Power Distribution/Allocation in Multirate Wideband CDMA Systems

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***Abstract*— A unified approach for power distribution and allocation in a multirate wideband CDMA system is investigated. It is shown that the traffic demand and the background disturbance fully govern the feasibility of the system and the optimal power distribution solutions, where the traffic demand is specified by the user QoS requirement, data rate, and spread spectrum bandwidth; the background disturbance includes the background noise and the intercell interference. Closed form expressions of the optimal power allocation, subject to power constraints in the practical system design, are derived. Convergent conditions are applied to evaluate the capacity region of the system. Numerical examples are provided to illustrate the applications of the obtained theoretical results.**

***Index Terms*— CDMA, power allocation, multirate, system feasibility, capacity region.**

I. Introduction

THIRD generation (3G) wireless systems and beyond are expected to provide a wide variety of services, *e.g.*, voice, data, facsimile, video. Direct-sequence code division multiple access (DS-CDMA) communication systems have been shown to exhibit great flexibility in supporting multi-class services and have been adopted as a multiple access method for 3G systems.

In a CDMA system, different approaches can be used to control the information rate and performance of different services, *e.g.*, by varying the chip rate [1], the processing gain [2], [3], the number of codes [4], and the modulation format [5]. Here, we focus our attention on a variable processing gain technique for the multirate transmission, although the results obtained can be extended to systems using other techniques.

It is well known that CDMA is interference limited [6]-[9]. Multiple-access interference (MAI) is the dominant parameter that governs the system capacity. High target received power requires high transmit power, which leads to increased MAI. Therefore, effective power allocation can minimize the effect of MAI, thereby enhancing the system utilization.

In this paper, we explore effective methods to perform the power distribution and allocation to support multiclass services in a wideband CDMA system, based on the framework established in [10]-[13]. The results obtained in this paper are generic and beyond those presented in [10]-[13].

Power distribution and allocation as a research problem has been receiving much attention in recent years, see e.g., [9], [14], [15]. In these earlier works, relatively simple approaches have been used to solve the power distribution problem for the CDMA uplink model where a uniform background disturbance[1] is observed at the base station. In [16], transmission power control is investigated, incorporating throughput maximization. In [17], the geometrical and topological properties of the capacity region are investigated in context of optimal power allocation. In [18], the optimal power allocation problem is investigated using a utility function approach.

In what follows, we present a unified approach to solve the power distribution problem using matrix analysis. It is shown that finding a solution for the power allocation involves the inversion of the traffic matrix. This is done by invoking the well-known Sherman-Morrison inverse formula for rank-one updated matrices in linear algebra to derive closed form expressions for the power distribution and the corresponding convergence conditions. Our results give the necessary and sufficient conditions to solve the power control problem. The approach and the obtained results are general in the sense that they are equally applicable to both uplink and downlink transmissions. In the latter case, the background disturbance seen by the individual mobile users is normally different. In addition, explicit expressions of optimal solutions for cases with different disturbance scenarios, with and without power constraints, are derived. The resultant power distribution convergence conditions are then used to evaluate the capacity region of the system. Our results show that any form of power control or power distribution is indeed a function of the spread spectrum bandwidth, user data rates, and user quality of service (QoS) specifications [10]-[13]. Our results are quite generic, special cases of which are consistent with works reported in the literature. The feasibility analysis using the traffic demand approach instead of the conventional spectral analysis (eigenvalue problem) is shown to be more tractable, easier, and possesses good physical interpretations.

The remainder of the paper is organized as follows. The power distribution problem under consideration is stated and described in Section II. It is shown that the key to power allocation involves the inversion of the traffic matrix. An explicit expression of the inverse of the traffic demand matrix needed to solve the power allocation problem is obtained in Section III. The results are applied in Section IV to solve the

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[1]The term disturbance is used here to represent the intercell interference plus the ambient noise.

power allocation problem and its convergence conditions for a system under different operating conditions. Numerical results to illustrate the applications of our approach and algorithm are presented in Section V. Finally, concluding remarks are given in Section VI.

II. Statement of the Power Distribution Problem

We are concerned with a generic cell in a wideband CDMA cellular system, which supports M users. The ensuing results are applicable to both uplink and downlink transmissions. For the uplink case, the transmitters are the mobiles and the receiver is the base station (BS), and for the downlink, the transmitter is the BS and the receivers are the mobiles. In the uplink case, the parameter S_j denotes the received power at the BS from mobile j. In the downlink, it denotes the transmission power dedicated to mobile j by the BS. Besides the power level, the jth mobile is specified by its required data rate, R_j, and a target signal-to-interference ratio (SIR), or equivalently a target bit energy-to-interference spectral density ratio (EISDR), $(E_b/I_o)^*_j$. Throughout this paper, we will use EISDR as the QoS measure and let $\gamma^*_j = (E_b/I_o)^*_j$ as the bit energy-to-interference spectral density ratio specification. QoS satisfaction requires that the actual $\gamma_j \geq \gamma^*_j$, where the received γ_j can be expressed as

$$\gamma_j = \frac{W}{R_j} \cdot \frac{S_j}{I_j}. \tag{1}$$

Here W is the spread spectrum bandwidth, and I_j is the total interference imposed on the desired signal.

For given values of R_j and W, a specification on γ_j is equivalent to a specification on S_j/I_j. Let S^*_j be the target received power at the receiver to achieve the γ^*_j value. Then S^*_j can be expressed as

$$S^*_j = \frac{1}{W} R_j \gamma^*_j I_j. \tag{2}$$

For satisfactory operation, the received signal power for the jth user must satisfy the inequality

$$S_j \geq S^*_j = \frac{1}{W} R_j \gamma^*_j I_j = \Gamma_j I_j, \tag{3}$$

where

$$\Gamma_j = \frac{1}{W} R_j \gamma^*_j \tag{4}$$

is the traffic demand of user j. The total interference on the jth user's signal, I_j, can be expressed as

$$I_j = \sum_{l=1, l\neq j}^{M} S_l + n_j = \sum_{l=1}^{M} S_l - S_j + n_j, \tag{5}$$

where n_j is the aggregate disturbance consisting of additive white Gaussian noise (AWGN) and intercell MAI. For the downlink transmission, different users do not necessarily have the same AWGN level. Furthermore, different users are expected to experience a different level of intercell interference due to their different physical locations. As a result, the disturbance seen at different mobile users is normally different [19], [20]. For the uplink transmission, the BS is the common receiver for all M mobile transmitters, leading the disturbance, n_j, to be the same for all $j = 1, \cdots, M$. However, for notational convenience for the unified matrix operation, we still use a heterogeneous vector, $\mathbf{n}$, to denote the disturbances. For the downlink, the jth element, n_j, denotes the disturbance seen at the jth mobile; for the uplink, it denotes the disturbance seen at the BS for the signal from the jth mobile. Substituting (5) into (3) and manipulating, we have the following inequalities:

$$\begin{aligned} S_1 - \Gamma_1 S_2 - \cdots - \Gamma_1 S_M &\geq \Gamma_1 n_1 \\ -\Gamma_2 S_1 + S_2 - \cdots - \Gamma_2 S_M &\geq \Gamma_2 n_2 \\ \vdots \quad & \quad \vdots \\ -\Gamma_M S_1 - \Gamma_M S_2 - \cdots + S_M &\geq \Gamma_M n_M. \end{aligned} \tag{6}$$

The right-hand sides of (6) represent the disturbance, while the left-hand sides represent signal strengths and intracell interferences. The inequalities mean that, in order for the signals to be discernable, the total strength generated by the transmitted signals must exceed the total disturbance. In matrix form (6) becomes

$$\mathbf{\Gamma}_S \mathbf{S} \geq \mathbf{\Gamma}_D \, \mathbf{n} \tag{7}$$

which implies that

$$\mathbf{S} \geq (\mathbf{\Gamma}_S)^{-1} \mathbf{\Gamma}_D \, \mathbf{n} \tag{8}$$

if the matrix $\mathbf{\Gamma}_S$ is invertible and $(\mathbf{\Gamma}_S)^{-1}$ is nonnegative, where

$$\mathbf{\Gamma}_S = \begin{bmatrix} 1 & -\Gamma_1 & \cdots & -\Gamma_1 \\ -\Gamma_2 & 1 & \cdots & -\Gamma_2 \\ & & \vdots & \\ -\Gamma_M & -\Gamma_M & \cdots & 1 \end{bmatrix}, \tag{9}$$

with $\Gamma_1, \Gamma_2, \cdots, \Gamma_M$ all positive, is the traffic matrix,

$$\mathbf{\Gamma}_D = \text{diag}[\Gamma_1, \Gamma_2, \cdots, \Gamma_M], \tag{10}$$

$$\mathbf{S} = [S_1, S_2, \cdots, S_M]^T \tag{11}$$

is the power vector that needs to be specified, and

$$\mathbf{n} = [n_1, n_2, \cdots, n_M]^T \tag{12}$$

is the nonnegative disturbance vector.

