Abstract—Classic network utility maximization problems are usually solved assuming all information is available, implying that information not locally available is always truthfully reported. This may not be practical in all scenarios, especially in distributed/semi-distributed networks. In this paper, incentive mechanisms for truthful reporting in network optimizations with local information are studied, and the impact of information and incentive on the solutions is revealed. A novel general model for extending network utility maximization (NUM) problems to incorporate local information is proposed, which allows each user to choose its own objective locally and/or privately. Two specific problems, i.e., a user-centric problem (UCP) and a network-centric problem (NCP), are studied. In the UCP, a network center aims to maximize the collective benefit of all users, and truthful reporting from the users regarding their local information is necessary for finding the solution. We show that the widely-adopted dual pricing cannot guarantee truthful information reporting from a user unless the resource is over-supplied or the price is too high for this user to afford. In the NCP, the network center has its own objective and preferred solution, and incentives are needed for the users to adopt the solution it prefers. Truthful reporting from users are necessary for the center to determine the incentives and achieve its solution. For two-user and multiuser cases, we propose two mechanisms to incentivize truthful reporting from users while guaranteeing nonnegative utility gains for both the users and the center. A trade-off between the chance of achieving the target solution and the utility gain for the center is shown. To illustrate the application of the UCP and NCP, an underlay D2D communication case study is conducted, where each D2D link has its own objective, such as data rate or energy efficiency. Simulations are conducted for the D2D application to validate the analysis results and demonstrate the proposed mechanisms.

Index Terms—incentive mechanism, network utility maximization, truthful local information reporting, underlay D2D

I. INTRODUCTION

A. Motivation

Network utility maximization (NUM) problems have been studied in many applications including sensor networks, vehicular networks, cellular networks, and smart grids [1]-[5]. These studies mostly adopt an optimization perspective, which focuses on the properties of a problem such as its convexity and decomposability, e.g., [6]. Meanwhile, it is generally assumed that information required for finding the solution is available to the corresponding entities (e.g., the nodes running the optimization algorithm) either locally or through truthful communications. These assumptions are well-justified for networks with a central administrator, e.g., cellular networks, or those with simple capability-limited nodes, e.g., sensor networks.

However, with the increasing popularity of heterogeneous networks and Internet of things (IoT) [7],[8], more applications tend to be based on a distributed or a semi-distributed network structure [9],[10]. Meanwhile, network nodes are expected to have increasingly more computation, communication, and/or other capabilities. As a result, a network node may simultaneously possess local information, computation capability for processing information and making decisions, and an individual benefit-maximizing objective. In such scenarios, it can be argued that assuming the availability of local information while finding the solution to an optimization problem is less practical. After all, a node is not motivated to share its true information if it cannot expect a larger benefit by doing so. As a result, the solution that can be found from an optimization perspective may not be achieved due to the existence of local information.

The above scenario requires an incentive mechanism, which can motivate individual users to truthfully report their local information while finding the solution.

B. Related Works

Mechanism design is an analytical framework in the field of game theory that considers the implementation of desirable system-wide solutions to problems when a solution depends on self-interested agents and the information for the agents to make decisions is dispersed and locally held [11]. Game theoretic perspectives for modeling decision making have been adopted in many research works, e.g., [12]-[13]. While game theory typically studies the outcome of a group of players interacting with each other, mechanism design concerns providing appropriate incentives for the players so that their interaction can yield a desirable outcome [16], e.g., the solution to the considered optimization problem. Mechanism design has been adopted in an increasing number of works, especially in the past five years. In [17], the classic Vickrey-Clarke-Groves (VCG) mechanism was adopted to maximize social welfare for a smart grid. Iosifidis et al. proposed an iterative double auction mechanism for efficient mobile data offloading [18]. A tax mechanism based on the classic Groves mechanism was
introduced in [19] to implement dual decomposition so that a user has a diminishing incentive to report falsely as the algorithm converges. Wu et al. used a game theoretic design in dynamic spectrum access and proposed a pricing mechanism for users to adopt the social welfare optimizing solution [20]. Jin et al. considered incentive for cloudlet service provisioning in mobile cloud computing and designed an auction-based mechanism for effective resource trading [21]. Considering that users have individual objectives and the network has a system-wide objective, Zheng et al. proposed a gradient-based incentive to motivate truthful information reporting in an algorithm which is based on alternating direction method of multipliers (ADMM) [22]. A price incentive based on dual variables, referred to as congestion price, was proposed in [23] for finding the optimal bandwidth allocation. Dual pricing (congestion price) has been adopted in many works, and a recent example is a study on social welfare maximization in a mobile crowdsensing scenario based on dual pricing [24].

