

Signals and Systems I

Lecture 4

Last Lecture

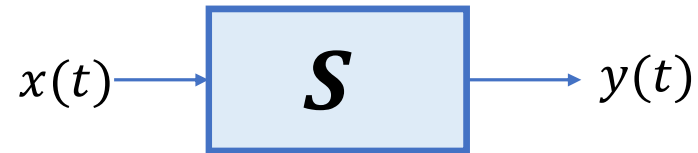
- more on $u(t)$ & $\delta(t)$
- Building Signals from other Signals
- Closed form expression for signals

“Closed form expression” is a mathematical expression that can be evaluated in a finite number of operations.

Today

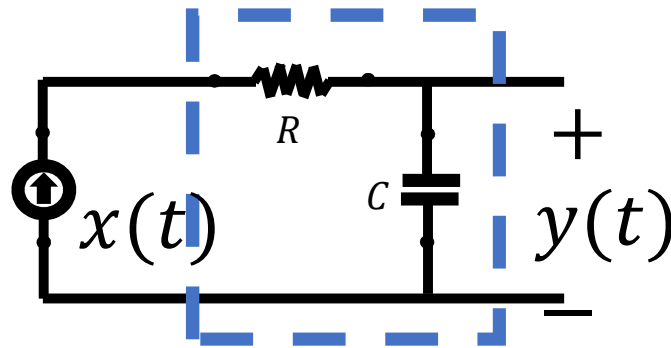
- System Classification:
 - Continuous Time/Discrete Time
 - Digital /Analog
 - Linear / Nonlinear
 - Time Invariant / Time Varying
 - Causal / Non-Causal
 - Memoryless/ With Memory
 - Invertible / Noninvertible

Systems: processes signals, operates on signals and generates signals



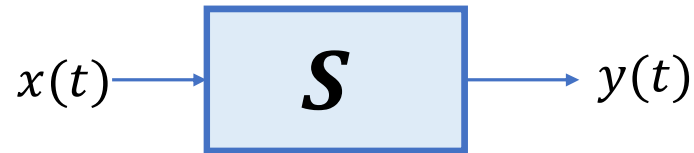
$$y(t) = \mathbf{S}(x(t))$$

Example: $y(t) = 3x(t - 7)$ or RC circuit:



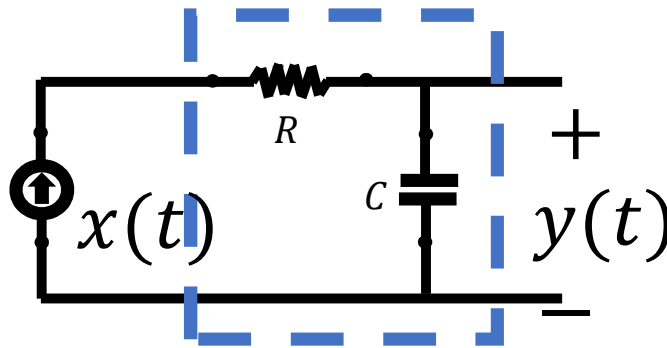
$$y(t) = \frac{1}{C} \int_{t_0}^t x(t) dt + V_C(t_0) \text{ where } V_C(t_0) \text{ is the initial condition and } t_0 \text{ is the start time}$$

Systems: processes signals, operates on signals and generates signals



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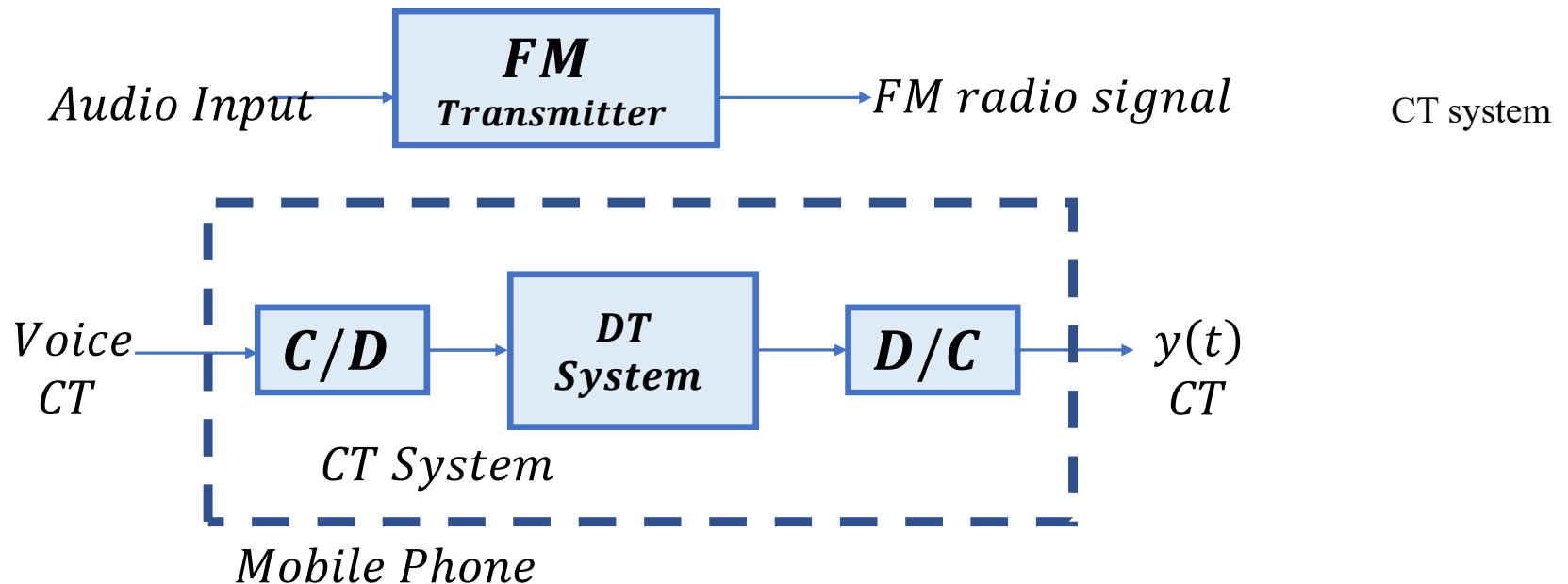
There are two types of systems around us, the existing ones in nature and the manmade ones. We study existing systems for the purpose of understanding and analyzing them and also to design new systems.

Systems Classification

Continuous Time (CT) or Discrete Time (DT)

- If input and output of a system are CT \longrightarrow the system is CT
- If input and output of a system are DT \longrightarrow the system is DT

Examples



Systems Classification

Analog or Digital Systems

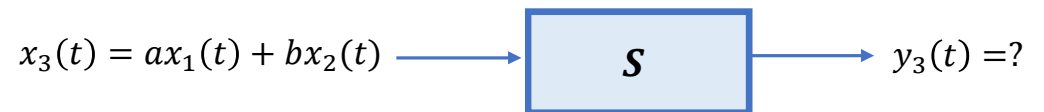
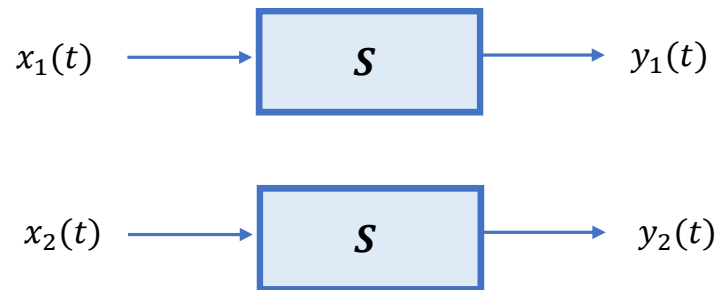
- If input and output of a system are Analog \longrightarrow the system is Analog
- If input and output of a system are Digital \longrightarrow the system is Digital

Analog example: Radio, Digital example: computers

Systems Classification

Linear or Non-Linear Systems

The system is called *Linear* if outputs of linear combinations of inputs is the same linear combination of linear combination of their outputs.

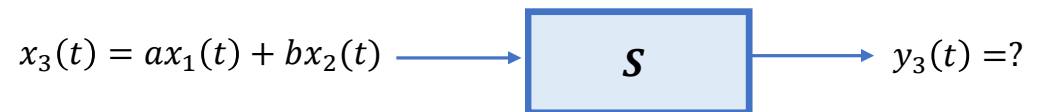
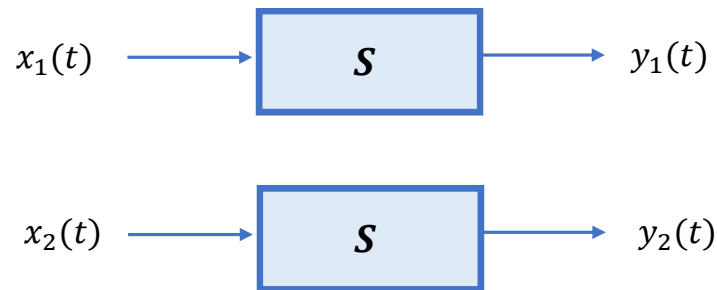


If the system is linear, then $y_3(t) = ay_1(t) + by_2(t)$

Systems Classification

Linear or Non-Linear Systems

The system is called *Linear* if outputs of linear combinations of inputs is the same linear combination of linear combination of their outputs.



If the system is linear, then $y_3(t) = ay_1(t) + by_2(t)$

If a system is linear, then output of the system to $x(t) = 0$ is always $y(t) = 0$.

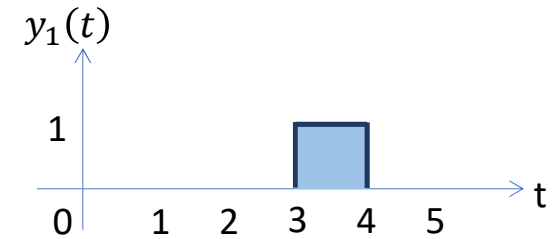
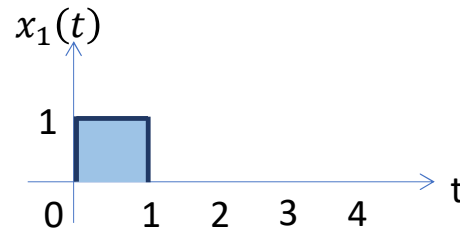
why? $x_1(t) = x_2(t)$ $a = 1$ $b = -1$ $x_3(t) = x_1(t) - x_1(t) = 0 \rightarrow y_3(t) = y_1(t) - y_1(t) = 0 \checkmark$

Can linear systems have initial conditions that generate nonzero output? no

Linear/ Nonlinear Systems

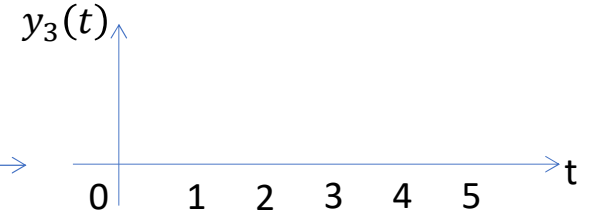
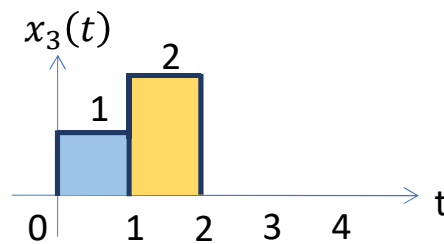
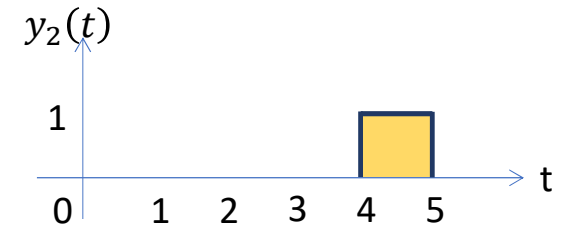
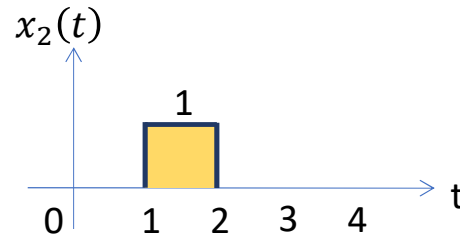
Example:

$$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t - 3)$$



$x_3(t) = x_1(t) + 2x_2(t)$
 linear combination of $x_1(t)$ and $x_2(t)$.