The objective of power control is (a) to find a solution for $\mathbf{S}$ in (7) to specify the power distribution among the M users in the cell, and (b) to minimize the total power in the cell, *i.e.*, minimize $\sum_{j=1}^{M} S_j$. As indicated in (8), finding a solution for $\mathbf{S}$ may involve the inversion of the traffic matrix $\mathbf{\Gamma}_S$.

Let $\mathbf{I}$ be the $M \times M$ identity matrix and $\mathbf{\Gamma}_P = \mathbf{I} - \mathbf{\Gamma}_S$ be defined as the complementary traffic matrix. It turns out that solving the power distribution problem can be transformed into one of examining the spectral radius, $r(\mathbf{\Gamma}_P)$, of $\mathbf{\Gamma}_P$. Let $\mathbf{e} = [1, 1, \cdots, 1]^T$ be an M-dimensional column vector of all 1's. The nonnegative complementary traffic matrix can be expressed as

$$\mathbf{\Gamma}_P = \mathbf{\Gamma}_D(\mathbf{e}\mathbf{e}^T - \mathbf{I}) = \begin{bmatrix} 0 & \Gamma_1 & \cdots & \Gamma_1 \\ \Gamma_2 & 0 & \cdots & \Gamma_2 \\ & & \vdots & \\ \Gamma_M & \Gamma_M & \cdots & 0 \end{bmatrix}. \tag{13}$$

Then (7) can be written as

$$(\mathbf{I}-\mathbf{\Gamma}_P)\mathbf{S} \geq \mathbf{\Gamma}_D\mathbf{n}. \tag{14}$$

Since $\mathbf{\Gamma}_P$ is a nonnegative matrix, the classical Perron-Frobenius theorem implies that the spectral radius $r(\mathbf{\Gamma}_P)$ of $\mathbf{\Gamma}_P$ is an eigenvalue of $\mathbf{\Gamma}_P$ with a nonnegative eigenvector. Moreover, since $\mathbf{\Gamma}_S$ has negative off-diagonal entries and positive diagonal entries, the theory of M-matrices [21] implies that if $r(\mathbf{\Gamma}_P) < 1$, then a nonnegative solution of (14) is given by [11]

$$\mathbf{S}^* = (\mathbf{I}-\mathbf{\Gamma}_P)^{-1}\mathbf{\Gamma}_D\mathbf{n}. \tag{15}$$

In this case any solution $\mathbf{S}$ of (14) is bounded from below by $\mathbf{S}^*$, *i.e.*, $\mathbf{S} \geq \mathbf{S}^*$. Hence, $\mathbf{S}^*$ gives the *minimal solution* of inequality (14). Now it is natural to study the spectrum of the nonnegative matrix $\mathbf{\Gamma}_P$ to solve inequality (14). For the simplest case of $M = 2$, the two eigenvalues of $\mathbf{\Gamma}_P$ were obtained in [11], but even for $M = 3$, the zeros of the resulting characteristic polynomial

$$p(\lambda) = \lambda^3 - (\Gamma_1\Gamma_2 + \Gamma_2\Gamma_3 + \Gamma_3\Gamma_1)\lambda - 2\Gamma_1\Gamma_2\Gamma_3$$

are difficult to find. Using the concept of M-matrices, [13] succeeded in exploring the spectrum for the $M \times M$ matrix $\mathbf{\Gamma}_P$ and obtained an equivalent condition for $r(\mathbf{\Gamma}_P) < 1$ in terms of the traffic demand for all the users in the system.

In this paper we present a complete solution to the power distribution/allocation problem for both downlink and uplink transmissions using an approach based on the Sherman-Morrison formula in linear algebra to find a necessary and sufficient condition for the nonnegativity of the inverse of $\mathbf{\Gamma}_S$. We avoid the intractable eigenvalue problem for a more friendly traffic demand manipulation, and provide explicit expressions for the solution of the power control problem (7) with or without power constraints. We then apply the convergence conditions to specify the capacity region of the system.

III. The Inverse of $\mathbf{\Gamma}_S$

To calculate the inverse of $\mathbf{\Gamma}_S$, we use the following Sherman-Morrison formula [21], the proof of which is via direct computation.

Lemma 1: Let $\mathbf{A}$ be an invertible $n \times n$ matrix and let $\mathbf{x}$ and $\mathbf{y}$ be n-dimensional vectors. Then $\mathbf{A}+\mathbf{x}\mathbf{y}^T$ is invertible if and only if $\mathbf{y}^T\mathbf{A}^{-1}\mathbf{x} \neq -1$. Moreover, if $\mathbf{y}^T\mathbf{A}^{-1}\mathbf{x} \neq -1$, then

$$(\mathbf{A}+\mathbf{x}\mathbf{y}^T)^{-1}\mathbf{A}^{-1} - \frac{1}{1+\mathbf{y}^T\mathbf{A}^{-1}\mathbf{x}}\mathbf{A}^{-1}\mathbf{x}\mathbf{y}^T\mathbf{A}^{-1}. \tag{16}$$

Let

$$c \equiv 1 - \sum_{i=1}^{M}\frac{\Gamma_i}{\Gamma_i+1} \tag{17}$$

be a parameter that captures the traffic demand. The parameter c governs the invertibility of $\mathbf{\Gamma}_S$.

Theorem 1: $\mathbf{\Gamma}_S$ is invertible if and only if $c \neq 0$. Moreover, if $c \neq 0$,

$$(\mathbf{\Gamma}_S)^{-1} = \left[\mathbf{I} + \frac{1}{c}(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^\mathbf{T}\right](\mathbf{I}+\mathbf{\Gamma}_D)^{-1}. \tag{18}$$

Proof. Since

$$\mathbf{\Gamma}_S = \mathbf{I}-\mathbf{\Gamma_P} = \mathbf{I}-\mathbf{\Gamma_D}(\mathbf{e}\mathbf{e}^\mathbf{T}-\mathbf{I}) = \mathbf{I}+\mathbf{\Gamma_D}-\mathbf{\Gamma_D}\mathbf{e}\mathbf{e}^\mathbf{T},$$

using Lemma 1 with $\mathbf{A} = \mathbf{I}+\mathbf{\Gamma_D}$, $\mathbf{x} = -\mathbf{\Gamma_D}\mathbf{e}$, and $\mathbf{y} = \mathbf{e}$ so that $\mathbf{\Gamma}_S = \mathbf{A}+\mathbf{x}\mathbf{y}^T$, we see that $\mathbf{\Gamma}_S$ is invertible if and only if $\mathbf{e^T}(\mathbf{I}+\mathbf{\Gamma_D})^{-1}(-\mathbf{\Gamma_D}\mathbf{e}) \neq -\mathbf{1}$ or equivalently, $c \neq 0$. Now, if $c \neq 0$, then formula (16) gives

$$\begin{aligned}
&(\mathbf{\Gamma}_S)^{-1} = \left[(\mathbf{I}+\mathbf{\Gamma}_D) - \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^\mathbf{T}\right]^{-1}\\
&= (\mathbf{I}+\mathbf{\Gamma}_D)^{-1} - \frac{(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}(-\mathbf{\Gamma}_D\mathbf{e})\mathbf{e}^\mathbf{T}(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}}{1+\mathbf{e}^\mathbf{T}(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}(-\mathbf{\Gamma}_D\mathbf{e})}\\
&= (\mathbf{I}+\mathbf{\Gamma}_D)^{-1} + \frac{(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^\mathbf{T}(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}}{1-\sum_{i=1}^{M}\frac{\Gamma_i}{\Gamma_i+1}}\\
&= \left[\mathbf{I} + \frac{1}{c}(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^\mathbf{T}\right](\mathbf{I}+\mathbf{\Gamma}_D)^{-1}.
\end{aligned}$$