An application of interest for the incentive perspective studies is device-to-device (D2D) communications, in which there can be no traditional network center for D2D links and each user has its own interest. Su et al. proposed a contract based mechanism to motivate truthful reporting in an overlay D2D resource allocation scenario with D2D channel quality held as local information [25]. Zhao and Song proposed incentive mechanisms to motivate base station data offloading through underlay D2D assisted content distribution [26]. A randomized reverse auction was proposed to incentivize content distribution via D2D communications and offload traffic from the base station [27]. Li et al. designed a double auction-based mechanism to incentivize cellular users with alternative D2D communication capabilities to use the D2D mode such that users waiting for the cellular access can be accommodated [28]. Wang et al. proposed a resource block exchange mechanism in an overlay D2D scenario to reduce interference experienced by D2D users through a sequence of exchanges [29]. Xu et al. exploited an iterative combinatorial auction for resource allocation among D2D links with the objective of sum-rate maximization [30]. Xu et al. applied mechanism design in [31] for coordinating D2D users to participate in traffic offloading from the BS to minimize the energy consumption of the base station. Kebrïai et al. proposed a double auction on bandwidth to allocate resource in a cellular network with a cognitive D2D user [32]. Hajiesmaili et al. designed a multi-slot online auction to achieve load balancing with minimum social cost considering battery limits of D2D devices [33].

C. Our Contributions

Despite the increasing number of related works, there is no general framework that can model a NUM problem with local information, to the best of our knowledge. A close example is [22], in which general forms of objective functions are used for the network and users. However, the model does not capture local information and adopts an optimization perspective. The objective of this work is to model and study the impact of local information and incentive in solution finding. Two problems are considered. The first is a user-centric problem (UCP), a NUM problem with local information in which the network aims to maximize the collective benefit of users (nodes). The second is a network-centric problem (NCP), in which the network aims to maximize its own benefit representing the preference of the network while the users prefer a different solution, e.g., the solution to the UCP. After investigating the UCP and the NCP in general forms, the proposed model is applied to underlay D2D communications. The contributions of this work include the followings.

First, we propose a novel model to capture a NUM problem with local information, in which optimization and incentive perspectives are connected. The proposed model characterizes the interest of a user through two mappings, i.e., an objective function and a valuation function. The objective function maps the optimization variable to a metric such as data rate, energy efficiency, etc. It represents the part that is usually considered from an optimization perspective. The valuation function maps the above metric to its corresponding value to a user, which can be local information. It represents the part that should be handled from an incentive perspective. The model also extends classic NUM problems by allowing each individual user to determine and adopt its own objective locally and, if needed, privately.

Second, dual pricing is studied for the UCP and the insights on when and why dual pricing cannot be used to guarantee truthful reporting are obtained. Dual pricing is of interest since it relates to both a pricing mechanism from the incentive perspective and the dual decomposition technique from the optimization perspective. It is observed in [19] and [34] that dual pricing cannot guarantee truthful reporting in the corresponding problems. However, the results of this work provide more insights on dual pricing in general resource allocation problems. It is shown that dual pricing guarantees truthful reporting only when the resource is oversupplied or when a user would not afford to obtain any resource.