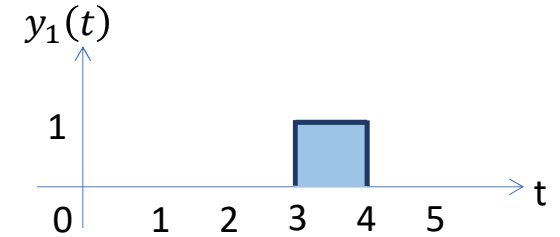
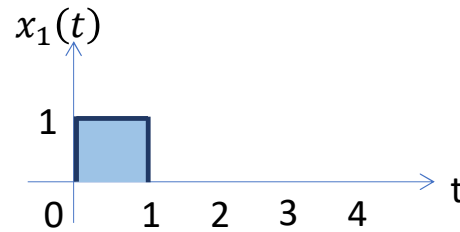
what is $y_3(t)$?



Linear/ Nonlinear Systems

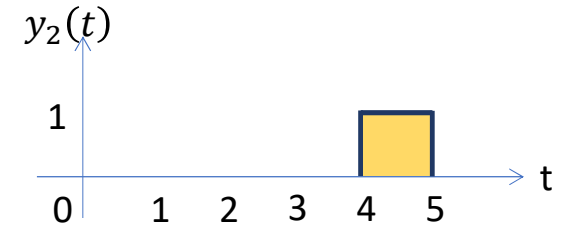
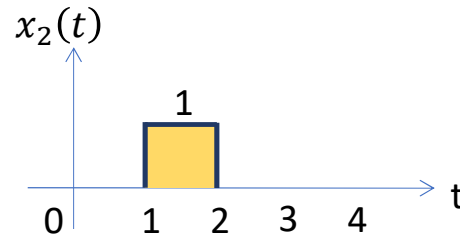
Example:

$$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t - 3)$$

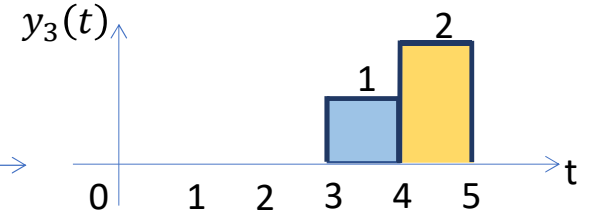
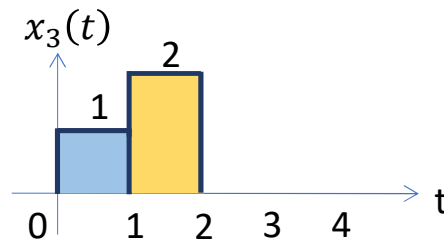


$x_3(t) = x_1(t) + 2x_2(t)$
linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?



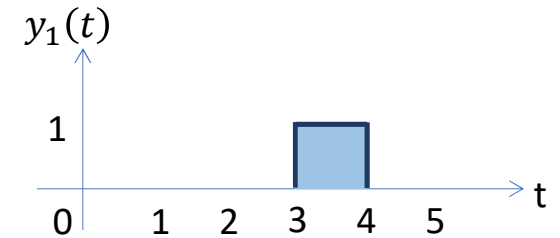
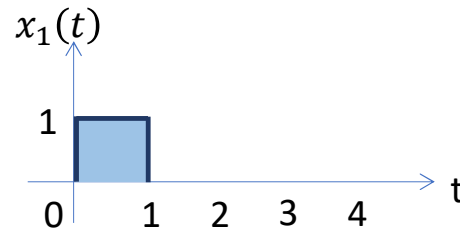
$$y_3(t) = x_3(t - 3)$$



Linear/ Nonlinear Systems

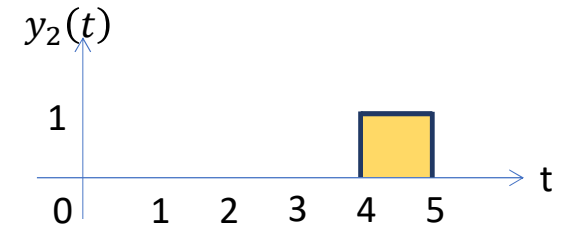
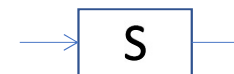
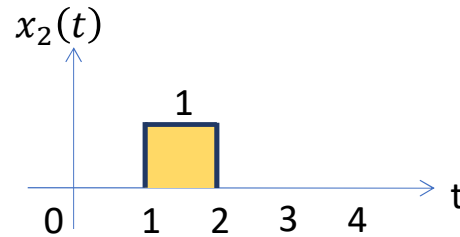
Example:

$$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t - 3)$$



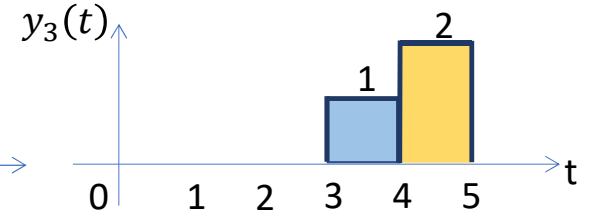
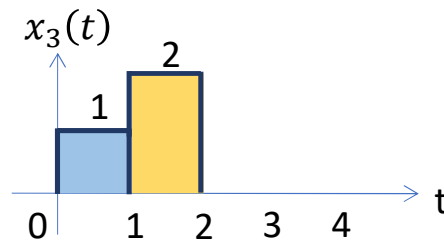
$x_3(t) = x_1(t) + 2x_2(t)$
linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?



$$y_3(t) = x_3(t - 3)$$

$$= y_1(t) + 2y_2(t)$$

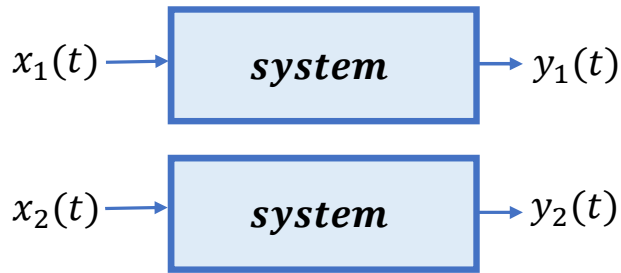


Can we conclude that the system is linear?

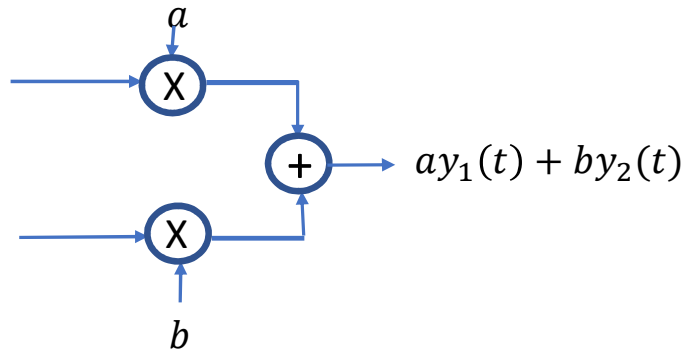
Linear/ Nonlinear Systems

Five steps for checking whether a system is linear or not

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



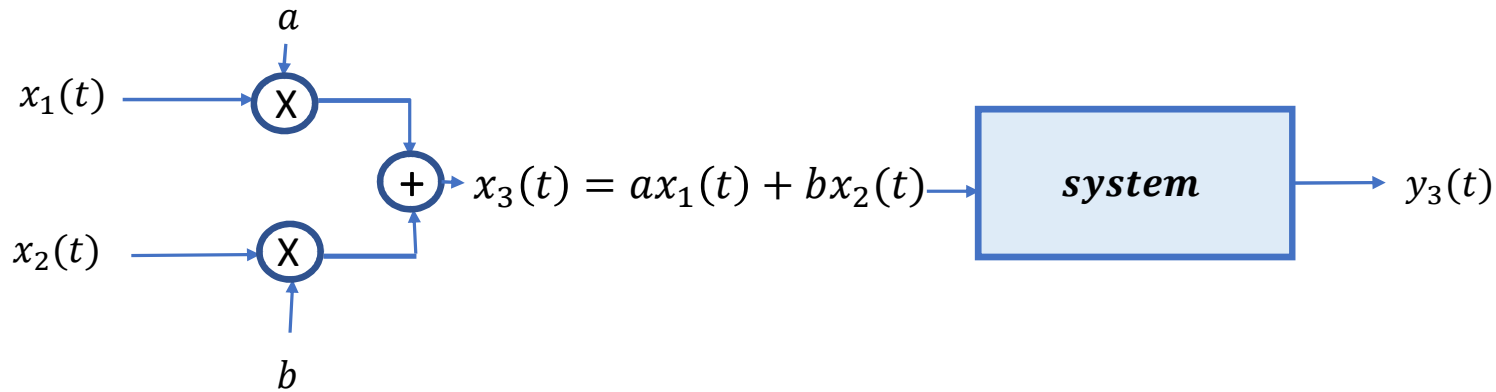
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
Check if result of step 2 is
the same as result of step 4:

If the answer is yes,
the system is **linear**

Step 3: Build $x_3(t)$

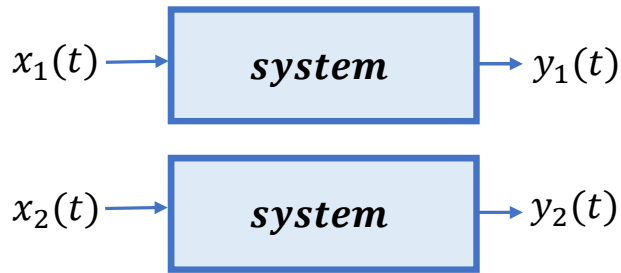


Step 4: Find the out put of the system to $x_3(t)$

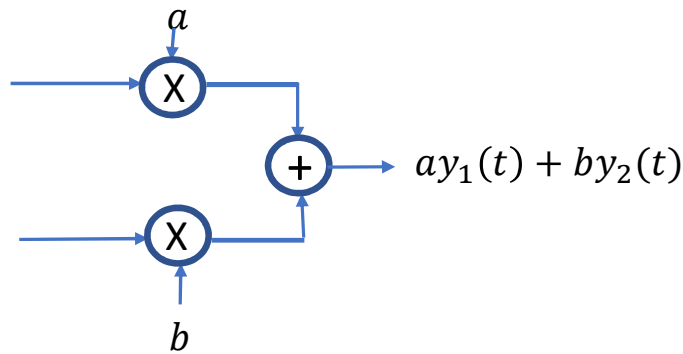
Linear/ Nonlinear Systems

Check the steps for $y(t) = x(t - 3)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$

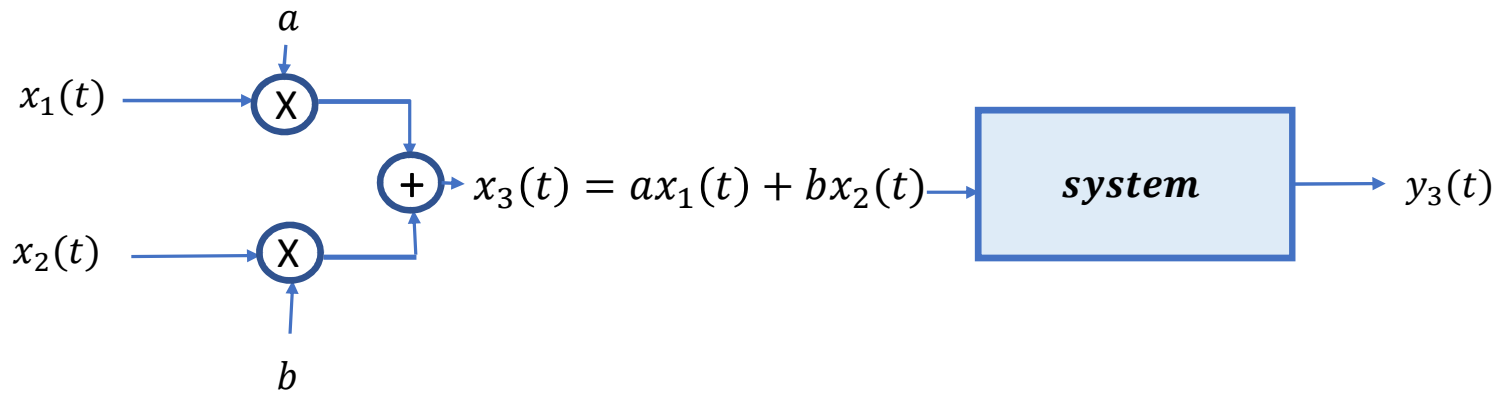


Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
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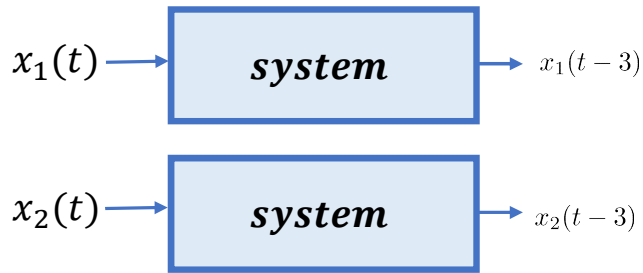
Step 3: Build $x_3(t)$