Multiplying out the right-hand side of (18), we obtain an explicit expression for $(\mathbf{\Gamma}_S)^{-1}$ as

$$(\mathbf{\Gamma}_S)^{-1} = \frac{1}{c}\begin{bmatrix} (\frac{\Gamma_1}{\Gamma_1+1}+c)\frac{1}{\Gamma_1+1} & \cdots & \frac{\Gamma_1}{(\Gamma_1+1)(\Gamma_M+1)} \\ \frac{\Gamma_2}{(\Gamma_2+1)(\Gamma_1+1)} & \cdots & \frac{\Gamma_2}{(\Gamma_2+1)(\Gamma_M+1)} \\ & \vdots & \\ \frac{\Gamma_M}{(\Gamma_M+1)(\Gamma_1+1)} & \cdots & (\frac{\Gamma_M}{\Gamma_M+1}+c)\frac{1}{\Gamma_M+1} \end{bmatrix}. \tag{19}$$

That is, the (i,j)th element of $(\mathbf{\Gamma}_S)^{-1}$ is $\frac{\Gamma_i}{c(\Gamma_i+1)(\Gamma_j+1)}$ for $i \neq j$ and the (i,i)th element of $(\mathbf{\Gamma}_S)^{-1}$ is $(\frac{\Gamma_i}{\Gamma_i+1}+c)\cdot\frac{1}{c(\Gamma_i+1)}$.
A consequence of Theorem 1 is the following:

Corollary 1: $\mathbf{\Gamma}_S$ is invertible and $(\mathbf{\Gamma}_S)^{-1}$ is a positive matrix if and only if $c > 0$.

From the theory of M-matrices, if the off-diagonal entries of an M-matrix $\mathbf{A}$ are increased to a new M-matrix $\mathbf{B}$, then $\mathbf{B}^{-1} \leq \mathbf{A}^{-1}$ (see, e.g., Theorem 2.4.10 in [21]). This gives the following result.

Proposition 1: Under the condition $c > 0$, if the Γ_i's are decreased, then so are the entries of $(\Gamma_S)^{-1}$.

Proposition 1 implies that the optimal power allocation is a "decreasing function" of the traffic demands.

IV. Convergence of Power Distribution Law

Now we apply the result in the previous section to solve (7) for the optimal power vector. In the following we investigate the solvability of (7) for different cases, including the cases where the system has (a) zero disturbance ($\mathbf{n} = \mathbf{0}$), (b) nonzero disturbance ($\mathbf{n} \neq \mathbf{0}$), and (c) different power constraints. We evaluate the capacity region based on the convergence conditions. We also investigate the "limiting case" in which the inequality becomes equality.

A. Analysis without Background Disturbance

If we ignore the background disturbance, the matrix inequality in (7) becomes

$$\mathbf{\Gamma}_S\mathbf{S} \geq \mathbf{0}. \tag{20}$$

Suppose we solve (20) with strict inequality. Then the existence of a positive power vector solution is equivalent to the condition $c > 0$ from Theorem 1.5.2 of [22] by using the theory of M-matrices. However, we give a direct proof in the following theorem in which an explicit solution is also obtained.

Theorem 2: $\mathbf{\Gamma}_S\mathbf{S} > \mathbf{0}$ for a positive power vector $\mathbf{S}$ if and only if $c > 0$. In this case, each solution $\mathbf{S}$ to the inequality $\mathbf{\Gamma}_S\mathbf{S} > \mathbf{0}$ is positive, and the vector $\mathbf{S} = (\mathbf{\Gamma}_S)^{-1}\mathbf{u}$ is a solution for each positive vector $\mathbf{u}$. In particular, a positive solution $\mathbf{S}$ is given by $\mathbf{S} = (\mathbf{\Gamma}_S)^{-1}\mathbf{e}$.

Proof. If $c > 0$, then $(\mathbf{\Gamma}_S)^{-1}$ exists and is a positive matrix by Corollary 1. Let $\mathbf{S} = (\mathbf{\Gamma}_S)^{-1}\mathbf{e}$. Then $\mathbf{S} > \mathbf{0}$ and $\mathbf{\Gamma}_S\mathbf{S} = \mathbf{\Gamma}_S(\mathbf{\Gamma}_S)^{-1}\mathbf{e}\mathbf{e} > \mathbf{0}$.

Now suppose that there is a positive power vector $\mathbf{S}$ such that $\mathbf{\Gamma}_S\mathbf{S} > \mathbf{0}$. Then $\mathbf{S} > \mathbf{\Gamma}_P\mathbf{S}$, which implies that $(\mathbf{I} + \mathbf{\Gamma}_D)\mathbf{S} > \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^T\mathbf{S} = (\mathbf{e}^T\mathbf{S})\mathbf{\Gamma}_D\mathbf{e}$. Thus, $(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} < (\mathbf{e}^T\mathbf{S})^{-1}\mathbf{S}$. It follows that

$$\mathbf{e}^T(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} < (\mathbf{e}^T\mathbf{S})^{-1}\mathbf{e}^T\mathbf{S} = 1,$$

which means that $c > 0$.

Finally, suppose that $c > 0$. If $\mathbf{\Gamma}_S\mathbf{S} > \mathbf{0}$ for a vector $\mathbf{S}$, then $\mathbf{S} = (\mathbf{\Gamma}_S)^{-1}\mathbf{\Gamma}_S\mathbf{S} > \mathbf{0}$ since $(\mathbf{\Gamma}_S)^{-1}$ is positive. Given any vector $\mathbf{u} > \mathbf{0}$, then $\mathbf{\Gamma}_S(\mathbf{\Gamma}_S)^{-1}\mathbf{u} = \mathbf{u} > \mathbf{0}$. This means that $\mathbf{S} = (\mathbf{\Gamma}_S)^{-1}\mathbf{u}$ is a solution. This completes the proof.

The following theorem gives more general results:

Theorem 3: The homogeneous inequality system (20) has a non-trivial nonnegative solution $\mathbf{S}$ if and only if $c \geq 0$. In the case of $c > 0$, all the vectors of the form $(\mathbf{\Gamma}_S)^{-1}\mathbf{u}$ with $\mathbf{u} \geq \mathbf{0}$ are solutions of (20). In particular, every column vector of $(\mathbf{\Gamma}_S)^{-1}$ is a solution of (20).

Proof. For the sufficiency part, it is enough to assume that $c = 0$ since the case $c > 0$ has been proved in Theorem 2. Then $\mathbf{S} = (\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e}$ gives a positive solution of (20) since $\mathbf{\Gamma}_S\mathbf{S} = (\mathbf{I} + \mathbf{\Gamma}_D - \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^T)(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} = \mathbf{\Gamma}_D\mathbf{e} - \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^T(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} = \mathbf{\Gamma}_D\mathbf{e} - \mathbf{\Gamma}_D\mathbf{e} = \mathbf{0}$.