Third, we design mechanisms to guarantee truthful reporting when the network has a preferred solution different from the one accepted by the users. We show that, unless this solution is also preferred by at least one user, no mechanism can simultaneously guarantee truthful reporting and, under any condition, motivate all users to adopt this solution. A subsidized exchange mechanism is proposed to motivate truthful reporting and implement the network-preferred solution in a two-user case. The mechanism provides nonnegative and fair utility gain for both users and also a nonnegative gain for the network. For the general multiuser case, a much more complicated situation in which more forms of untruthful reporting exist, we propose an iterative extended subsidized exchange mechanism to guarantee that truthful reporting is the best choice for any rational user.

II. System Model

Consider a network with \( N \) users, where a user can be a network node, a communication link, etc. The set of users

\[ \text{This is a typical case but it could depend on the specific approach of implementing D2D communications.} \]
is denoted as \( U = \{i\}_{i=1}^{N} \). A network center coordinates the resource allocation among users, where the resource allocated to user \( i \) is represented by \( x_i \).

A. Objective, Valuation, and Local Information

Each user has an individual objective, such as data rate, energy efficiency, etc. It is assumed that users can have different objectives. This reflects the fact that users may be associated with different applications in a network. For instance, users running throughput-sensitive applications can adopt data rate as the performance metric while users running delay-sensitive applications can adopt delay as the performance metric. Allowing different objectives at different users introduces flexibility in characterizing and coordinating multiuser networks.

Assume there are \( M \) different objectives and the set of objectives is \( O = \{o_i\}_{i \in \mathbb{Z}^+} \). Denote the objective chosen by user \( i \) as \( o(i) \). The mapping from \( x_i \) to the achieved result on \( o(i) \) is denoted as \( b_i(x_i) \). The mapping \( b_i \) is determined by standard formulas, e.g., Shannon capacity.

For users \( i \) and \( j \) with the same objective \( o_i \), it can be argued that they may value \( b_i(x_i) \) and \( b_j(x_j) \) differently even if \( b_i(x_i) = b_j(x_j) \). For example, a user on high-definition video streaming and another user browsing web pages, both adopting the data rate as their objectives, can value the same data rate of 50Mb/s differently. Similarly, the same energy efficiency of 100 bits/Hz/J can be of different values to users with 20% and 70% remaining battery power, respectively. This difference reflects different application requirements and other local factors such as its battery power, at the users. To characterize user \( i \)'s valuation of an achieved objective \( b_i(x_i) \), the valuation function \( v_i(b_i) \) is introduced. Unlike the mapping \( b_i \), \( v_i \) is determined individually.

As aforementioned, a user can choose its own objective and have its unique valuation function based on its application requirements and other local factors such as its battery power. Therefore, it is reasonable to consider the valuation function \( v_i(b_i) \) and/or the objective \( o(i) \) as local/private information. In addition, while \( b_i(x_i) \) is given by standard formulas such as the Shannon capacity, parameters in \( b_i(x_i) \), e.g., the channel gain, can also be local information. While such information is required for solving an optimization problem in the network, it is not available to the network center originally. As a result, truthful reporting on such information from all users can be necessary for finding the solution to a considered problem. The network center, which coordinates and manages the network, is assumed to be not selfish and always truthful.

The following two subsections introduce two problems considered in this work.

B. The User-Centric Problem (UCP)

The network coordinates the resource allocation to maximize the sum-valuation in the network. It is represented by the following problem:

\[
\begin{align*}
\max \quad & \sum_{i=1}^{N} v_i(b_i(x_i)) \\
\text{s.t.} \quad & \sum_{i} x_i \leq X^{max} \\
& 0 \leq x_i \leq x^{max}, \forall i \in \mathcal{U}.
\end{align*}
\] (1a) (1b) (1c)

Note that the adoption of the valuation \( v_i(b_i) \) includes the weighted-sum optimization as a special case when \( v_i(b_i(x_i)) = \alpha_i b_i(x_i), \forall i \). Problem (1) also includes the NUM problems with all users using the same objective, such as a sum-rate optimization problem, as special cases.