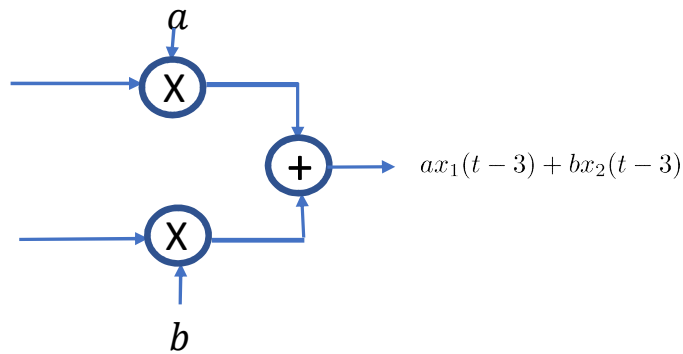
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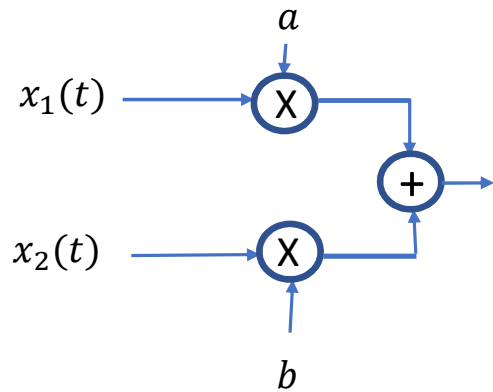
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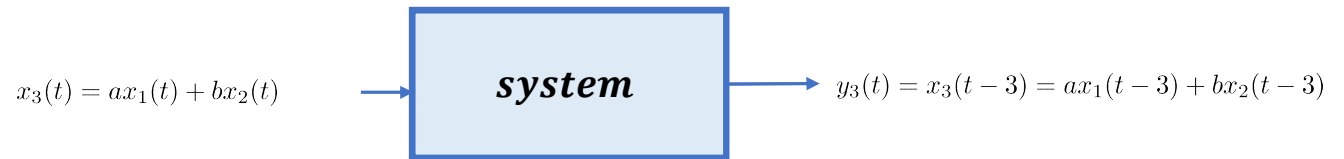
Step 5:
Check if result of step 2 is
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The two are the
same so the
system is **linear**

Step 3: Build $x_3(t)$



Step 4: Find the out put of the system to $x_3(t)$

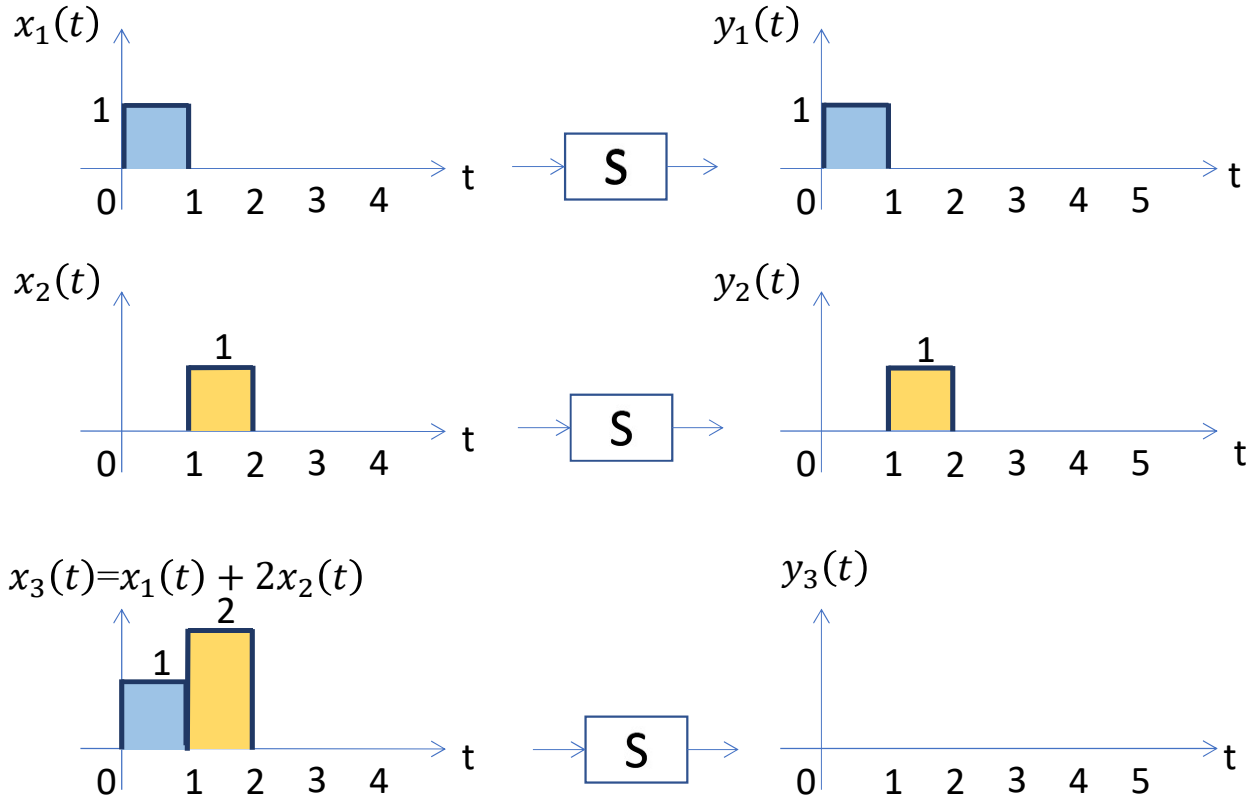


Linear/ Nonlinear Systems

Example 2: $y(t) = x^2(t)$

$x_3(t) = x_1(t) + 2x_2(t)$
linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?



Linear/ Nonlinear Systems

Example 2: $y(t) = x^2(t)$

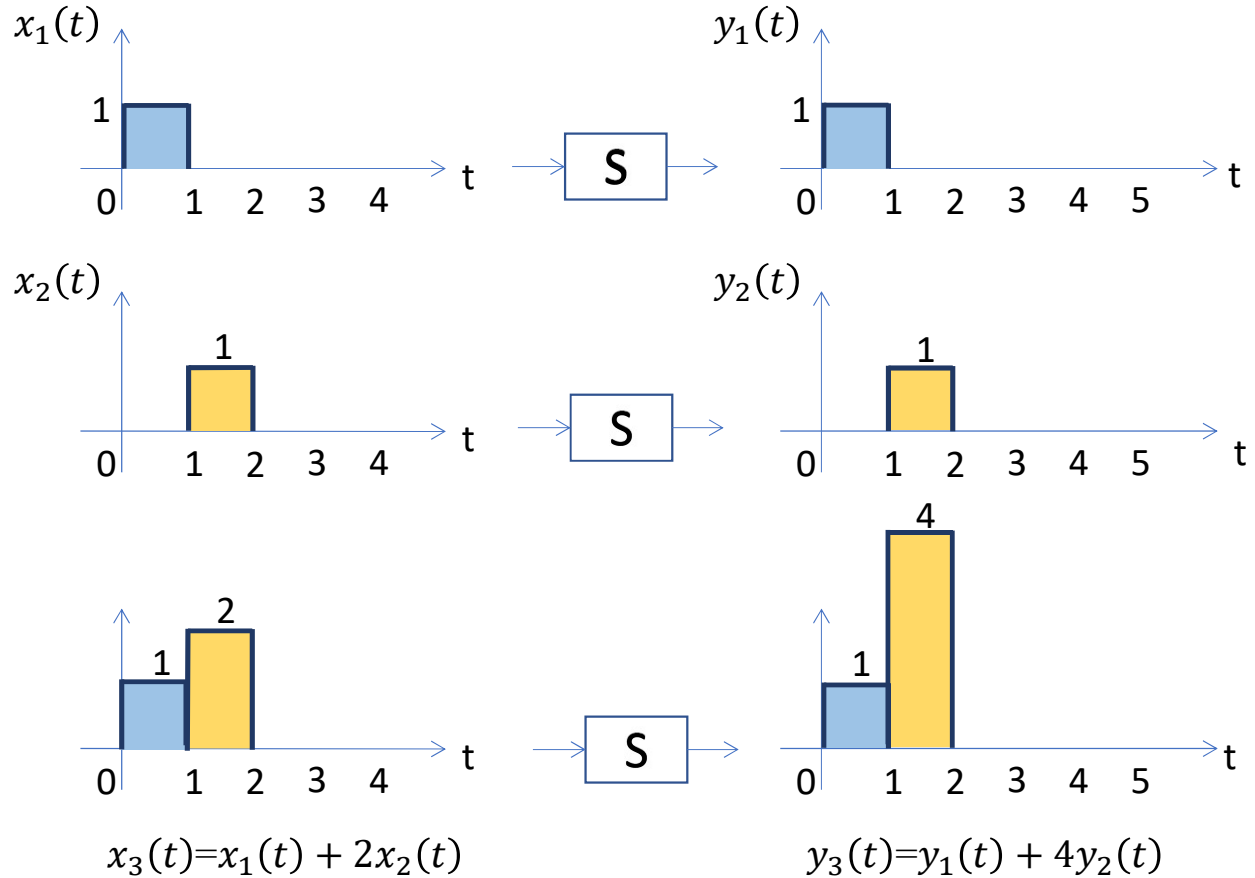
$x_3(t) = x_1(t) + 2x_2(t)$
 linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?

$$y_3(t) = x_3^2(t)$$

$$= y_1^2 + 4y_2(t)$$

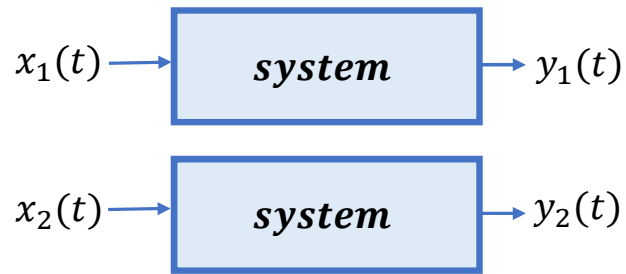
not linear!



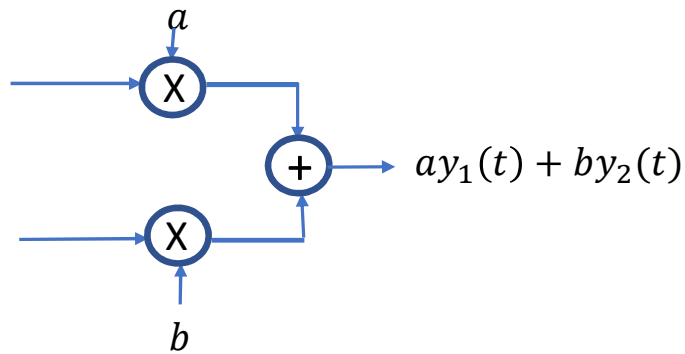
Linear/ Nonlinear Systems

Check the steps for $y(t) = x^2(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



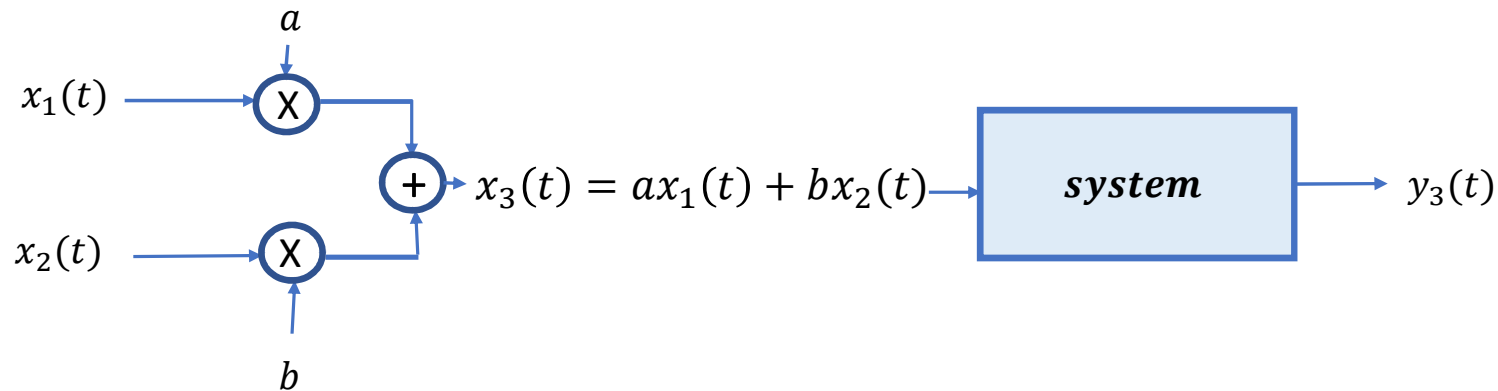
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
Check if result of step 2 is
the same as result of step 4:

If yes, the system is **Linear**

Step 3: Build $x_3(t)$

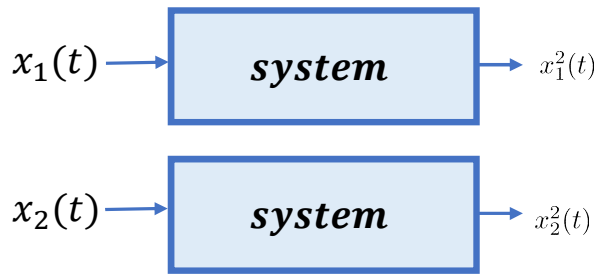


Step 4: Find the out put of the system to $x_3(t)$

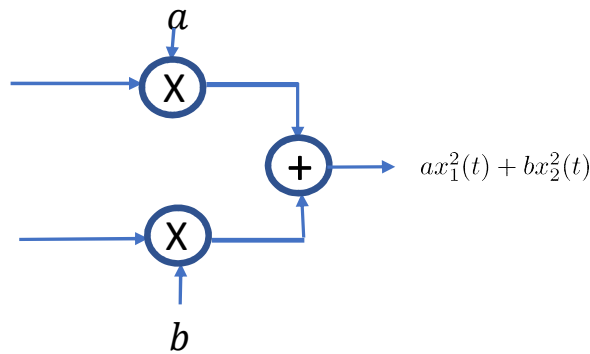
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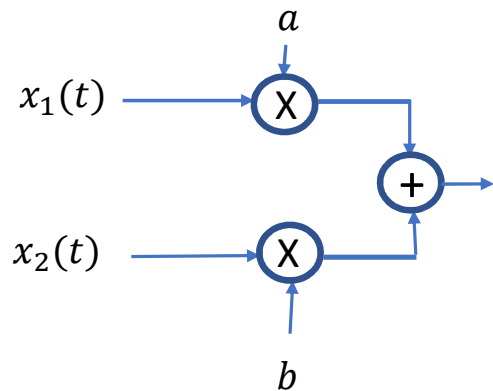
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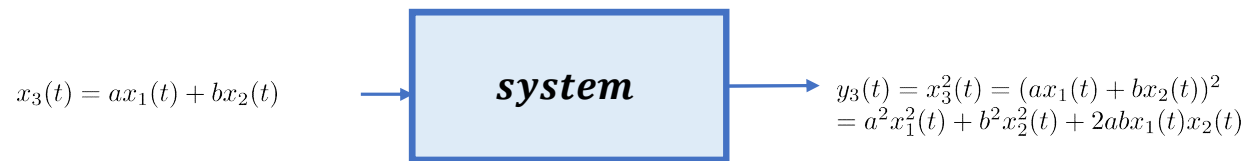
Step 5:
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The two are not
the same so the
system is **Not**
Linear

Step 3: Build $x_3(t)$



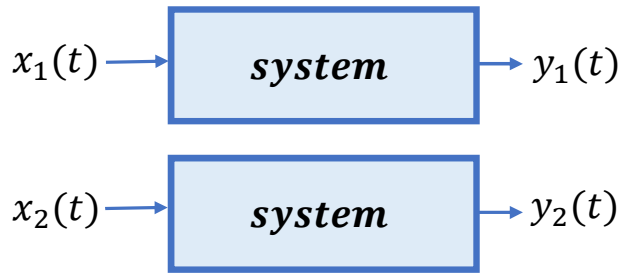
Step 4: Find the out put of the system to $x_3(t)$



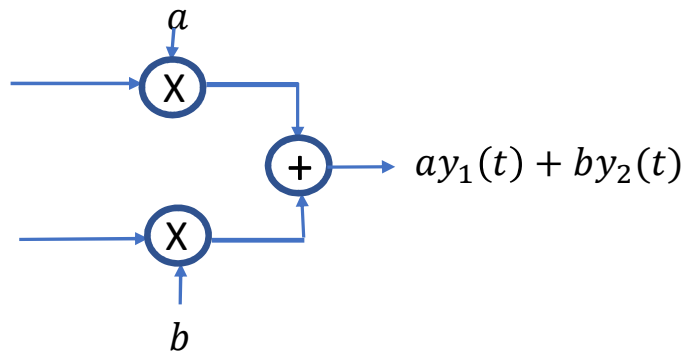
Linear/ Nonlinear Systems

Check the steps for $y(t) = x(t) - 3$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



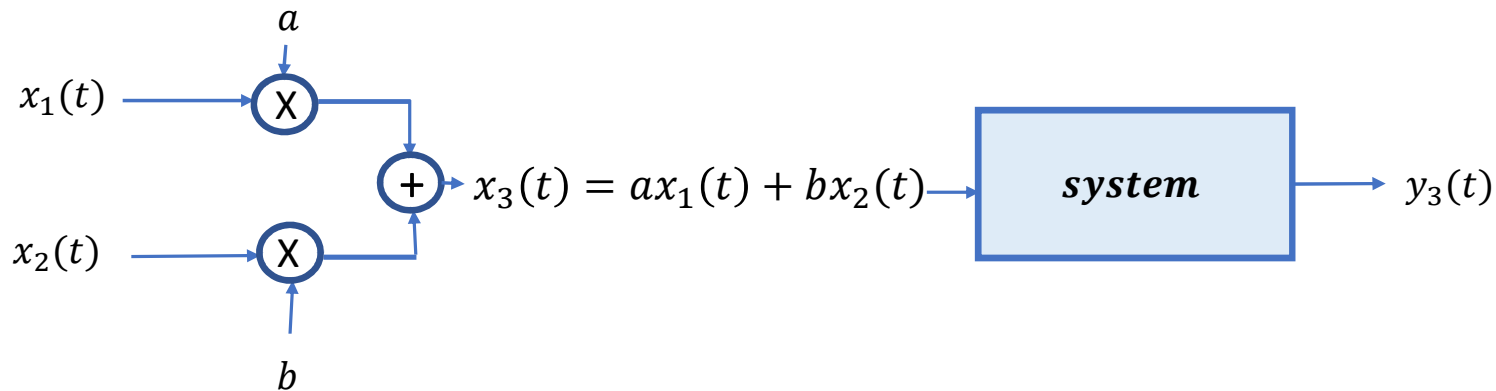
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
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If yes, the system is **Linear**

Step 3: Build $x_3(t)$

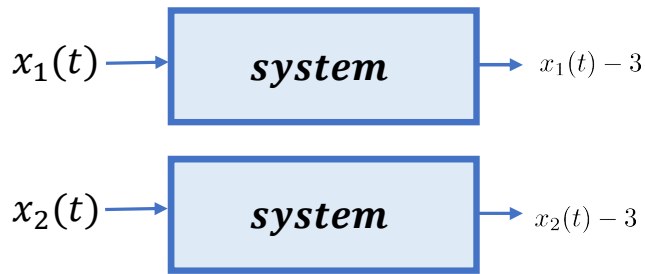


Step 4: Find the out put of the system to $x_3(t)$

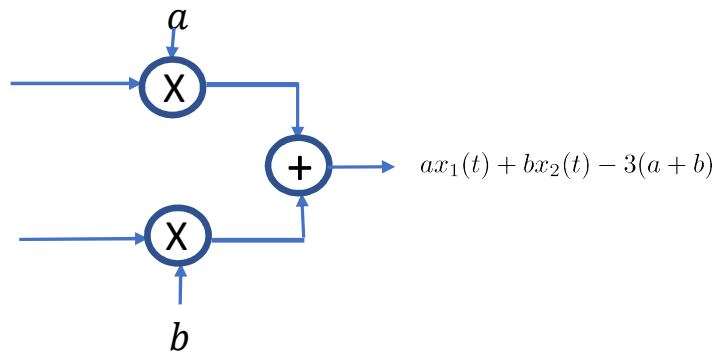
Linear/ Nonlinear Systems

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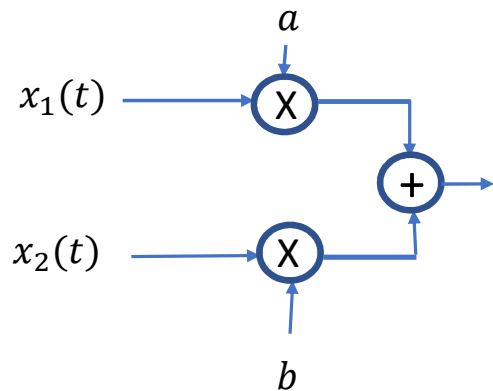
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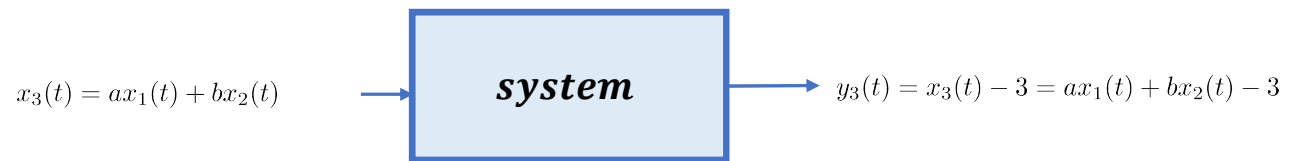
Step 5:
Check if result of step 2 is the same as result of step 4:

The two are not the same so the system is **Not Linear**

Step 3: Build $x_3(t)$



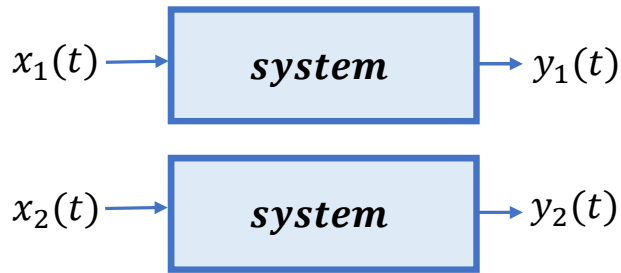
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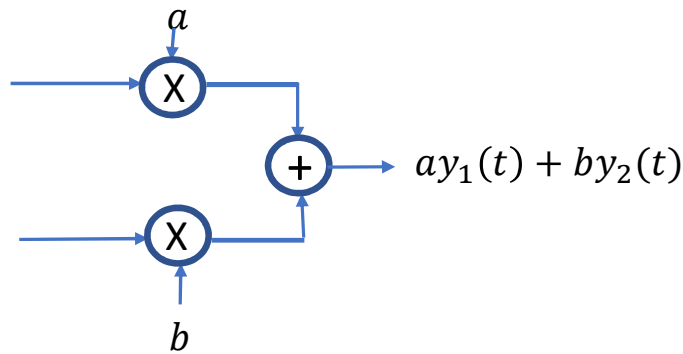
Linear/ Nonlinear Systems

Check the steps for $y(t) = x(2t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