Now suppose that there is a nonzero and nonnegative power vector $\mathbf{S}$ such that $\mathbf{\Gamma}_S\mathbf{S} \geq \mathbf{0}$. Then $(\mathbf{I} + \mathbf{\Gamma}_D)\mathbf{S} \geq \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^T\mathbf{S}$, which implies that $(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} \leq (\mathbf{e}^T\mathbf{S})^{-1}\mathbf{S}$ since $\mathbf{e}^T\mathbf{S} > 0$, so

$$\mathbf{e}^T(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} \leq (\mathbf{e}^T\mathbf{S})^{-1}\mathbf{e}^T\mathbf{S} = 1.$$

That is, $c \geq 0$. The last conclusion of the theorem is obvious from the proof of Theorem 2.

Remark 1. Theorem 3 is consistent with Proposition 1 of [15].

Strictly speaking, there is no minimal solution to the homogeneous inequality (20). Theorems 2 and 3 allow us to formulate the following procedure to perform the power allocation.

Power Allocation Procedure

1) From the traffic matrix, calculate $(\mathbf{\Gamma}_S)^{-1}$ using (19) and (17).
2) Let g_{ij} denote the value at the ith row and jth column of $(\mathbf{\Gamma}_S)^{-1}$.
3) Sum over the columns,
$$G_j = \sum_{i=1}^{M} g_{ij} \qquad j = 1, \cdots, M$$
4) Find the minimal value of $G_j, j = 1, \cdots, M$, say $j = k$ and return the index k.
5) Then the kth column is the optimal power solution:
$$S_i^* = g_{ik}.$$

If s_0 is the minimal required power level for a proper detection of the desired signal, then the power vector can be scaled as:

$$\mathbf{S} = s_0 \cdot \frac{\mathbf{S}^*}{S^*_{\min}} \tag{21}$$

where $S^*_{\min}$ is the minimum value of the components of the vector $\mathbf{S}^*$.

B. Analysis with Nonzero Disturbance Vector

As an immediate application of the result in Section III, we have an improvement of Theorem 1 in [10]. For notational simplicity, we define the *normalized traffic demand* as

$$\Phi_i = \frac{\Gamma_i}{\Gamma_i + 1}, \ i = 1, 2, \cdots, M. \tag{22}$$

It is readily seen that Φ_i is a monotonically increasing function of Γ_i. When Γ_i changes in the range of $(0, \infty)$, Φ_i has a corresponding value in the range of $(0, 1)$. The rationale of Φ_i will be explained in Subsection IV.D.

The following result gives an equivalent condition for inequality (7) to have a positive solution and an explicit expression of its optimal solution.

Theorem 4: Suppose $\mathbf{n} \neq \mathbf{0}$. Then (7) has a positive solution if and only if $c > 0$. If $c > 0$, then any solution $\mathbf{S}$ to (7) satisfies the inequality $\mathbf{S} \geq \mathbf{S}^*$, where the minimal positive solution $\mathbf{S}^*$ of (7) has the expression

$$\mathbf{S}^* = \left[\mathbf{I} + \frac{1}{c}(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^\mathbf{T}\right](\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{n}, \tag{23}$$

and so its ith component is

$$S_i^* = \frac{\Phi_i}{c}\left[(\Phi_i + c)\, n_i + \sum_{j \neq i} \Phi_j n_j\right], \ \ i = 1, 2, \cdots, M. \tag{24}$$

Proof. If $\mathbf{S}$ is a positive solution of (7), then $(\mathbf{I} + \mathbf{\Gamma}_D - \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^\mathbf{T})\mathbf{S} \geq \mathbf{\Gamma}_D\mathbf{n} \geq 0$. Thus,

$$(\mathbf{I} + \mathbf{\Gamma}_D)\mathbf{S} \geq \mathbf{\Gamma}_D\mathbf{e}\mathbf{e}^T\mathbf{S} = (\mathbf{e}^T\mathbf{S})\mathbf{\Gamma}_D\mathbf{e},$$

and the strict inequality holds for at least one component in the above inequality since $\mathbf{\Gamma}_D\mathbf{e} \neq \mathbf{0}$, that is,

$$[(\mathbf{I} + \mathbf{\Gamma}_D)\mathbf{S}]_i > \left[(\mathbf{e}^T\mathbf{S})\mathbf{\Gamma}_D\mathbf{e}\right]_i$$

for at least one i. It follows that $(\mathbf{I}+\mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} \leq (\mathbf{e}^T\mathbf{S})^{-1}\mathbf{S}$ and $\left[(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e}\right]_i < \left[(\mathbf{e}^\mathbf{T}\mathbf{S})^{-1}\mathbf{S}\right]_i$. Therefore

$$c = 1 - \mathbf{e}^\mathbf{T}(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D\mathbf{e} > 1 - (\mathbf{e}^\mathbf{T}\mathbf{S})^{-1}\mathbf{e}^\mathbf{T}\mathbf{S} = 0.$$

Conversely, suppose that $c > 0$. Then $(\mathbf{\Gamma}_S)^{-1}$ is a positive matrix by Corollary 1. Let $\mathbf{S}^* = (\mathbf{\Gamma}_S)^{-1}\mathbf{\Gamma}_D\mathbf{n}$. Then $\mathbf{S}^*$ is a positive solution of (7) since $\mathbf{\Gamma}_S\mathbf{S}^* = \mathbf{\Gamma}_S(\mathbf{\Gamma}_S)^{-1}\mathbf{\Gamma}_D\mathbf{n} = \mathbf{\Gamma}_D\mathbf{n}$, and the expressions (23) and (24) are direct results from Theorem 1.

Finally, if $c > 0$ and $\mathbf{S}$ is any solution of (7), then

$$\mathbf{S} = (\mathbf{\Gamma}_S)^{-1}\mathbf{\Gamma}_S\mathbf{S} \geq (\mathbf{\Gamma}_S)^{-1}\mathbf{\Gamma}_D\mathbf{n} \equiv \mathbf{S}^*,$$

so $\mathbf{S}$ is actually a positive solution and $\mathbf{S}^*$ is the minimal positive solution of (7).

Remark 2. The minimal solution $\mathbf{S}^*$ is called the *optimal power allocation*. For the uplink transmission, the disturbance vector can be written as $\mathbf{n} = \xi\mathbf{e}$ for some positive number

ξ, *i.e.*, all the signals are experiencing the same level of disturbance, then the minimal power solution is

$$\mathbf{S}^* = \frac{\xi}{c}(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{e},$$

a result which is consistent with Lemma 3 in [15].

C. Analysis with Power Constraints

In practice we often need to solve (7) with some constraints. Here we give two results for solving (7) with different power constraints on the uplink and downlink transmissions. We will show that with the addition of these power constraints, the necessary and sufficient condition for the existence of a feasible power solution becomes more stringent. However, as long as the system is feasible, the optimal power levels are allocated in the same way as those without these power constraints.

1) Uplink Transmission: For the uplink transmission, the maximal transmit power of a mobile is normally constrained as

$$S_i \leq \bar{S}_i, \;\; 1 \leq i \leq M, \tag{25}$$

where $\bar{S}_i$ is the maximal allowable power for the ith user. The optimal power allocation constrained by (25) is given by the following theorem.

Theorem 5: Suppose $\mathbf{n} \neq \mathbf{0}$. Then system (7) constrained by (25) has a positive solution if and only if for $i = 1, 2, \cdots, M$,

$$0 < \frac{\sum_{j=1}^{M} \Phi_j n_j}{1 - \sum_{j=1}^{M} \Phi_j} \leq \frac{\bar{S}_i}{\Phi_i} - n_i. \tag{26}$$

Proof. Let $\mathbf{S} > \mathbf{0}$ satisfy both (7) and (25). Then $c > 0$ by Theorem 4. Since $\mathbf{S}^*$ given by (23) is the minimal solution of (7), $\mathbf{S}^* \leq \mathbf{S} \leq \bar{\mathbf{S}}$. Thus,

$$\left[\mathbf{I} + \frac{1}{c}(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{e}\mathbf{e}^{\mathbf{T}}\right](\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{n} \leq \bar{\mathbf{S}}.$$

Now (26) follows immediately from the definition of c given by (17). Conversely, if (26) is true, then $c > 0$. It is clear that $\mathbf{S}^*$ is a positive solution of (7) subject to the constraint of (25).