Unlike the case in classic NUM problems, local information is captured in this model through the mapping \( v_i \) and/or the objective \( o(i) \). Evidently, solving problem (1), in either a centralized or distributed approach, requires all users to truthfully reveal their local information. To solve (1) in a centralized approach, user \( i \) needs to report the objective \( o(i) \), the mapping \( v_i(b_i) \), and other related local information (e.g., parameter set) to the center. To solve (1) in a distributed approach, assuming that the problem is convex and dual decomposition is used, user \( i \) should report its solution to the problem of maximizing \( v_i(b_i(x_i)) - \lambda x_i \) in each iteration, where \( \lambda \) is the Lagrange multiplier associated with constraint (1c).

If any user reports untruthfully, the solution, assuming that it can be found, might not be the solution to the original problem, i.e., problem (1). For example, if user \( i \) reports \( v_i(b_i) \) instead of \( v_i(b_i) \) while other users report truthfully, the obtained solution is the solution to the following problem:

\[
\begin{align*}
\max \quad & \sum_{i=1}^{N} v_i(b_i(x_i)) + \sum_{j \neq i} v_j(b_j(x_j)) \\
\text{s.t.} \quad & \sum_{i} x_i \leq X^{max} \\
& 0 \leq x_i \leq x^{max}, \forall i \in \mathcal{U}.
\end{align*}
\] (2a) (2b) (2c)

As each user has its own objective and valuation, assuming truthful reports from the users can be impractical. Indeed, as long as the solution to (2) is better than the solution to (1) from the perspective of user \( i \), it can be motivated to report untruthfully. Therefore, a mechanism is required to incentivize users to report truthfully. For the UCP, a common pricing mechanism, i.e., dual pricing \([24], [38], [42]\), will be studied in details.

C. The Network-Centric Problem (NCP)

While problem (1) considers system performance from a user perspective, a different solution can be preferred from a network perspective. Assume the network prefers the solution

\[2\] This corresponds to the case in a centralized optimization approach. In a distributed optimization approach, user \( i \) reports its local solution based on \( v_i(b_i) \) instead of directly reporting \( v_i(b_i) \) to the center.
to the following problem based on its own objective function \( v_i(b_i(x)) \):

\[
\begin{align*}
\max_x & \quad v_i(b_i(x)) \\
\text{s.t.} & \quad \sum_i x_i \leq X^{\text{max}} \\
& \quad 0 \leq x_i \leq x_i^{\text{max}}, \forall i \in U,
\end{align*}
\]

(3a) (3b) (3c)

where \( x = [x_1, \ldots, x_N] \). Denote the solution to (3) as \( x_i^* \) and the solution to (1) or (any other solution accepted by the users) as \( x_i^\dagger \). While \( x_i^* \) could be found without help from the users, convincing the users to adopt \( x_i^\dagger \) instead of \( x_i^* \) requires incentive. \[\] Providing incentive to user \( i \) for switching from \( x_i^* \) to \( x_i^\dagger \) requires user \( i \) to reveal local information regarding \( v_i(b_i(x_i^*)) \) and \( v_i(b_i(x_i^\dagger)) \). A proper mechanism is necessary for the users to report truthfully. For the NCP, mechanisms are designed to provide incentives for the users and guarantee truthful reporting while switching from \( \{x_i^*\} \) to \( \{x_i^\dagger\} \).

In both the UCP and the NCP, the valuation function \( v_i(b_i), \forall i \) are assumed to be twice differentiable, strictly concave, nondecreasing, and satisfy \( v_i(0) = 0 \). Such valuation models the case of decreasing marginal valuation and is widely used in both classic mechanism design and its applications in the networking field \([35]\). Examples of such utilities can be found in \([36]\). The objective function \( b_i(x_i) \) is twice differentiable and can be either concave or unimodal and concave in \([0, x^*]\), where \( x^* \) maximizes \( b_i(x_i) \). Section VI will provide detailed examples.

In the remainder of this paper, the focus will be on mechanism analysis and mechanism design based on the following two assumptions: i). All users are selfish but not malicious, which implies that they avoid causing an algorithm to diverge; and ii). A user avoids being identified as untruthful, which can happen only if its reports contradict the properties of \( v_i(b_i) \) mentioned above.

