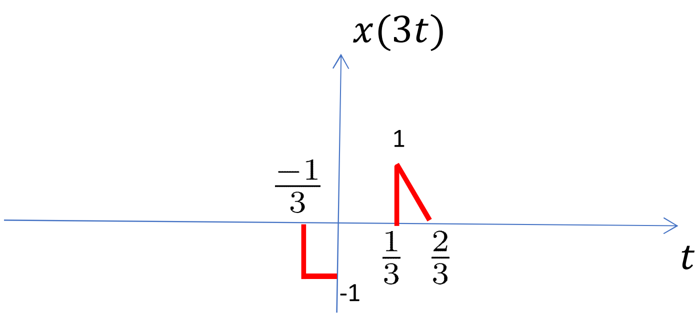
Example: *Plot* $x(3t)$, $x(t+2)$, $-4x(3t+2)$, *and* $x(\frac{-t}{2}-3)$



Answer:

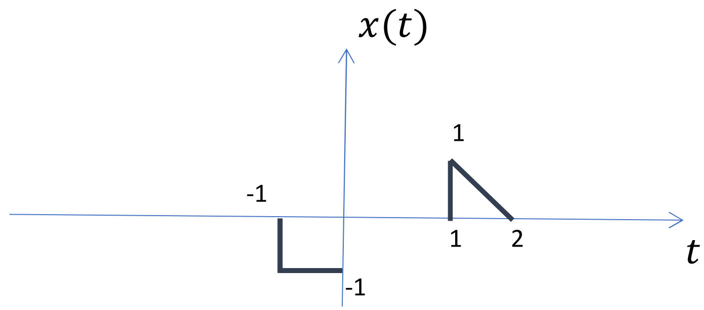
- $x(3t)$





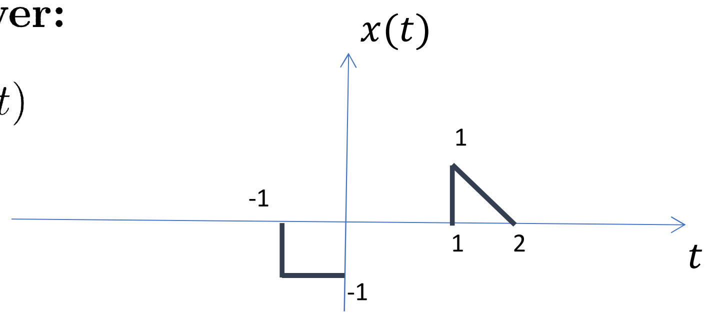
Combined Operations:

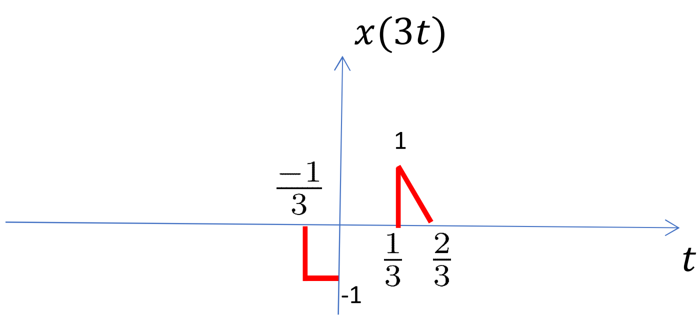
Example: *Plot* $x(3t)$, $x(t+2)$, $-4x(3t+2)$, *and* $x(\frac{-t}{2}-3)$



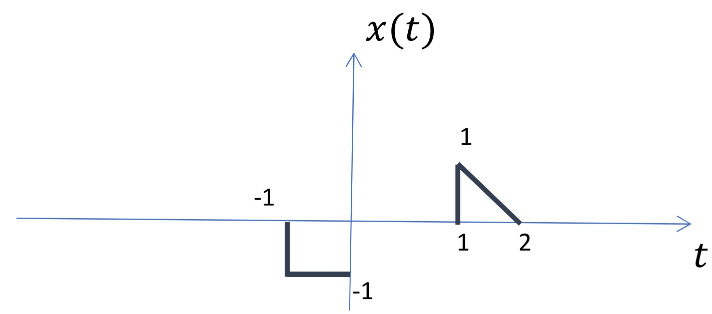
Answer:

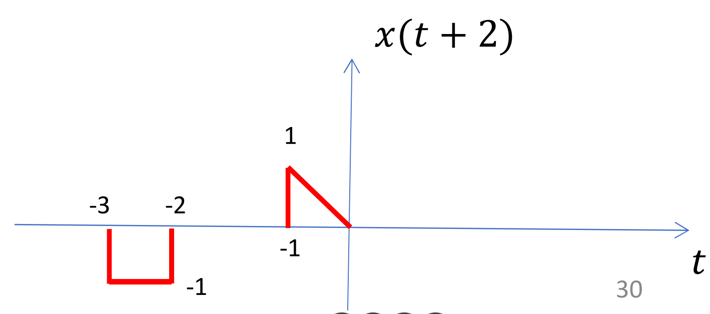
- $x(3t)$





- $x(t+2)$





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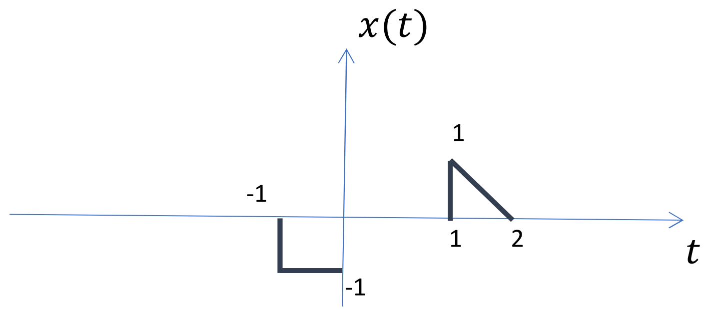
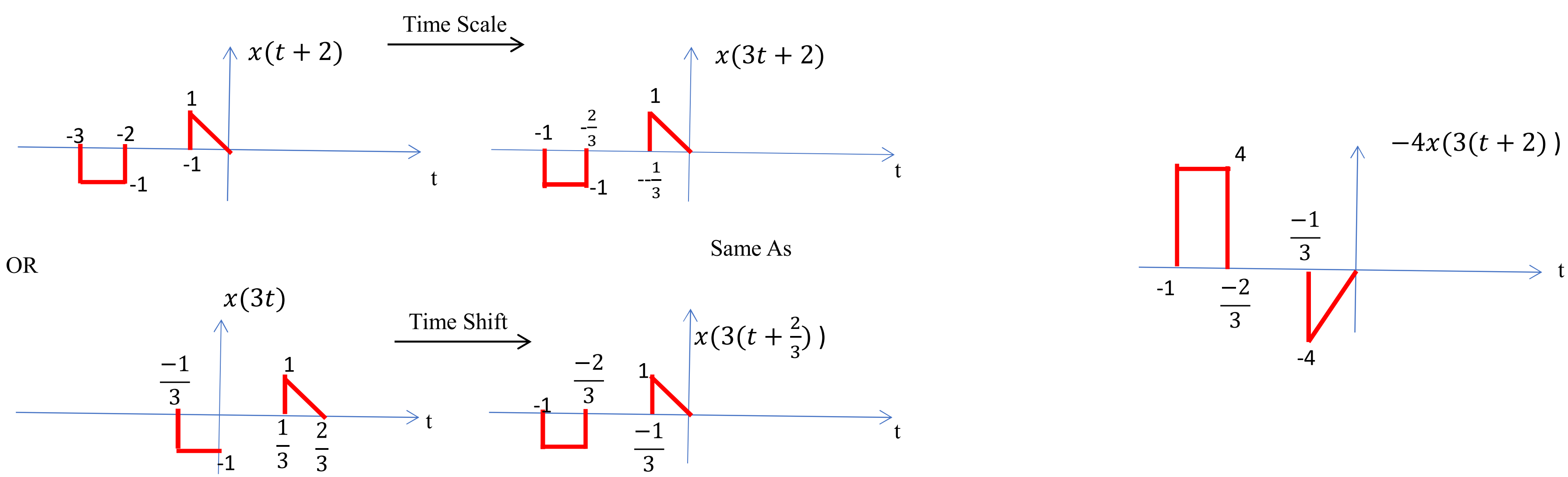


Answer:

- $-4x(3t+2)$



Combined Operations:

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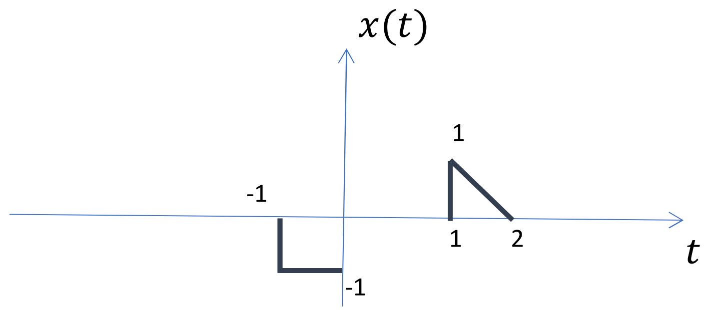
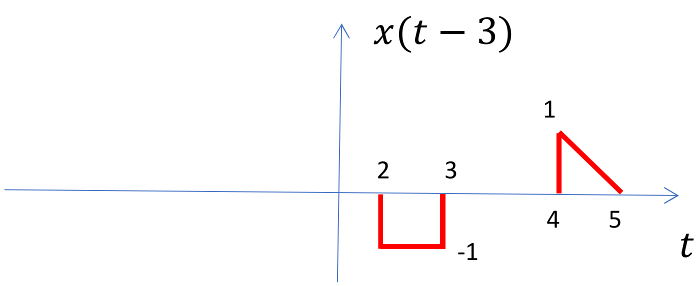
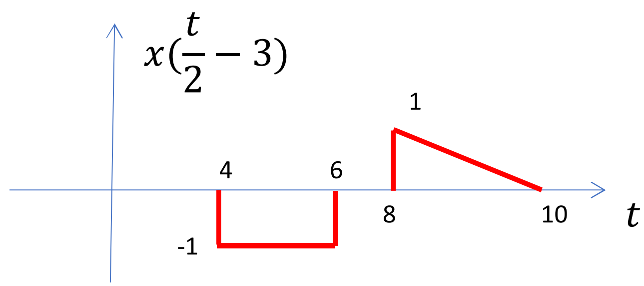


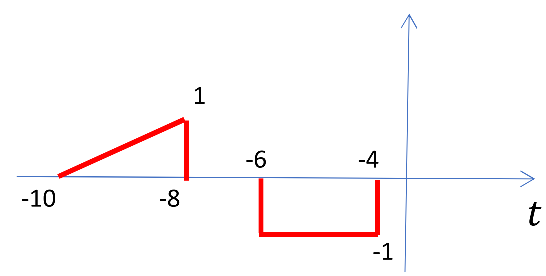
Answer:

- $x(\frac{-t}{2}-3)$

Time Shift → Time Scale → Flip $x(-\frac{t}{2}-3)$





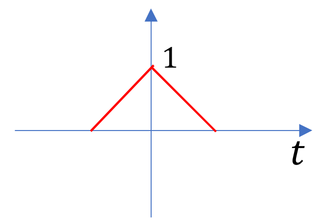


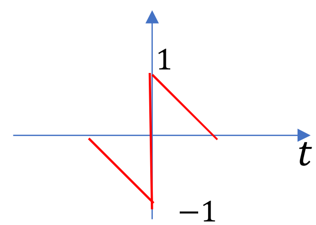
Odd and Even Functions (Signals):

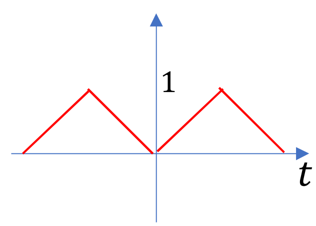
Even functions	odd functions
$x_e(-t) = x_e(t)$	$x_o(-t) = -x_o(t)$
$\int_{-\infty}^{\infty} x_e(t)dt = 2\int_{0}^{\infty} x_e(t)dt$	$\int_{-\infty}^{\infty} x_o(t)dt = 0$

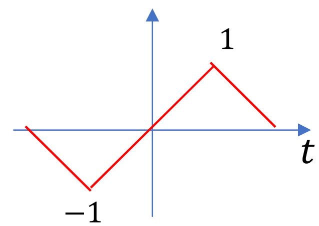
$$Examples$$

$\cos(t)$	$\sin(t)$









Odd and Even Functions (Signals):

In general signals can be neither odd nor even. However, all signals can be represented as sum of their even & odd components!