Remark 3. When we apply the condition $\mathbf{n} = \xi \mathbf{e}$ for the uplink transmission, the necessary and sufficient condition for (7) and (25) to have a positive solution is reduced to

$$\sum_{j=1}^{M} \Phi_j \leq 1 - \max_i \left(\frac{\xi \Phi_i}{\bar{S}_i}\right), \qquad i = 1, 2, \cdots, M. \tag{27}$$

It is intuitive to observe the role of the normalized traffic demand Φ_i in (27). The right-hand side of the inequality is constrained by the maximal ratio of $\Phi/\bar{S}$, while the left-hand side is the sum of the normalized traffic demand. The higher the normalized traffic demand, the higher system resources it occupies, leaving a smaller space for the system to support other users.

Remark 4. An equivalent statement of Theorem 5 is that the combined inequalities (7) and (25) have a positive solution if and only if $c > 0$ and $\mathbf{S}^* \leq \bar{\mathbf{S}}$.

2) Downlink Transmission: For the downlink transmission, the available total transmit power of a BS is limited by

$$\sum_{i=1}^{M} S_i \leq S_T, \tag{28}$$

where $S_T > 0$ is the maximal total power of a BS. Then, the necessary and sufficient condition for the existence of a feasible solution is given by the following theorem.

Theorem 6: Suppose $\mathbf{n} \neq \mathbf{0}$. Then system (7) constrained by (28) has a positive solution if and only if

$$0 < \frac{\sum_{i=1}^{M} \Phi_i n_i}{1 - \sum_{i=1}^{M} \Phi_i} \leq S_T. \tag{29}$$

In particular, if $\mathbf{n} = \xi \mathbf{e}$, then (29) becomes

$$\sum_{i=1}^{M} \Phi_i \;\; \leq \;\; \frac{S_T}{S_T + \xi}. \tag{30}$$

Proof. Let $\mathbf{S} > \mathbf{0}$ satisfy both (7) and (28). Then $c > 0$ by Theorem 4. Since $\mathbf{S}^*$ given by (23) is the minimal solution of (7), $\mathbf{e}^T\mathbf{S}^* \leq \mathbf{e^T S} \leq \mathbf{S_T}$. Thus,

$$\mathbf{e}^T \left[\mathbf{I} + \frac{1}{c}(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{e}\mathbf{e}^{\mathbf{T}}\right](\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{n} \leq S_T.$$

Since the left-hand side of the above inequality equals

$$\mathbf{e}^T(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{n}\left[1 + \frac{1}{c}(1 - c)\right] = \frac{\mathbf{e}^T(\mathbf{I} + \mathbf{\Gamma}_D)^{-1}\mathbf{\Gamma}_D \mathbf{n}}{c},$$

(29) is true. Conversely, if (29) is satisfied, then $c > 0$, and so the minimal solution $\mathbf{S}^*$ of (7) is well defined. It is obvious that $\mathbf{S}^*$ satisfies (28).

Remark 5. Similarly, Theorem 6 says that the combined inequalities (7) and (28) have a positive solution if and only if $c > 0$ and $\mathbf{e}^T\mathbf{S}^* \leq S_T$.

Remark 6. It is clear that if the above constrained power allocation problems have a solution, then the minimal solution is still $\mathbf{S}^*$ given by Theorem 4.

D. Capacity Analysis

In this subsection, we first explain the rationale of the definition of the normalized traffic demand, Φ_i, for the ith user. In Section III, we determined that the necessary and sufficient condition for the existence of a positive inverse matrix for Γ_S is that $c > 0$, which implies that

$$\sum_{i=1}^{M} \Phi_i < 1.$$

If all the M users in the system have the same traffic demand, *i.e.*, $\Phi_i = \Phi$ for all i, then the above condition implies that $\Phi < 1/M$. Therefore, for this system, each user is allowed to occupy less than $1/M$ of the system resource. This fact leads us to define Φ as the normalized traffic demand.

Suppose that there are K classes among the M users. The traffic demand (Φ or Γ) is the same for all the users within each class. We use Φ_k to denote the normalized traffic demand for the kth traffic class when the summation index changes from 1 to K. The capacity in our study is defined as the K-dimensional space spanned by the maximum number of users supported in each class, denoted by a vector

$$\mathcal{C} = [N_1, N_2, ..., N_K]^T,$$

where N_k is the number of users in the kth class. For simplicity, we further suppose that in the downlink transmission, all the users are experiencing the same disturbance level ξ, the same situation as in the uplink. With the condition $c > 0$, (27) and (30) can be used to evaluate the system capacity under different situations.

- When there are no power constraints, the power control convergence condition $c > 0$ is equivalent to the inequality

$$\sum_{k=1}^{K} N_k \Phi_k < 1. \tag{31}$$

  As a special case, for a dual-class system, the above condition can be written as

$$N_2 \leq -\frac{\Phi_1}{\Phi_2} N_1 + \frac{1}{\Phi_2}.$$

- When the system is constrained by an upper power level as in (25) for the uplink transmission, by applying (27), we have

$$\sum_{k=1}^{K} N_k \Phi_k < 1 - \max_j \left(\frac{\xi \Phi_j}{\bar{S}_j} \right), \qquad 1 \leq j \leq K. \tag{32}$$

  For a dual-class system, the above condition can be written as

$$N_2 \leq -\frac{\Phi_1}{\Phi_2} N_1 + \frac{1}{\Phi_2} - \max\left(\frac{\xi}{\bar{S}_1} \frac{\Phi_1}{\Phi_2}, \frac{\xi}{\bar{S}_2} \right).$$

- When the system is constrained by the maximal power sum as in (28) for the downlink transmission, by using (30), we can obtain

$$\sum_{k=1}^{K} N_k \Phi_k \leq \frac{S_T}{S_T + \xi}. \tag{33}$$

  Similarly, for a dual class,

$$N_2 \leq -\frac{\Phi_1}{\Phi_2} N_1 + \frac{S_T}{(S_T + \xi)\Phi_2}.$$

It is clear that the right-hand side of (32) and (33) is less than that of (31). Furthermore, (32) and (33) reduce to (31) as $\bar{S}$ and S_T tend to infinity, respectively. Therefore, power constraints have the effect of reducing the system capacity.

E. The Limiting Case

In this subsection, we consider the limiting case studied by Zhu and Mark in [10]. In this case, the thermal noise and the MAI from other cells are ignored. Further, inequality (7) becomes the following equation,

$$\mathbf{\Gamma}_S \mathbf{S} = \mathbf{0}, \tag{34}$$

which is equivalent to the fixed point problem for $\mathbf{\Gamma}_P$:

$$\mathbf{\Gamma}_P \mathbf{S} = \mathbf{S}. \tag{35}$$

The following result about nonnegative eigenvalues of $\mathbf{\Gamma}_P$ will be used here.

Lemma 2: Any positive eigenvalue λ of $\mathbf{\Gamma}_P$ satisfies the following rational equation

$$\sum_{i=1}^{M} \frac{\Gamma_i}{\Gamma_i + \lambda} = 1. \tag{36}$$

Moreover, the corresponding eigenvector

$$\mathbf{v} = \left[\frac{\Gamma_1}{\Gamma_1 + \lambda}, \frac{\Gamma_2}{\Gamma_2 + \lambda}, \cdots, \frac{\Gamma_M}{\Gamma_M + \lambda} \right]^T \tag{37}$$

is positive.