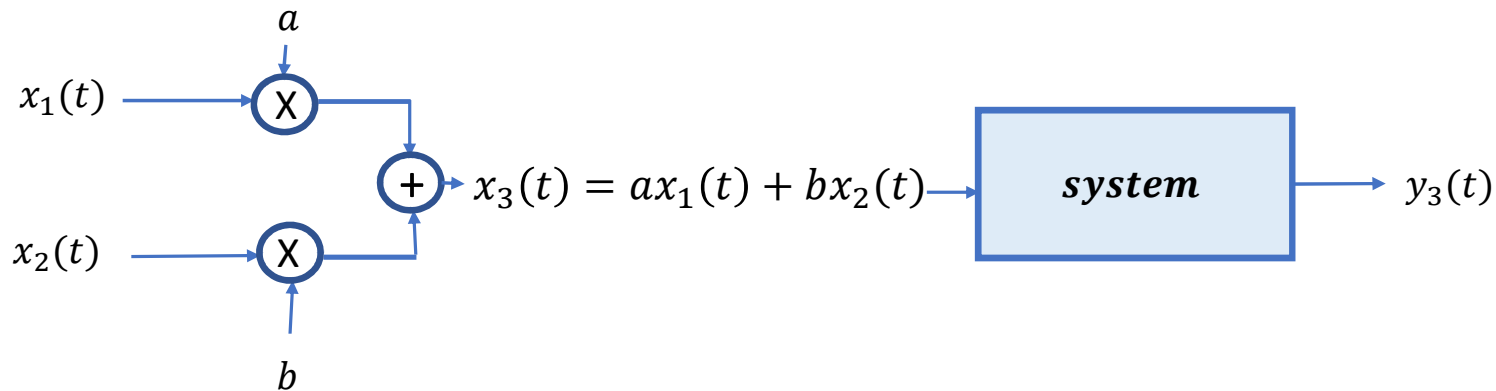
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
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If yes, the system is **Linear**

Step 3: Build $x_3(t)$

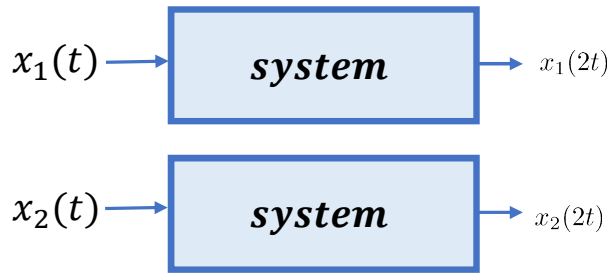


Step 4: Find the out put of the system to $x_3(t)$

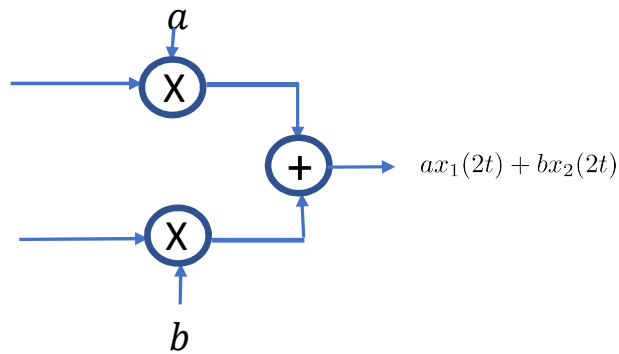
Linear/ Nonlinear Systems

Check the steps for $y(t) = x(2t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



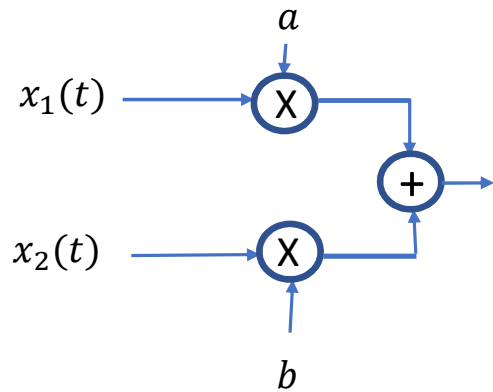
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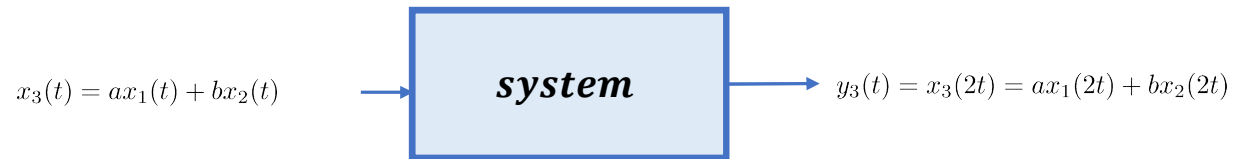
Step 5:
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The two are the
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system is
Linear

Step 3: Build $x_3(t)$



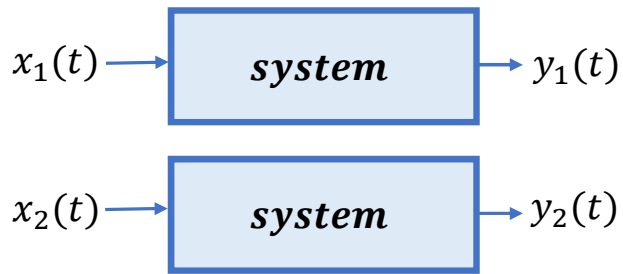
Step 4: Find the out put of the system to $x_3(t)$



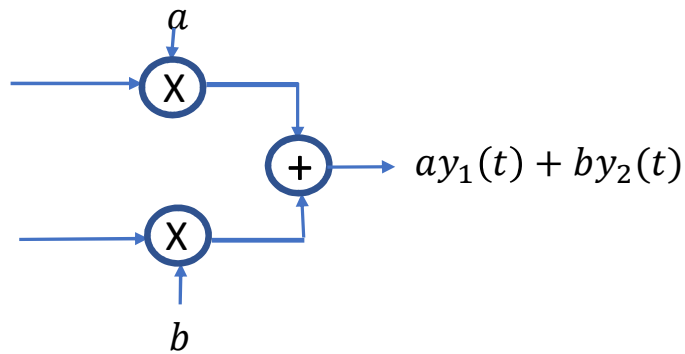
Linear/ Nonlinear Systems

Check the steps for $y(t) = t^2x(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



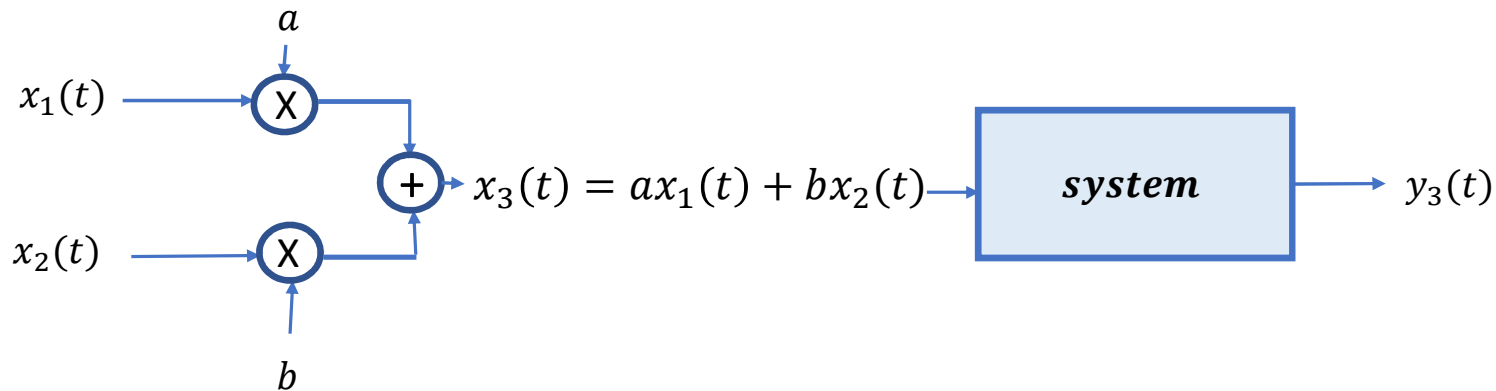
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
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If yes, the system is **Linear**

Step 3: Build $x_3(t)$

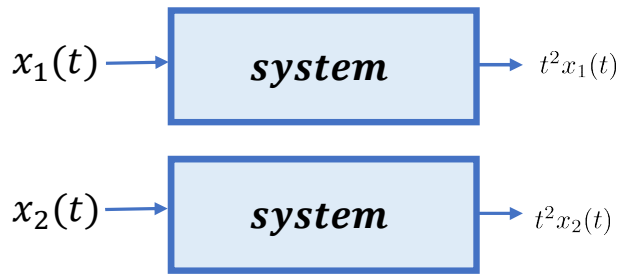


Step 4: Find the out put of the system to $x_3(t)$

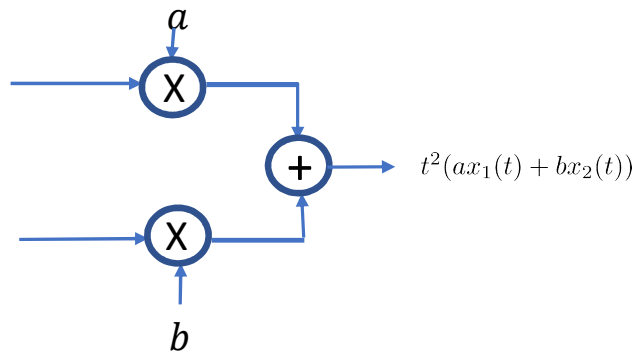
Linear/ Nonlinear Systems

Check the steps for $y(t) = t^2 x(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



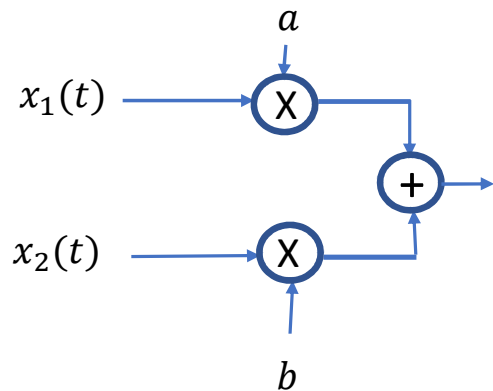
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



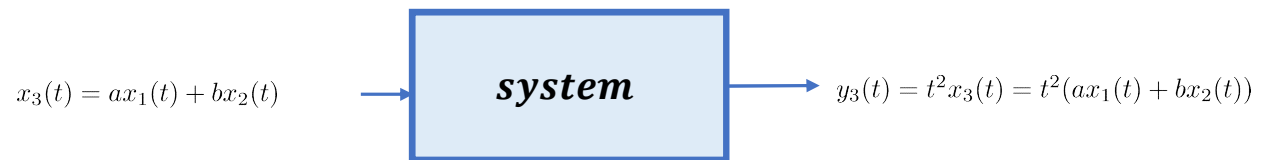
Step 5:
Check if result of step 2 is
the same as result of step 4:

The two are the
same so the
system is
Linear

Step 3: Build $x_3(t)$



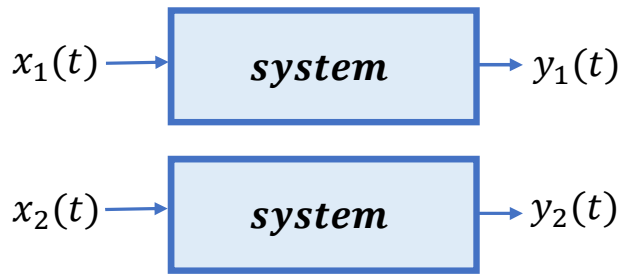
Step 4: Find the out put of the system to $x_3(t)$



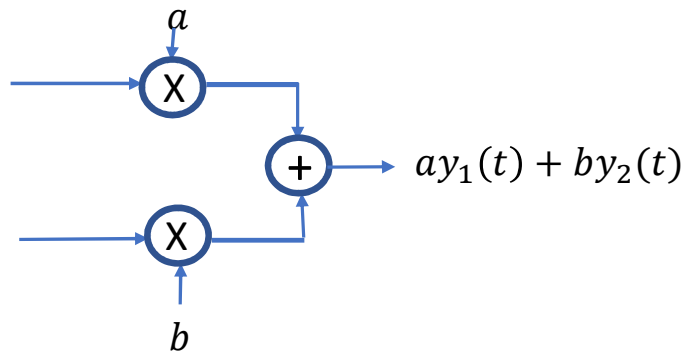
Linear/ Nonlinear Systems

Check the steps for a system with following equation $\frac{dy(t)}{dt} + t^2y(t) = 2tx(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