III. MECHANISM AND INCENTIVE COMPATIBILITY

The concept of mechanism and incentive compatibility is briefly introduced in this section while strategies and incentives in the considered system model will be described.

A mechanism is a tuple \( \{S_1, \ldots, S_N, \Gamma(\cdot)\} \) that consists of a set of possible actions \( S_i \) for each user and specifies an outcome based on a social choice function \( \Gamma(\cdot) \) for each possible combination of actions of users \([16]\). Specifically, in a quasilinear mechanism, \( \Gamma(\cdot) \) specifies a transfer \( t_i \) (a payment if \( t_i > 0 \) or charge if \( t_i < 0 \)) to user \( i \), which renders user \( i \)'s final utility \( u_i \) to be \( u_i = v_i + t_i \) (valuation of the outcome plus the transfer).

User \( i \) chooses its action from \( S_i \) based on its local information and the knowledge on how the transfer \( t_i \) is decided. A common action is to report a value (e.g., a solution, a demand, a quote, etc.). A mechanism is dominant strategy incentive compatible (DSIC) if truthful reporting is the best strategy (or one of the best strategies) for each user regardless of strategies used by other users \([16]\). A mechanism is ex post incentive compatible (EPIC) if truthful reporting is the best strategy for any user which knows that other users also report truthfully.

Several metrics are used to evaluate a mechanism. A mechanism is efficient if the outcome maximizes the sum valuation of all users. A mechanism is individually rational if each user is guaranteed to have a nonnegative utility. A quasi-linear mechanism is considered budget balanced if \( \sum_i t_i \leq 0 \).

For the UCP problem, reporting occurs in each iteration of a distributed optimization procedure while solving problem \( \textbf{1} \). Specifically

- Iteration \( j = 0 \). Network (center) chooses an initial value \( \lambda^0 \) for the Lagrange multiplier \( \lambda \) and broadcast \( \lambda^0 \).
- User \( i \), \( \forall i \): expected to report the solution to \( \max \{v_i(x_i) - \lambda x_i\} \) subject to \( \textbf{1c} \).
- Network (center) sets \( \lambda^{j+1} = \lambda^j + \delta(\sum_i x_i - X^{\text{max}}) \) and broadcast \( \lambda^{j+1} \).
- Repeat the above two steps until \( x^*_i, \forall i \) converge.

where \( v_i(b_i(x_i)) \) is denoted as \( v_i(x_i) \) for brevity.

The sequence of reports \( \{x_i^{j}\}, \forall i \) decides whether the algorithm can converge and whether the solution found is the optimal solution to the original problem \( \textbf{1} \). The strategy of a user is to choose a report \( x_i^{j} \) given the true \( x_i^{j} \) and \( \lambda^{j} \) in each iteration \( j \). An example mechanism specifies the resource allocation outcome and the transfer as below

\[
\left\{ \begin{array}{ll}
\lambda^j \rightarrow x^j, t_i = f_i(x^j) & \text{if} \{x_i^{j}\} \text{converge to } x^j, \forall i \\
0 = t_i = 0 & \text{otherwise}
\end{array} \right.
\]

(4)

where \( f_i(x) \) is a function to determine transfers.

A common pricing mechanism for distributed solution finding in the literature is dual pricing \([24], [38] - [42]\), which interprets and applies the dual variable \( \lambda \) in the aforementioned distributed optimization procedure as a per-unit price. Equivalently, \( t_i = f_i(x^j) = -\lambda^j x_i \) in \( \textbf{4} \). It can be seen

\[\] that the "converge" here refers to "not diverge" but not necessarily "converge to the optimal solution".

\[\]
that dual pricing is individually rational and budget-balanced. However, it is shown by a two-user example in [19] that dual pricing is not incentive compatible when one user is a leader and another is a follower. It is also shown in [34] that a dual pricing based auction is not incentive compatible for improving secrecy capacity. The next section will study the dual pricing mechanism and the incentive of users under dual pricing in the considered general scenario.