Proof. Let λ be a positive eigenvalue of $\mathbf{\Gamma}_P$ with a corresponding eigenvector $\mathbf{v}$. Then from $\mathbf{\Gamma}_D(\mathbf{e}\mathbf{e}^\mathbf{T} - \mathbf{I})\mathbf{v} = \lambda \mathbf{v}$, we have

$$(\mathbf{e}\mathbf{e}^\mathbf{T} - \mathbf{I})\mathbf{v}\lambda(\mathbf{\Gamma}_D)^{-1}\mathbf{v}.$$

Since $\mathbf{e}\mathbf{e}^\mathbf{T}\mathbf{v} = (\mathbf{e}^\mathbf{T}\mathbf{v})\mathbf{e}$, we have

$$\left[\mathbf{I} + \lambda(\mathbf{\Gamma}_D)^{-1}\right] \mathbf{v}(\mathbf{e}^\mathbf{T}\mathbf{v})\mathbf{e}. \tag{38}$$

Therefore, for $i = 1, 2, \cdots, M$,

$$\left(1 + \lambda \Gamma_i^{-1}\right) v_i = \sum_{k=1}^{M} v_k. \tag{39}$$

Since $\lambda > 0$ and $\mathbf{v} \neq 0$, the right-hand side of (39) is nonzero. We may normalize the vector $\mathbf{v}$ such that

$$\left(1 + \lambda \Gamma_i^{-1}\right) v_i = 1, \quad i = 1, 2, \cdots M. \tag{40}$$

That is,

$$v_i = \frac{1}{1 + \lambda \Gamma_i^{-1}} \frac{\Gamma_i}{\Gamma_i + \lambda}, \quad i = 1, 2, \cdots M, \tag{41}$$

which yields (37). Now (36) follows from (39) and (41).

Remark 7. Actually the spectral radius $r(\mathbf{\Gamma}_P)$ of $\mathbf{\Gamma}_P$ is the only positive eigenvalue of $\mathbf{\Gamma}_P$. Thus, we always have

$$\sum_{i=1}^{M} \frac{\Gamma_i}{\Gamma_i + r(\mathbf{\Gamma}_P)} = 1. \tag{42}$$

In normal cases, we need the spectral radius $r(\mathbf{\Gamma}_P) < 1$. Considering the monotonically decreasing property of the function $f(t) = \Gamma_i/(\Gamma_i + t)$, we can also see that $r(\mathbf{\Gamma}_P) < 1$ if and only if

$$\sum_{i=1}^{M} \frac{\Gamma_i}{\Gamma_i + 1} < 1,$$

which is our earlier condition $c > 0$.

Since $\mathbf{\Gamma}_S \mathbf{S} = \mathbf{0}$ if and only if $\mathbf{\Gamma}_P \mathbf{S} = \mathbf{S}$, the existence of a nontrivial solution to equation (34) is equivalent to the existence of an eigenvalue equal to 1 for the matrix $\mathbf{\Gamma}_P$. Lemma 2 immediately implies the following result on the existence of a nontrivial solution of (34).

Theorem 7: (34) has a positive solution if and only if

$$\sum_{i=1}^{M} \frac{\Gamma_i}{\Gamma_i + 1} = 1, \tag{43}$$

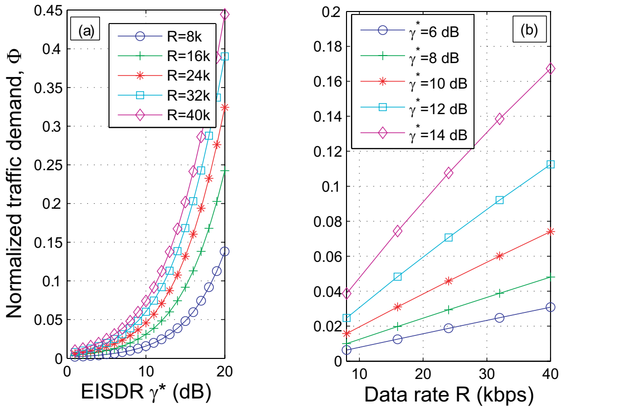

Fig. 1. Normalized traffic demand as a function of the QoS requirements.

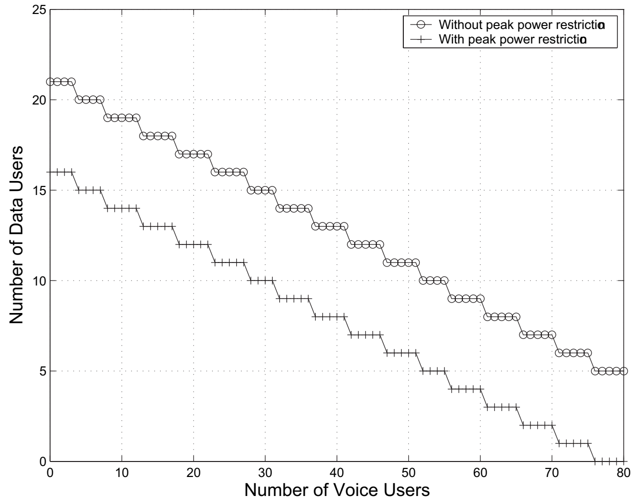

Fig. 2. Capacity curves for cases with and without peak power constraint.

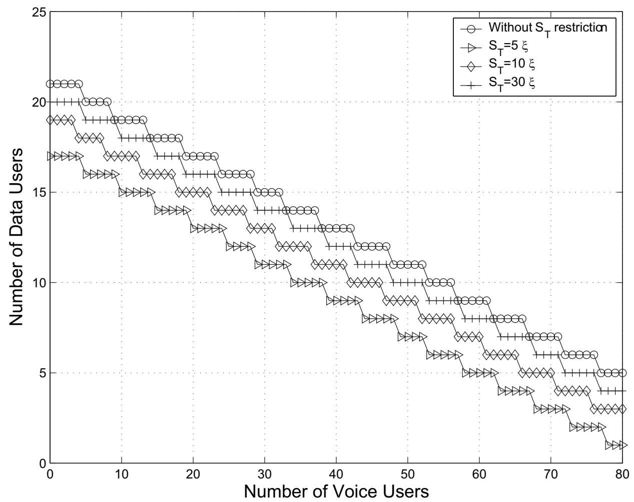

Fig. 3. Capacity curves for cases with and without sum power constraint.

or equivalently, if and only if $c = 0$. In this case the normalized solution $\mathbf{S}$ is given by

$$S_i = \frac{\Gamma_i}{\Gamma_i + 1} = \Phi_i, \; i = 1, 2, \cdots, M. \tag{44}$$

Remark 8. For $M = 2$ or 3, condition (43) with a different form was obtained in [10].

Corollary 2: [10] In particular, if $\Gamma_i \equiv \Gamma$ for all i, then the necessary and sufficient condition (43) is reduced to $\Gamma = 1/(M-1)$ or equivalently, $\Phi = 1/M$.

V. Numerical Results

Since the necessary and sufficient condition for the existence of a feasible solution is that the sum of the normalized traffic demand, Φ, from all the users is less than 1, we first illustrate how Φ_i varies with the traffic QoS requirements. Fig. 1(a) shows the normalized traffic demand vs. the target EISDR requirement with a spread bandwidth of 5 MHz. From the bottom to the top, the five curves show the data rate requirement changes from 8 kbps to 40 kbps. Similarly, Fig. 1(b) shows the normalized traffic demand vs. the data rate with the target EISDR γ^* as a parameter. From the figures, we can find that Φ exhibits a nearly exponentially increasing trend with respect to γ^*, and a linearly increasing trend with respect to the target data rate R. This is because γ^* is expressed in the dB scale. The more stringent the QoS requirement or the data rate of a connection, the higher value the normalized traffic demand, and the more radio resource needs to be allocated to this mobile user.