For any signal $x(t)$ we have:

$$x(t) = x_e(t) + x_o(t) \qquad (1)$$

How to find these components (?)

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

To prove the above claim we need to show the following facts:

1) $x_e(t) = x_e(-t)$ 2) $x_o(t) = -x_o(-t)$ 3) $x(t) = x_e(t) + x_o(t)$

showing 3):

$$x_e(t) + x_o(t) = \frac{x(t)+x(-t)}{2} + \frac{x(t)-x(-t)}{2} = 2\frac{x(t)}{2} = x(t)$$

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How about 1) and 2)?

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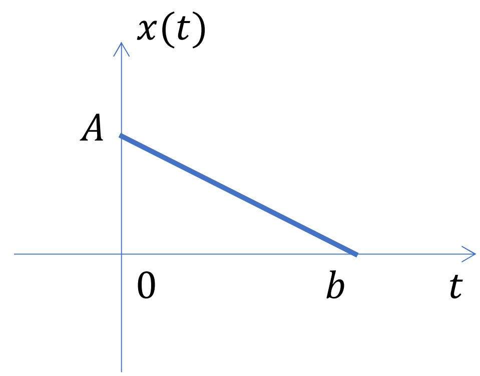
Odd and Even Signals:

Example: *Find odd & even parts of the following signal.*



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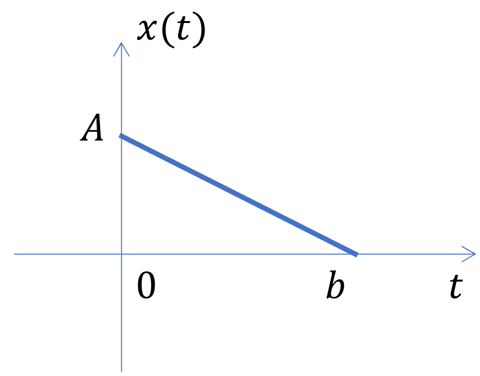
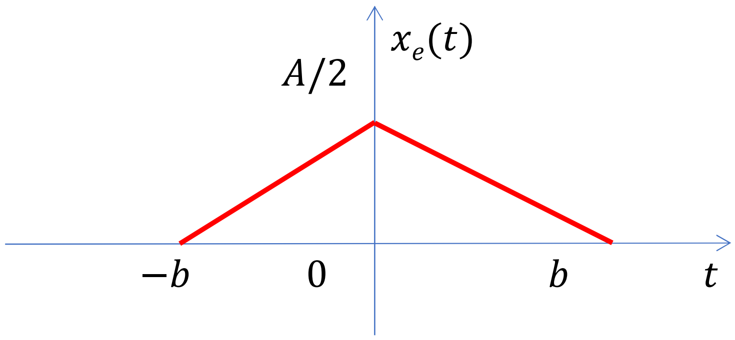
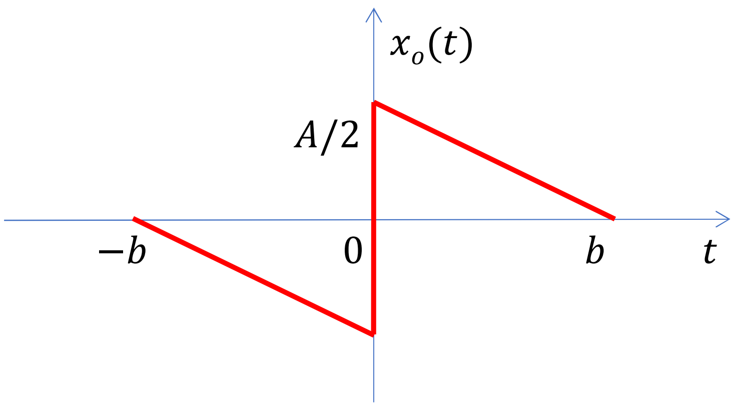
Example: *Find odd & even parts of the following signal.*