It should be noted that strategies of a user can be extremely rich, i.e., represented by all mapping from \( \{x_i^j\}_{k=0}^\infty \) to \( x_i^j \) in iteration \( j \). Even if a user makes a decision in each iteration without using historical information, its strategy space still consists of all mappings from \( \{x_i^j, \lambda_i^j\} \) to \( x_i^j \) in each iteration. However, most of the strategies would trivially lead an optimization algorithm to diverge. Thus, we limit our consideration to the case that users report based on a valuation function \( v_i(b_i) \) that possesses the same properties as \( v_i(b_i) \): strictly concave, twice differentiable, nondecreasing, and \( v_i(0) = 0 \). The reporting is truthful if \( v_i = v_i \). Such reporting strategy is consistent over iterations, allow a distributed algorithm to converge, and is impossible to be identified as untruthful.

IV. INCENTIVE FOR TRUTHFUL REPORTING IN THE UCP: A STUDY ON DUAL PRICING

In this section, dual pricing, as a pricing mechanism for solution finding in distributed optimization, will be studied.

The focus is on whether a user would report truthfully, i.e., \( v_i = v_i \), so that the solution to the original problem \( \text{UCP} \) can be found. Notations used in this section are given in Table I. Based on the assumptions on \( v_i(b_i) \) and \( b_i(x_i) \) in section I, \( v_i(x_i) \) is twice differentiable in \([0, x^{\text{max}}]\) and either concave in \([0, x^{\text{max}}]\) or unimodal and concave in \([0, x^*]\), where \( x^* \) corresponds to the maximum of \( v_i(x_i) \), for any \( i \) and any objective function \( b_i(x_i) \).

**Lemma 1:** The utility of user \( i \) satisfies \( u_i(\{s_i, s^{TR}_{-i}\}) \leq u_i(\{\tilde{s}^i, s^{TR}_{-i}\}) \) for any reporting strategy \( s_i \), such that \( \lambda^*(\{s_i, s^{TR}_{-i}\}) > \lambda^*(\{\tilde{s}^i, s^{TR}_{-i}\}) \), where the strict inequality holds when \( x_i^* > 0 \).

**Proof:** Please refer to Section I in Appendix.

**Lemma 2:** User \( i \) can unilaterally affect the price through untruthful reporting such that \( \lambda^*(\{s_i, s^{TR}_{-i}\}) < \lambda^*(\{s^i, s^{TR}_{-i}\}) \) with \( s_i \neq s^i \) as long as \( x_i^* \neq 0 \).

**Proof:** Please refer to Section I in Appendix.

**Lemma 3:** Unless \( \sum_i x_i^* < X^{\text{max}} \), truthful reporting is not optimal for any user \( i \) with \( x_i^* \neq 0 \), for any given combination of valuation functions \( \{v_i(x_i)\}_i \), assuming others report truthfully. A \( x_i^* \in (0, x^*_i) \) and a corresponding \( \lambda_i^* \) always exist such that

\[
u_i(s_i, s^{TR}_{-i}) = v_i(x_i^*) - \lambda_i^* x_i^* > v_i(x_i) - \lambda^* x_i^* = u_i(s_i, s^{TR}_{-i}),
\]

where \( x_i^* \), \( \lambda_i^* \), and \( \lambda^* \) are abbreviations of \( x_i^* \{s_i, s^{TR}_{-i}\} \), \( \lambda_i^* \{s_i^*, s^{TR}_{-i}\} \), and \( \lambda_i^* \{s_i^{TR}_{-i}, s^{TR}_{-i}\} \), respectively.