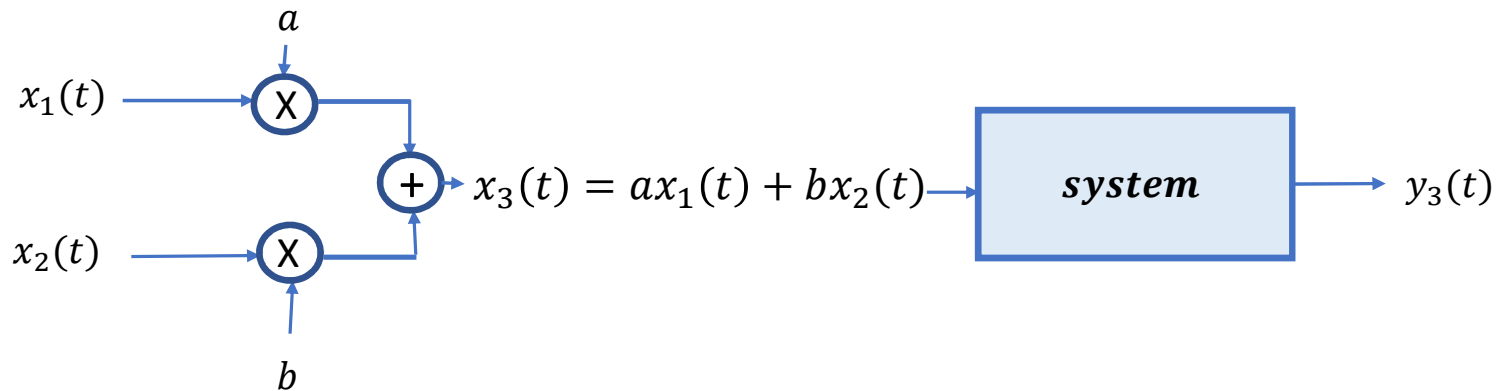
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
Check if result of step 2 is
the same as result of step 4:

If yes, the system is **Linear**

Step 3: Build $x_3(t)$

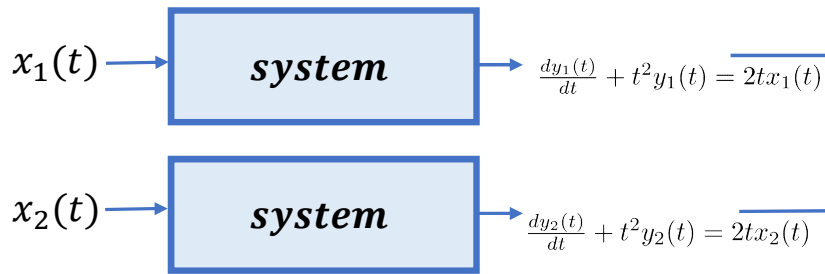


Step 4: Find the out put of the system to $x_3(t)$

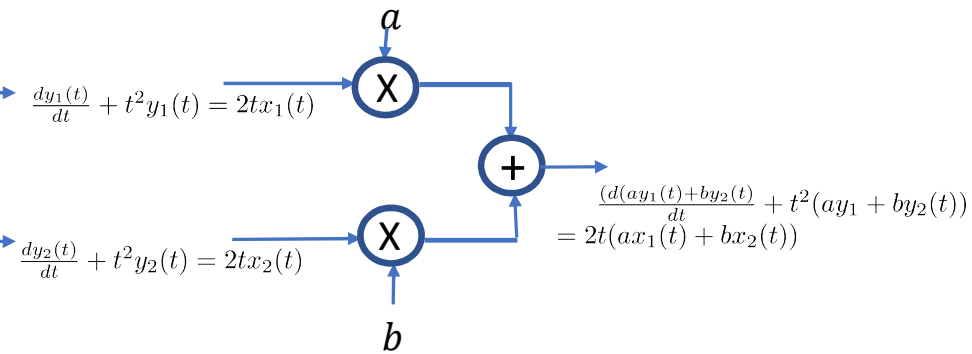
Linear/ Nonlinear Systems

Check the steps for a system with following equation $\frac{dy(t)}{dt} + t^2y(t) = 2tx(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$

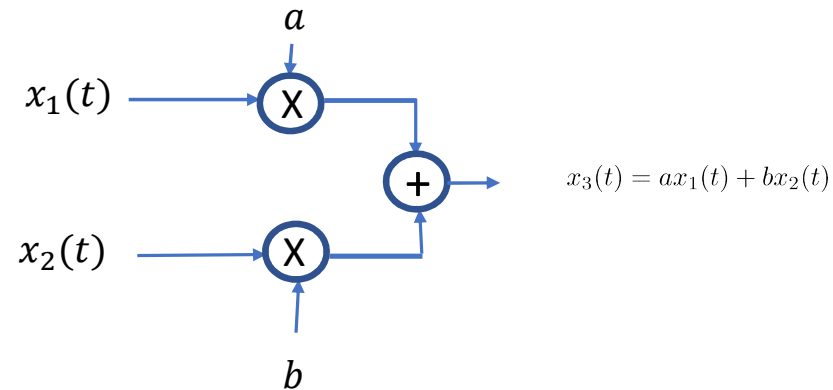


Step 5:
Check if result of step 2 is the same as result of step 4:

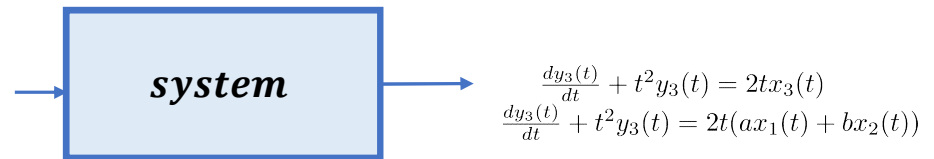
Here we can replace $y_3(t)$ is step 4 with $ay_1(t) + by_2(t)$ in step 2 as both equations then become identical.

so the system **Linear**

Step 3: Build $x_3(t)$



Step 4: Find the out put of the system to $x_3(t)$



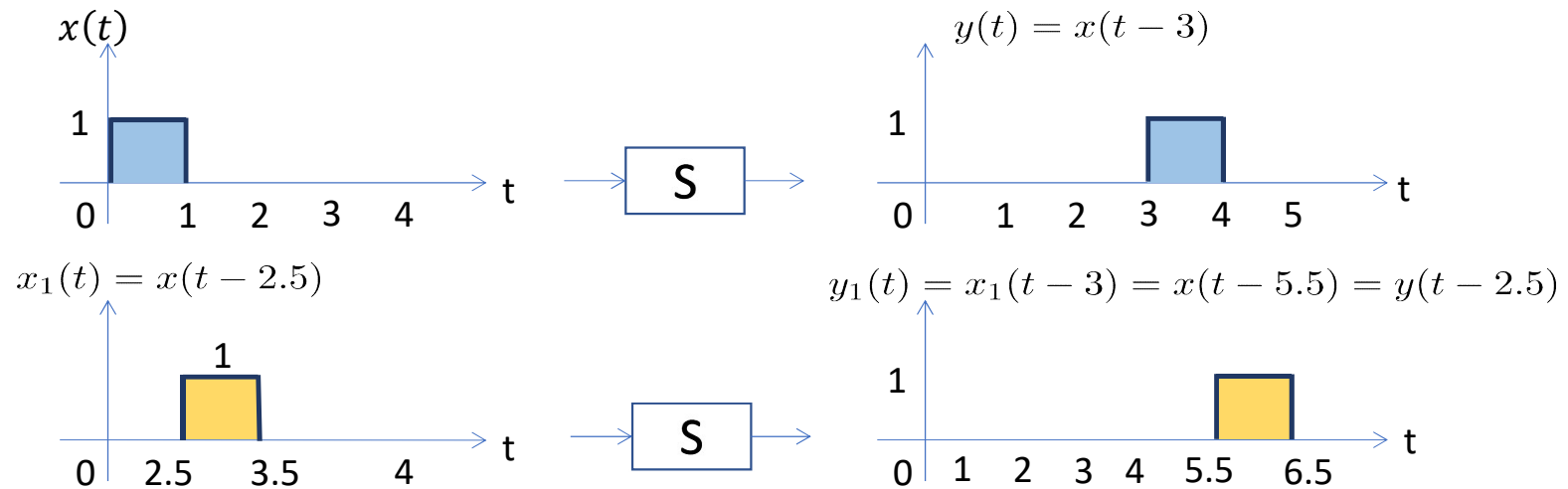
Time Invariant(TI) / Time Variant (TV) Systems

System S with output $y(t) = S(x(t))$ is *TI* if and only if

$$y(t - T) = S(x(t - T))$$

“Time shift T in input results in time shift T in output.”

Example:



Time Invariant(TI) / Time Variant (TV) Systems

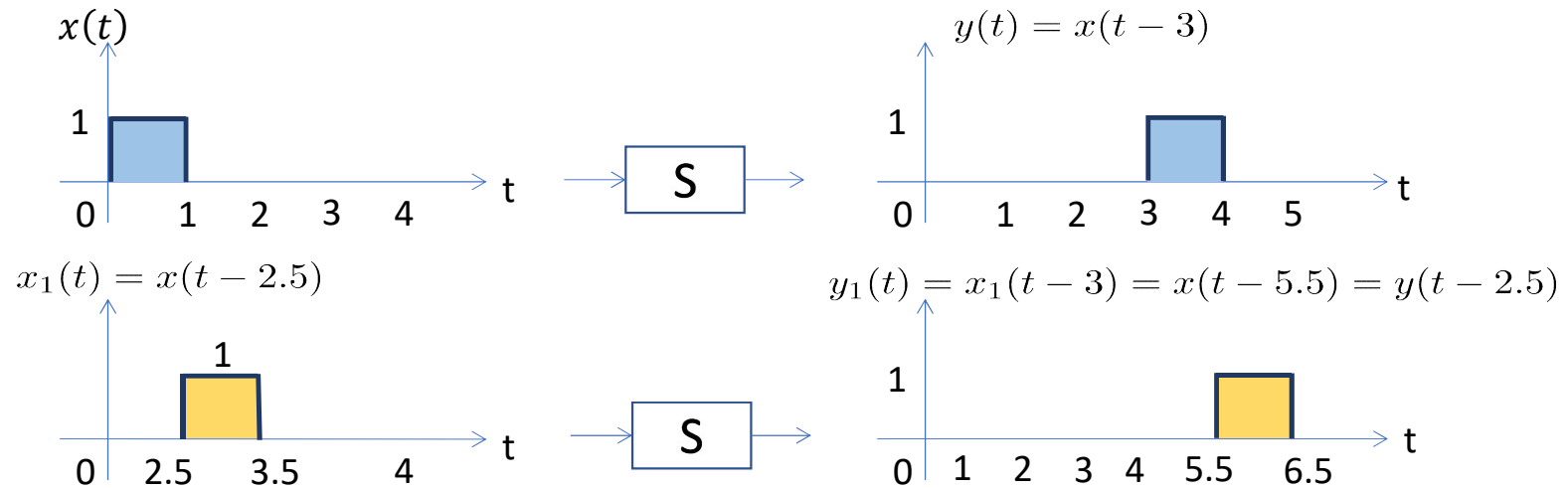
System S with output $y(t) = S(x(t))$ is *TI* if and only if

$$y(t - T) = S(x(t - T))$$

“Time shift T in input results in time shift T in output.”

Holds for $T = 2.5$
is the system TI?