As a numerical example, consider a system supporting two classes of services: voices and data. The parameters used in the calculation are: spread bandwidth $W = 5$ MHz, noise power spectral density $N_0 = 10^{-6}$, leading to a background noise power, $\xi = N_0 \cdot W = 5$ watts seen by each user if background noise is considered, where the intercell interference has been neglected. For the voice users, the transmission rate is 8 kbps, the target EISDR is $\gamma_v^* = 6$ (corresponding to 7.8 dB), and the maximal power level is 0.5 watt. For the data users, the data rate is 24 kbps, the target EISDR is $\gamma_d^* = 10$ (corresponding to 10 dB), and the maximal power level is 1 watt.

Fig. 2 shows the capacity curves for the cases with and without the peak power constraint. The result for the unconstrained case, shown by the circled markers, is obtained by using the condition $c > 0$, given by (31); that for the constrained case, shown by plus markers, is obtained by using (32). It is observed that the capacity curves are parallel to one another. The capacity loss is 5 data users for a given number of voice users with the peak power restriction. It is expected that with the increase in the peak power limit, the bottom curve will shift upwards and eventually approach the top curve without constraints.

We can also observe that the capacity for the voice traffic is much higher than that of the data traffic. Consider the normalized traffic demand: $\Phi_v = 0.0095$ and $\Phi_d = 0.0458$ for voice and data, respectively. That is, Φ_d is roughly 5 times greater than Φ_v, leading to roughly 5 times capacity loss for data user.

Fig. 3 shows the capacity curves under the no constraint case and the total power constraint case using (31) and (33). From the top to the bottom, the curves represent the results for no constraint, $S_T = 30\xi, 10\xi$, and 5ξ, respectively, where ξ is the noise level. We can observe that all the curves are parallel to each other. This is true since the capacity conditions (31) and (33) have the same slope. Fig. 3 shows that with increasing

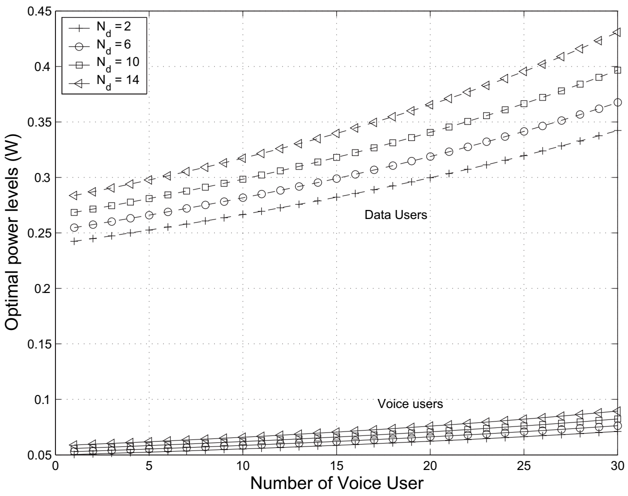

Fig. 4. Optimal allocated power levels to voice and data users as a function of the number of voice users, with $N_d = [2, 6, 10, 14]$. Upper group curves: data user power level, lower group curves: voice user power level.

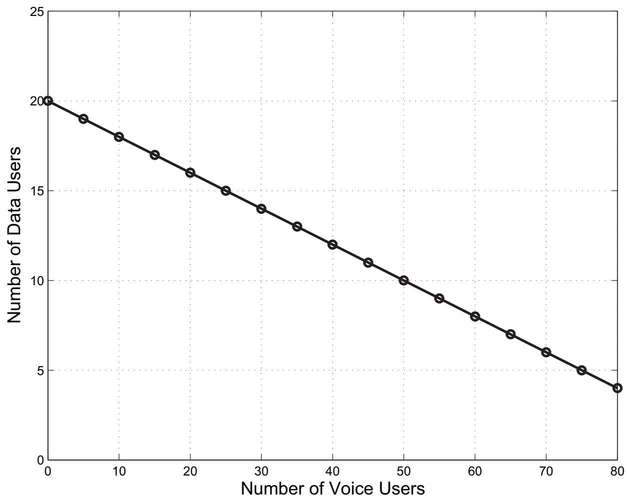

Fig. 6. Capacity for the limiting case, $\Phi = [0.01, 0.05]$.

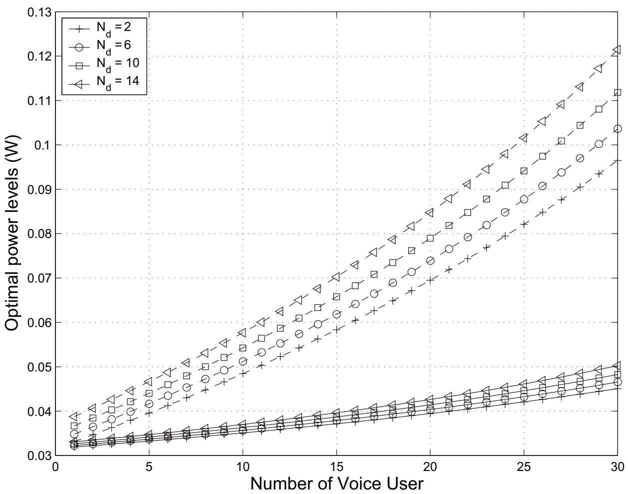

Fig. 5. Optimal allocated power levels to voice and data users as a function of the number of voice users, with $N_d = [2, 6, 10, 14]$. Upper group curves: data user power level, lower group curves: voice user power level, where background disturbance has been ignored.

sum power restrictions, the capacity curves shift upwards.

Fig. 4 shows the optimal allocated power levels for voice (lower group) and data (upper group) users as a function of the number of voice users, with the number of data users, $N_d = [2, 6, 10, 14]$, as a parameter. The results are obtained by using Theorem 4 with a nonzero background disturbance, where the background noise seen by each active user is $\xi = 5$ watts. It is clear that data users need a much higher power because the data users have a higher traffic demand than that of voice users. It also shows that when the number of voice users increases, the required power for data users increases more quickly than that for voice users. Moreover, it can be observed that the imposition of power restriction reduces the capacity, but does not affect the power allocation. Within the capacity of the lower curve of Fig. 2, the power allocation is the same for both restricted and unrestricted cases. This is consistent with Remark 6 in Section IV.C.

Fig. 5 shows the optimal power allocation for the voice and data traffic when the background disturbance is ignored. The results are obtained by using Theorem 2 with a minimal required power level for a proper detection to be 0.0316 W (corresponding to 15 dBm). From Figs. 4 and 5, we can observe that the data users require a higher power level than that of voice users. With an increase in the number of active users, the allocated power level for both voice and data users increases and the data users exhibit a faster increasing trend.

Next we present results for the limiting case. In this situation, it is required that each user achieves the target EISDR values with strict equality, ignoring the background disturbance. Without loss of generality, we consider two classes with normalized traffic demand $\Phi = [0.01, 0.05]$, leading to the corresponding traffic demand $\Gamma = [0.0101, 0.0526]$, which is close to the voice and data traffic parameters presented above.

Fig. 6 shows the corresponding capacity curve by using Theorem 7, equation (43). Since strict equality is required for the necessary and sufficient condition of a feasible power allocation solution, different from previous results on capacity staircase curves, the capacity for the limiting case is a straight line. The optimal power level for the two classes are $S^* = k \cdot \Phi$, where k is an arbitrary scale factor.

We arbitrarily select a point from the capacity curve at $M = [20, 16]$. The fixed point problem (35) ($\mathbf{\Gamma_P S = S}$) can be verified by a simple matrix operation. Investigating the maximal eigenvalue of matrix Γ_P also provides the feasibility information of the system. Fig. 7 shows the corresponding 36 eigenvalues for Γ_P. It is shown that there is only one positive eigenvalue, which equals 1. This observation is consistent with Remark 7 that the spectral radius of the matrix Γ_P is the only positive eigenvalue of Γ_P. For the limiting case, the value is 1, meaning that the system is fully loaded and that there is no space to accommodate any more traffic demand.