**Proof:** Please refer to Section I in Appendix.

Lemma 3 shows that an untruthful reporting strategy can result in a smaller share of resource but yield a larger utility for a user due to a lower price. Such a strategy is related to “demand reduction”, and discussion on a simpler discrete case in which resources are multiple indivisible items for sale can be found in [43]. An illustration of Lemma 3 is shown in Fig. 2, where \( v_i \) and \( \lambda_i \) are the truthful valuation function and the cost with truthful reporting, respectively, and \( v_i^\star \) and \( \lambda_i^\star \) are an untruthful valuation reported and the corresponding cost, respectively. The variables \( u_i \), \( \tilde{u}_i \), and \( u_i \) are the utility of user \( i \) with truthful reporting, the utility of user \( i \) if \( \tilde{u}_i \) were the truthful valuation, and the utility actually acquired by user \( i \) with untruthful reporting based on \( \tilde{u}_i \), respectively.

Lemma 1 to Lemma 3 provide insights regarding using dual pricing as a mechanism for a distributed resource allocation problem step by step. First, it is shown by Lemma 1 that dual pricing can prevent untruthful reporting that would intensify the competition and drive the price \( \lambda \) up. Next, Lemma 2 shows that, any user that would be allocated a nonzero share of resource when everybody report truthfully, i.e., \( x_i^* > 0 \), can unilaterally drive the price downward. Lemma 3 shows that such a user can benefit from driving the price down even if all other users report truthfully except for trivial cases when the resource is oversupplied, i.e., \( \sum_i x_i^* > X^{\text{max}} \). The condition that \( x_i^* \neq 0 \) in Lemmas 2 and 3 implies that truthful reporting is optimal for a user when it cannot afford any resource when dual pricing is applied. The above insights lead to the following remark.

**Remark 1:** Under dual pricing, reporting truthfully is optimal for a user only when the resource is oversupplied or when it cannot afford any resource.

**Theorem 1:** Dual pricing, when used as a pricing mechanism, is not EPIC.

**Proof:** Combining Lemma 1 to Lemma 3 proves this result.

The results in Lemma 1 to Lemma 3 are obtained without needing to know the specific local valuations of the users. It implies that a rational user can also obtain the above insights
need to be provided to the users for a consensus to adopt a solution to problem (3). Although the center is assumed to be the solution to problem (3), the one accepted by the users, i.e., \( x^\dagger \), and therefore truthful reporting is a concern.

From the users regarding their local valuations are necessary, which the center prefers a different solution, i.e., \( x^\star \). This section investigates incentive for truthful reporting while solving the UCP. This section investigates incentive for truthful reporting while solving the NCP, in which the center prefers a different solution, i.e., \( x^\dagger \), from the one accepted by the users, i.e., \( x^\star \). The solution \( x^\dagger \) is assumed to be the solution to problem (3). Although the center can solve problem (3) without help from the users, incentives need to be provided to the users for a consensus to adopt \( x^\dagger \) instead of \( x^\star \). To determine the incentives, reports from the users regarding their local valuations are necessary, and therefore truthful reporting is a concern.

Denote \( \nu(x) = v_i(b_c(x)) \). The study is based on the assumption that, while \( x^\dagger \) is optimal to the center, the center is also willing to adopt any intermediate solution \( x \) at the cost of providing incentives \( \{ t_i \} \) to the users as long as \( \nu(x) - \sum_i t_i(x_i) \geq \nu(x^\dagger) \). A mechanism can implement \( x^\dagger \) if users agree on using \( x^\dagger \) with incentives provided by the center, i.e., \( v_i(x_i) + t_i \geq v_i(x_i^\star) \), \( \forall i \). Properties such as convexity are not assumed for \( \nu(x) \) as long as the center can solve it from problem (3).

With the above assumptions, whether \( \{ x^\dagger \} \) can be implemented or not and which mechanism can be used to implement \( \{ x^\dagger \} \) are investigated.

**Lemma 4:** If \( x^\dagger_i < x^\star_i \), \( \forall i \), no mechanism can simultaneously be incentive compatible and, under any condition, guarantee the implementation of \( x^\dagger \).

**Proof:** Please refer to Section D in Appendix.

Lemma 4 shows that at least one user, in addition to the center, should prefer \( x^\dagger_i \) to \( x^\star_i \). Otherwise, no mechanism can be simultaneously incentive compatible and able to specify a condition under which the implementation of \( x^\dagger \) is guaranteed.