Example:



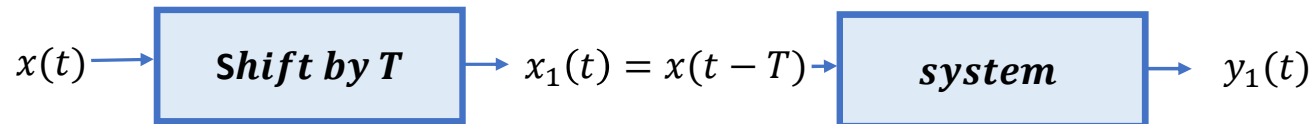
Time Invariant(TI) / Time Variant (TV) Systems

Three steps to check if a system is TI:

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

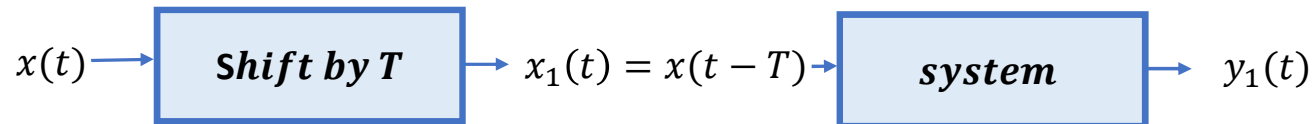
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t - 3)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



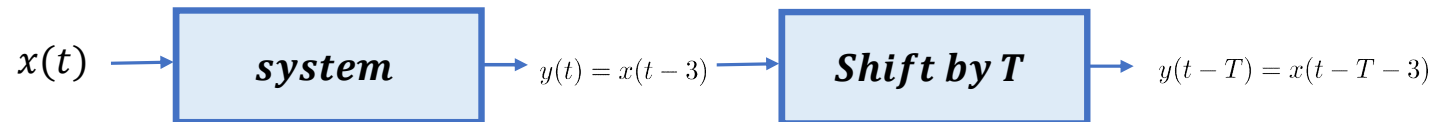
Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

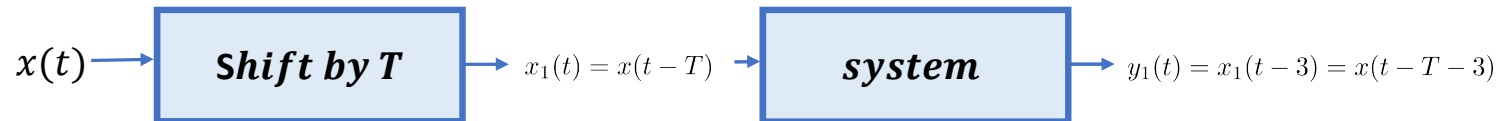
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t - 3)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



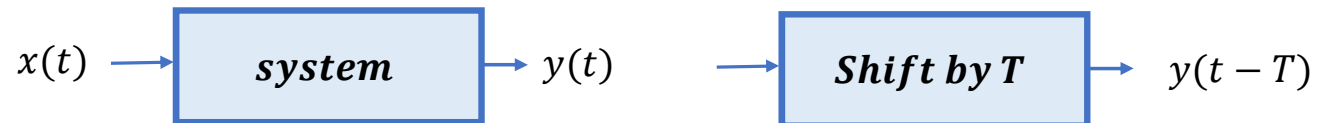
Step 3:
Check if result of step 1 is
the same as result of step 2:

Results of step 1 and step 2 are
the same the system is
Time Invariant

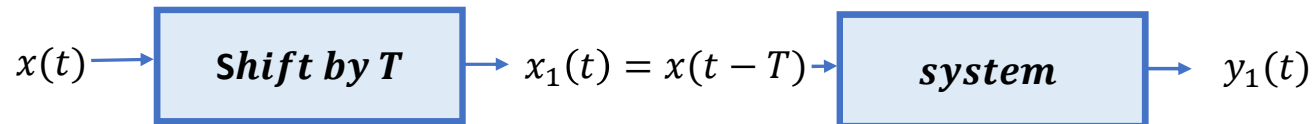
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x^2(t)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



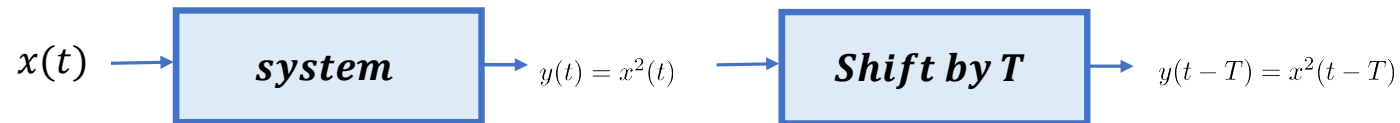
Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

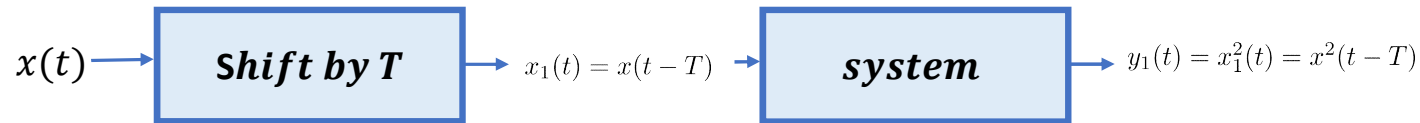
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x^2(t)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



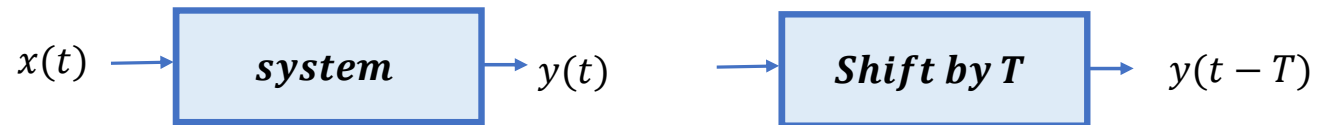
Step 3:
Check if result of step 1 is
the same as result of step 2:

Results of step 1 and step 2 are
the same the system is
Time Invariant

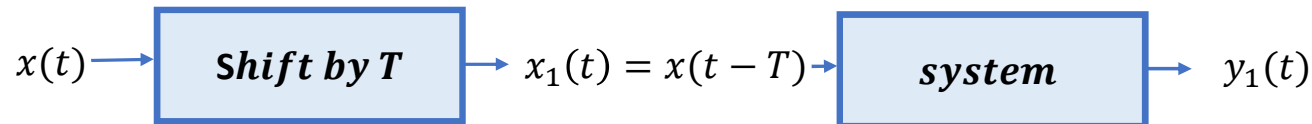
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t) + 3$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



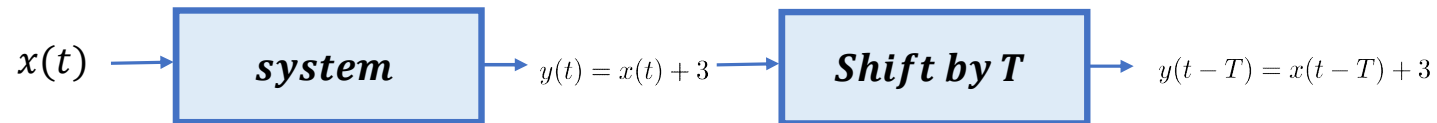
Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

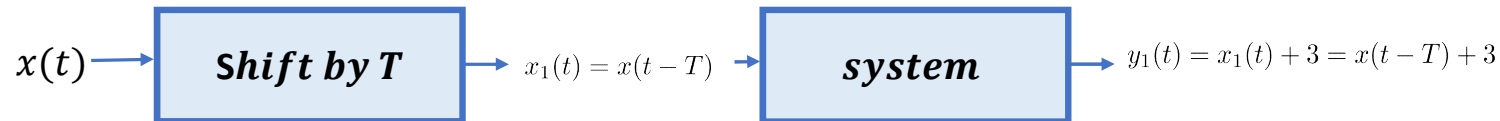
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t) + 3$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$

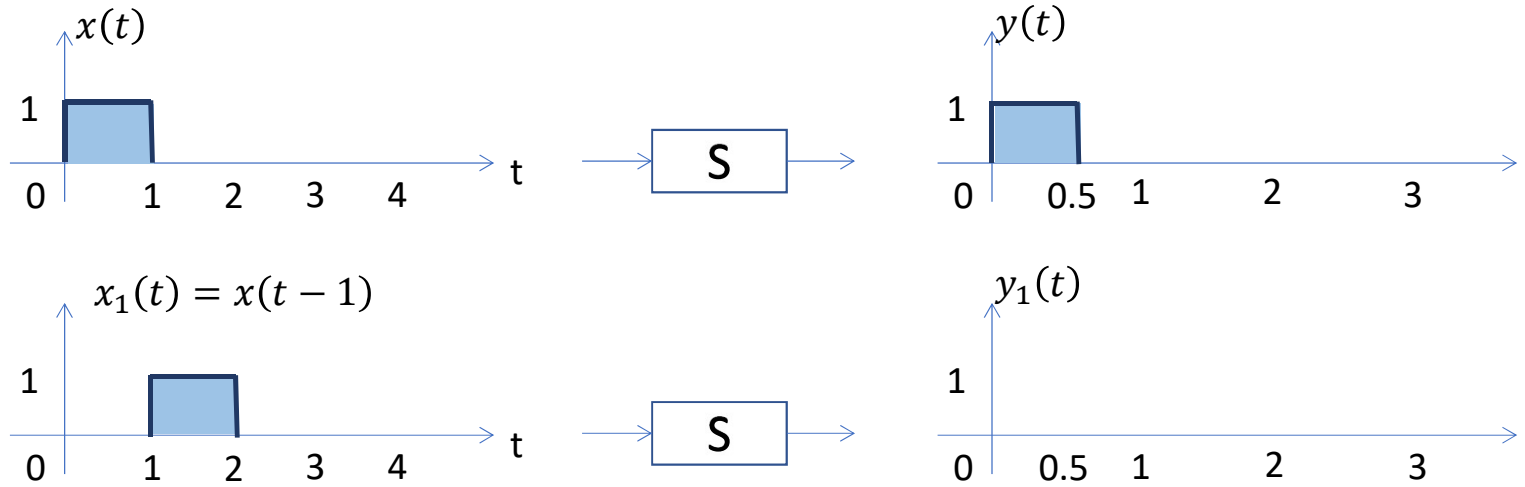


Step 3:
Check if result of step 1 is
the same as result of step 2:

Results of step 1 and step 2 are
the same the system is
Time Invariant

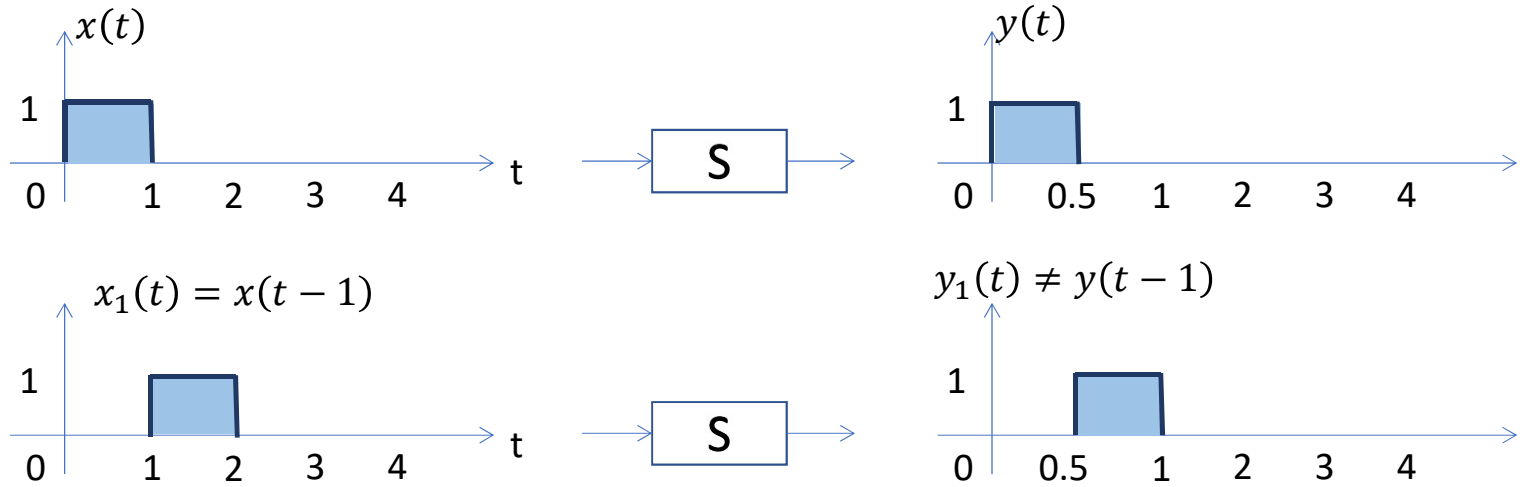
Time Invariant(TI) / Time Variant (TV) Systems

Is $y(t) = x(2t)$ Time Invariant?



Time Invariant(TI) / Time Variant (TV) Systems

Is $y(t) = x(2t)$ Time Invariant?



Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(2t)$

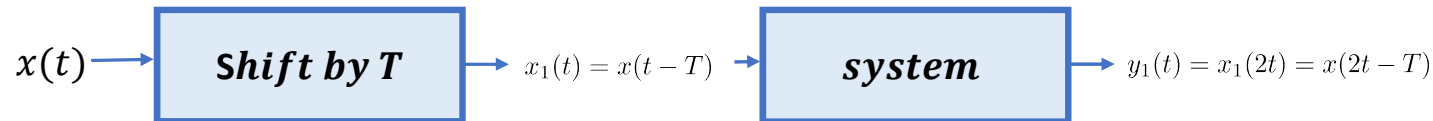
Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 3:
Check if result of step 1 is
the same as result of step 2:

Results of step 1 and step 2 are not
the same, the system is
Time Varying

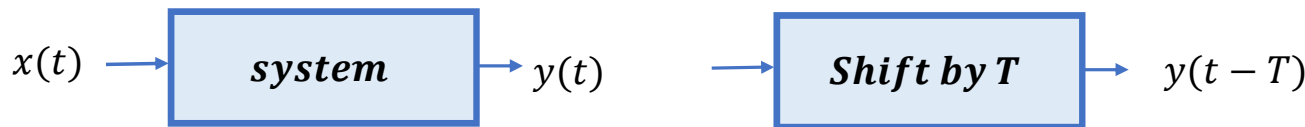
Step 2: Find the output of system to shifted version $x(t)$



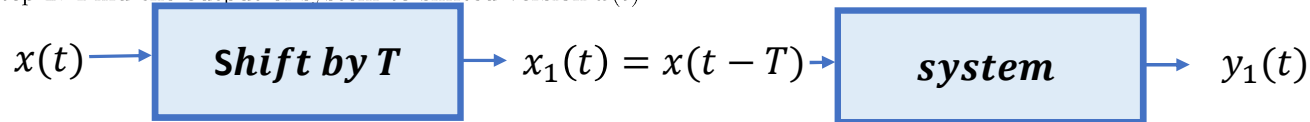
Time Invariant(TI) / Time Variant (TV) Systems

Is system $y(t) = x(-t)$ Linear? is it time invariant?

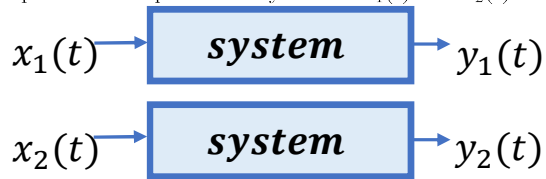
Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



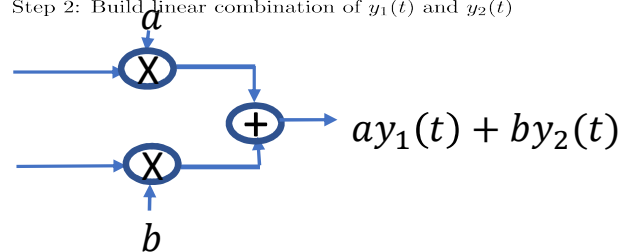
Step 2: Find the output of system to shifted version $x(t)$



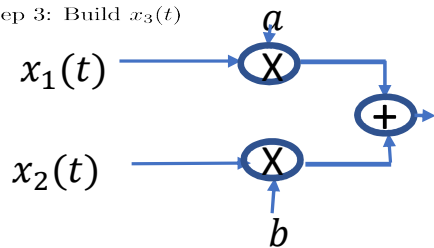
Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 3: Build $x_3(t)$



Step 4: Find the out put of the system to $x_3(t)$



Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

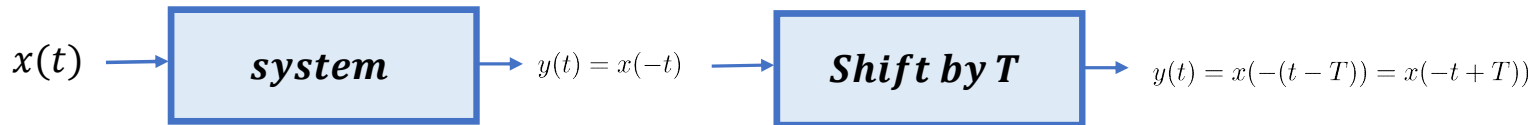
Step 5:
Check if result of step 2 is
the same as result of step 4:

If yes, the system is **Linear**

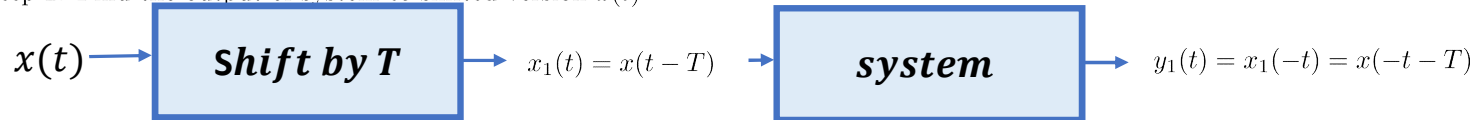
Time Invariant(TI) / Time Variant (TV) Systems

Is system $y(t) = x(-t)$ Linear? is it time invariant?

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



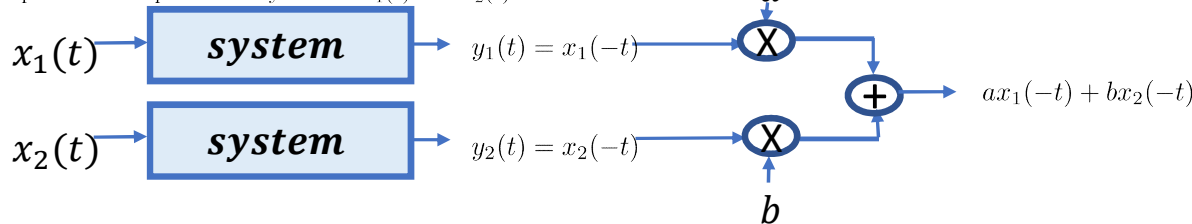
Step 2: Find the output of system to shifted version $x(t)$



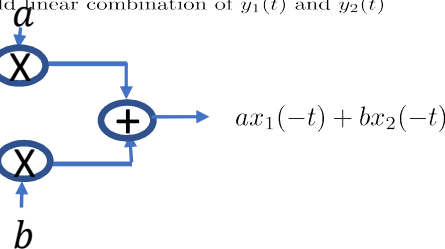
Step 3:
The result of step 1 is not the same as result of step 2:

The system is **Time Varying**

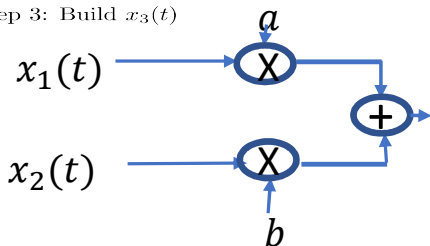
Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



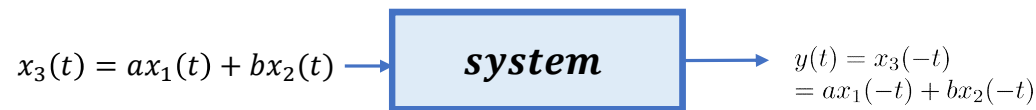
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 3: Build $x_3(t)$



Step 4: Find the out put of the system to $x_3(t)$



Step 5:
The result of step 2 is the same as result of step 4:

The system is **Linear**

System Classification: Memoryless (instantaneous)/with memory(dynamic)

Memoryless system's output at time t only depends on input at time t . Otherwise the system is with memory!

Which of these systems are memoryless?

- ✓ (a) $x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$
- (b) $x(t) \rightarrow \boxed{S} \rightarrow x(t + 2) = y(t)$
- (c) $x(t) \rightarrow \boxed{S} \rightarrow x(t - 1) + x(t)$
- ✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)}$
- (e) $x(t) \rightarrow \boxed{S} \rightarrow x(-t) = y(t)$
- (f) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(v)dv = y(t)$
- (g) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^t x(v)dv = y(t)$
- ✓ (h) $x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$

Any system that takes the derivative or integral of the input has memory!

System Classification: Causal/ Non-Causal Systems

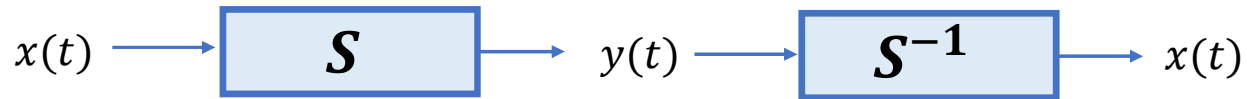
Causal system's output at time t_0 only depends on input values at time t_0 and at times before t_0 , i.e., only depends on $t \leq t_0$.

Which of these systems are causal?

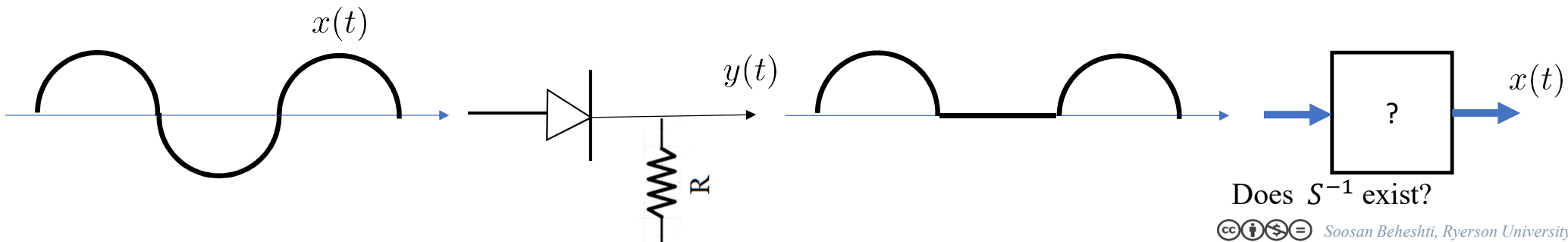
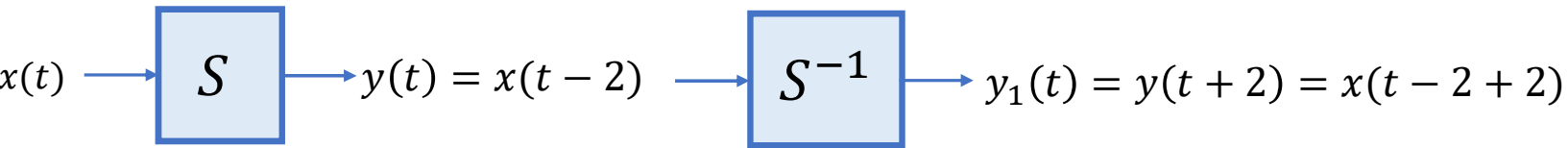
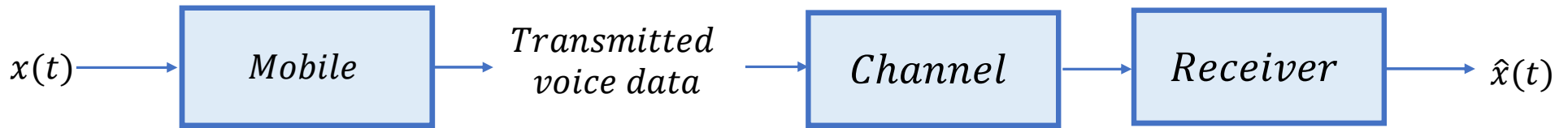
- ✓ (a) $x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$
- (b) $x(t) \rightarrow \boxed{S} \rightarrow x(t + 2) = y(t)$
- ✓ (c) $x(t) \rightarrow \boxed{S} \rightarrow x(t - 1) + x(t)$
- ✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)} = y(t)$
- (e) $x(t) \rightarrow \boxed{S} \rightarrow x(-t) = y(t)$
- (f) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(v)dv = y(t)$
- ✓ (g) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^t x(v)dv = y(t)$
- ✓ (h) $x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$

System Classification: Invertible/ Non-invertible Systems

System S is invertible if there exists a system S^{-1} such that:



(Important desired property in design of many systems such as communications systems).



System Classification: Invertible/ Non-invertible Systems

Which of these systems are invertible?

- ✓ (a) $x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$
- ✓ (b) $x(t) \rightarrow \boxed{S} \rightarrow x(t + 2)$
- (c) $x(t) \rightarrow \boxed{S} \rightarrow x(t - 1) + x(t)$
- ✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)}$
- ✓ (e) $x(t) \rightarrow \boxed{S} \rightarrow x(-t)$
- (f) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(v) dv$
- ✓ (g) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^t x(v) dv$
- (h) $x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$

System Classification: Stable/Unstable Systems

External stability or Bounded input/Bounded output (BIBO) stability

$$\text{If } |x(t)| < C_1, \forall t \rightarrow \exists c > 0 : |y(t)| < C \forall t$$

We will discuss Internal Stability later

Serious concepts in design of Control Systems (ELE639)

System Classification: Stable/Unstable Systems

Which of these systems are externally stable?

- ✓ (a) $x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$
- ✓ (b) $x(t) \rightarrow \boxed{S} \rightarrow x(t + 2)$
- ✓ (c) $x(t) \rightarrow \boxed{S} \rightarrow x(t - 1) + x(t)$
- ✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)}$
- ✓ (e) $x(t) \rightarrow \boxed{S} \rightarrow x(-t)$
- (f) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(v) dv$
- (g) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^t x(v) dv$
- ✓ (h) $x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$