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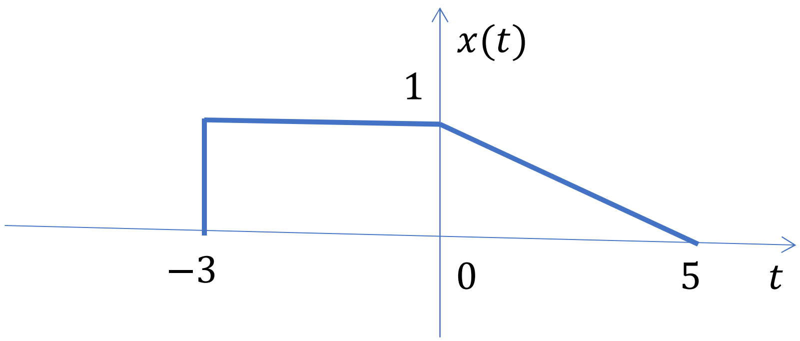
Example: *Find odd & even parts of the following signal.*



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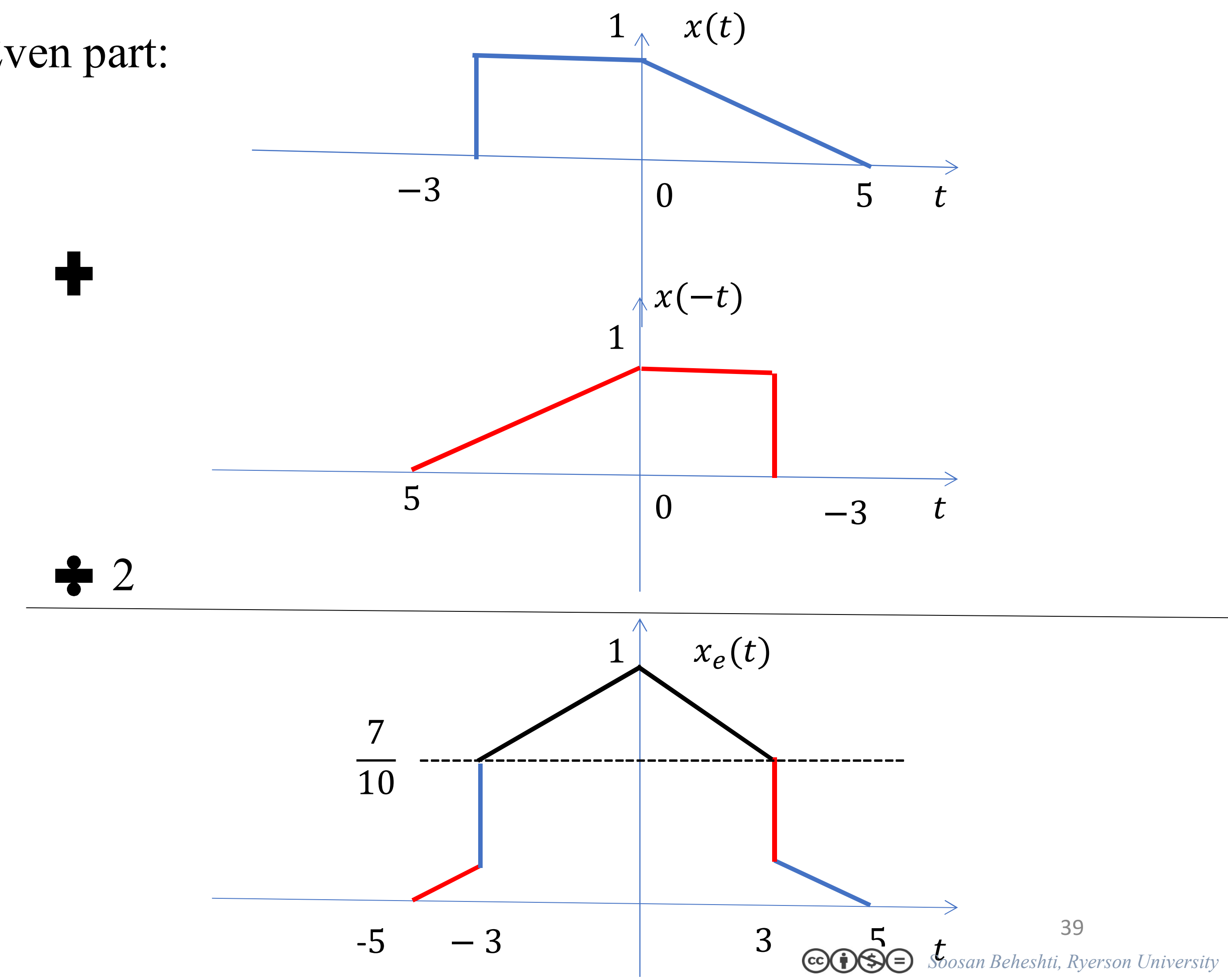