For the uniform user situation, for example, let the number of users be 20. From Corollary 2, for each user, the allocated normalized traffic demand is equally likely with $\Phi = 1/20 = 0.05$, leading to a traffic demand of $\Gamma = 0.0526$. Fig. 8(a) shows the corresponding 20 eigenvalues of the matrix Γ_P. It is interesting to note that in the limiting case, the only positive eigenvalue is 1 and the remaining negative eigenvalues are

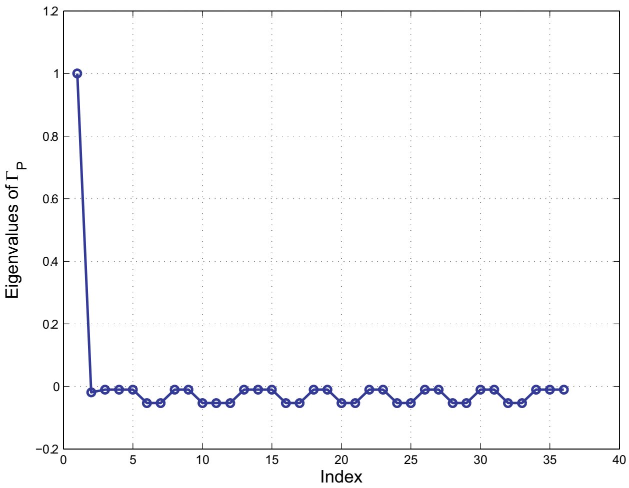

Fig. 7. Eigenvalues of Γ_P for the limiting case with number of users as $M = [20, 16]$, normalized traffic demand $\Phi = [0.01, 0.05]$.

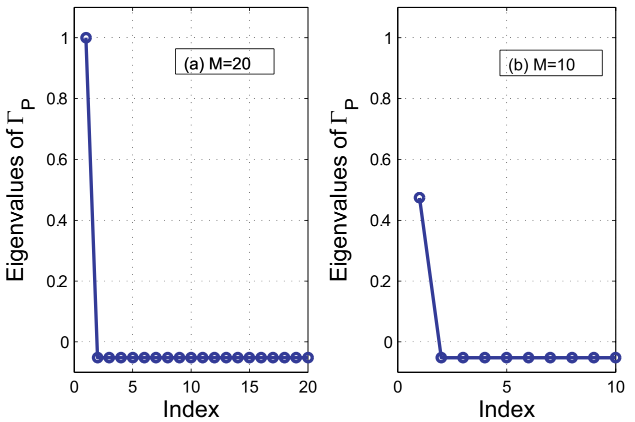

Fig. 8. Eigenvalues of Γ_P for uniform traffic distribution $\Phi = 0.05$. (a) limiting case with $M = 20$ users (b) $M = 10$ users.

-0.0526, which is exactly $-\Gamma$. It is interesting to note that if we reduce the number of users to $M < 20$, we can observe the same eigenvalue distribution, *i.e.*, a positive eigenvalue less than 1 and $M-1$ negative eigenvalues equal to $-\Gamma$, as shown in Fig. 8(b) where $M = 10$. The maximal eigenvalue is less than 1, meaning that there is some space for the system to accommodate more traffic demand.

In our treatment of the feasibility condition, we used $c > 0$, which implies $\sum \Phi_i < 1$ as an equivalent condition of $r(\mathbf{\Gamma}_P) < 1$, where $r(\mathbf{\Gamma}_P)$ is the spectral radius of the matrix $\mathbf{\Gamma}_P$. Figs. 9(a-d) compare the values of $\sum \Phi_i$ with those of $r(\mathbf{\Gamma}_P)$ when the numbers of users are $M = [10, 20, 30, 40]$. For a given number of users, we randomly generate the normalized traffic demand, Φ_i, to be uniformly distributed between $(0, 1)$. Then we properly scale the vector $\mathbf{\Phi}$, making $\sum \Phi_i$ uniformly distributed in $(0, 1)$. The spectral radius of $\mathbf{\Gamma}_P$ can be calculated based on the traffic demand of each user. In order to have a good visual effect, we finally sort the generated $\sum \Phi_i$ and $r(\mathbf{\Gamma}_P)$ and sketch them as a function of the simulation index. From the figures, we can observe that $\sum \Phi_i$ is always a little larger than $r(\mathbf{\Gamma}_P)$. With an increase of the number of users, the difference becomes smaller. In the simulation (300 runs), the mean difference between $\sum \Phi_i$

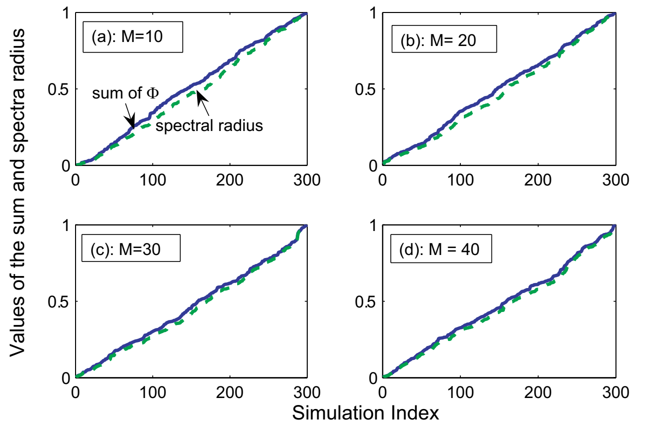

Fig. 9. Comparison of $\sum \Phi_i$ (solid curves) with spectral radius Γ_P, $r(\mathbf{\Gamma}_P)$ (dashed curves).

and $r(\mathbf{\Gamma}_P)$ is $[0.0351, 0.0296, 0.0225, 0.0218]$ when $M = [10, 20, 30, 40]$, respectively. We can also observe that when the value approaches 1, the difference becomes negligible, which is consistent with our analytical expectation.

By using the condition $c > 0$ ($\sum \Phi_i < 1$), instead of looking for all the eigenvalues, the power allocation and system feasibility problem is more tractable and has a good physical interpretation of the system demand and the service provisioning.

VI. Conclusions

We have investigated the power distribution problem for multiclass services in a wideband DS-CDMA system. The power vector is expressed as a function of the traffic demand, which depends on the target data rate, SIR or the EISDR requirement, and the spread spectrum bandwidth. The optimal power distribution is solved via a decomposition of the traffic matrix and applying the well-known Sherman-Morrison inverse formula for rank-one updated matrices, subject to some power constraints. We have presented a unified approach to solve the matrix inequality and generic results for the power distribution law, which applies well to both uplink and downlink transmissions. Under different special situations, our results are shown to be consistent with the results published in the literature. The convergence conditions for the existence of nonnegative power vector are applied to evaluate the capacity region of the system. With the introduction of power restrictions, the capacity regions shrink correspondingly. Within the capacity region, the optimal power allocation has the same solution as those without power restrictions. Our analysis shows the fact that the traffic demand fully determines the capacity region. We further compare the conventional eigenvalue approach and the equivalent traffic demand restrictions for the system feasibility condition. The traffic demand approach makes the system feasibility study easily tractable with a good physical interpretation.

One example of applications of the power allocation problem investigated in this paper is call admission control, conducted at the mobile switch center (MSC). When a new call is requesting admission, its target SIR or EISDR and data rate are specified. Therefore, the MSC can easily find the admissibility of the call request using the capacity results and the allocated target power levels. The target power levels can be used as the target for different kinds of transmit power control algorithms, for example, open-loop and closed-loop power control strategies. The complexity involved is relatively low.

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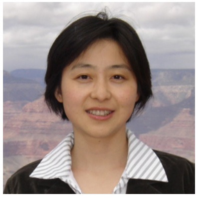

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