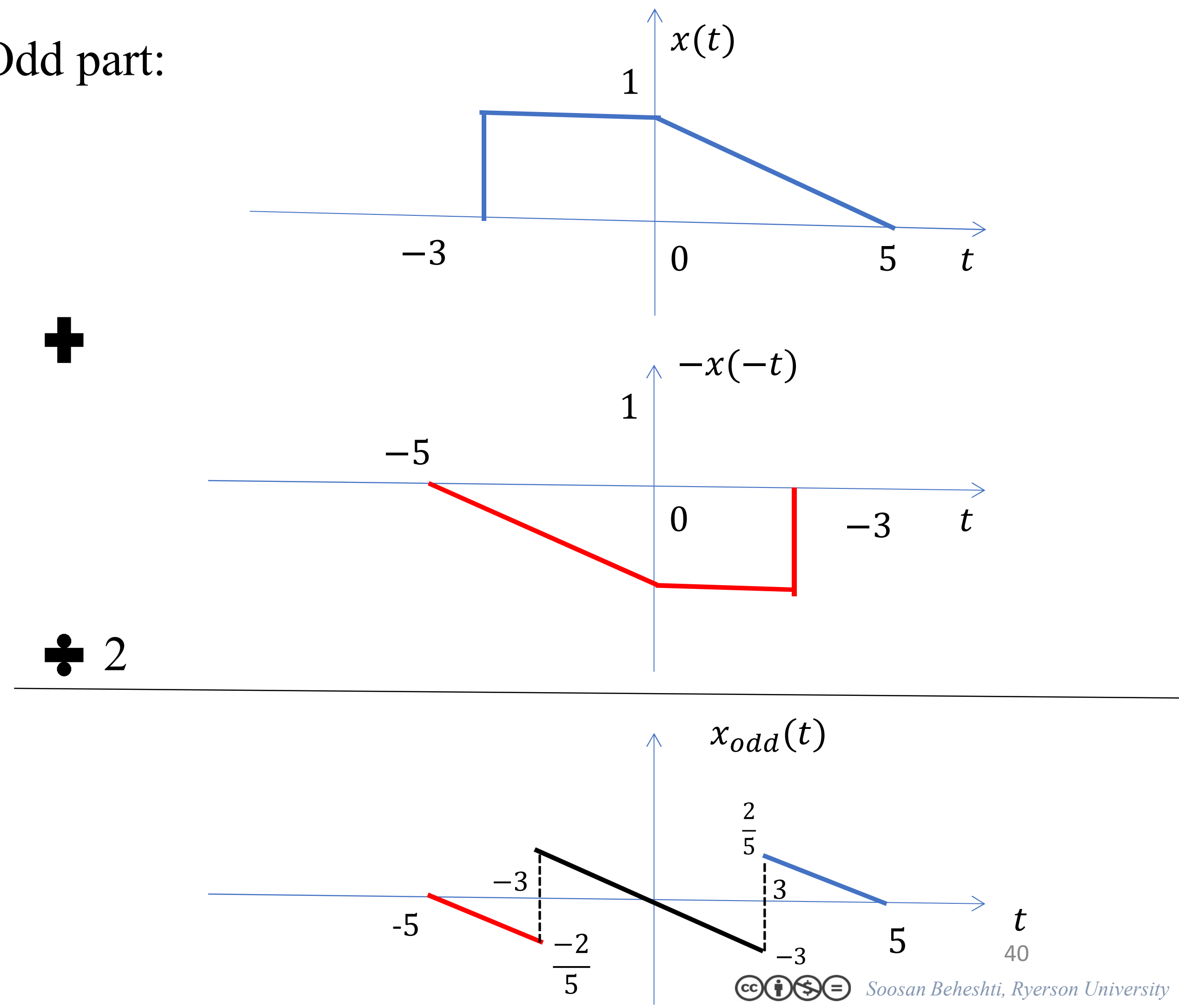
Odd and Even Signals:

Even part:



Odd and Even Signals:

Odd part:



Odd and Even Signals:

Odd part:

Try adding $x_e(t)$ and $x_o(t)$ to generate $x(t)$ itself!