Next consider the case that \( x^\dagger_i < x^\star_i \) for a set of \( i \) denoted as \( S_1 \) and \( x^\dagger_i > x^\star_i \) for a nonempty set of \( i \) denoted as \( S_2 \).

Consider a two-user example first. Assume \( v_1(x^\dagger_1) > v_1(x^\star_1) \) and \( v_2(x^\dagger_2) < v_2(x^\star_2) \). A subsidized exchange mechanism for incentivizing users to switch to \( x^\dagger \) from \( x^\star \) is proposed in Algorithm 1. In this algorithm, Line 3 and Line 4 are executed by the users while other lines are executed by the center.

**Algorithm 1 Subsidized Exchange Mechanism**

**Input:** \( x^\dagger, x^\star \)

**Output:** \( x \)

1: Calculate \( s_c = \nu(x^\dagger) - \nu(x^\star) \) and determine a threshold \( \alpha \in (0, 1/2] \).
2: Announce \( x^\dagger_1 \) and \( x^\dagger_2 \) to users 1 and 2, respectively.
3: User 1 reports \( \rho \), i.e., the compensation it requires to implement \( x^\dagger_1 \), to the center.
4: User 2 reports \( \varphi \), i.e., the price it is willing to pay to implement \( x^\dagger_2 \) to the center.
5: Compare \( \varphi + \alpha s_c \) with \( \rho \).
6: if \( \varphi + \alpha s_c < \rho \) then
   7:     abort with \( x = x^\star \)
   8: else
   9:     Charge user 2 the amount of \( \rho - \alpha s_c \) and pay user 1 the amount of \( \varphi + \alpha s_c \).
10: \( x = x^\dagger \)
11: end if
12: return \( x \)

**Lemma 5:** The mechanism in Algorithm 1 can implement \( \{ x^\dagger, x^\dagger_1 \} \) under the condition \( \alpha s_c \geq \rho - \varphi \). Moreover, the proposed mechanism is DSIC and individually rational.

**Proof:** Please refer to Section E in Appendix.

Since \( \{ x^\star \} \) maximizes \( \sum_i v_i(x_i) \), it can be seen that \( \rho^{TR} = v_1(x^\star_1) = v_1(x^\dagger_1) > v_2(x^\star_2) = v_2(x^\dagger_2) = \varphi^{TR} \). Consequently, \( \varphi + \alpha s_c \geq \rho \) is possible only if \( \alpha s_c > 0 \) given that \( \rho \geq \rho^{TR} \) and \( \varphi \leq \varphi^{TR} \) for any rational users. Therefore, the center must subsidize an exchange for it to be successful, i.e., \( \varphi + \alpha s_c \geq \rho \). In addition to being DSIC and individually rational, SEM also gives the center a
nonnegative gain after deducting the subsidy. Specifically, the gain of the center is
\[
\pi_c = \nu(x^{\dagger}) - \nu(x^\star)
\]
where \( \nu_c = \nu(x^\dagger) - \nu(x^\star) \) represents the value of a successful exchange to the center. The terms \( \varphi + \alpha s_c \) and \( \rho - \alpha s_c \) are the amounts that the center pays and receives in SEM, respectively, with the difference being the subsidy. The inequality follows from the fact that \( \rho \geq \rho^{TR} \geq \varphi^{TR} \geq \varphi \). As given in Algorithm 1, the choice of \( \alpha \) in \( (0,1/2] \) guarantees that \( \pi_c > 0 \) in an exchange.

Insights can be drawn from comparing the cases with and without considering information availability and incentive. If \( \rho^{TR} \) and \( \varphi^{TR} \) were information known to the center, the center could coordinate a successful exchange as long as \( s_c \geq \rho^{TR} - \varphi^{TR} \). By contrast, with \( \rho^{TR} \) and \( \varphi^{TR} \) being local information, a successful exchange occurs when \( \alpha s_c \geq \rho^{TR} - \varphi^{TR} \) in SEM. Since \( \alpha \leq 1/2 \), the condition that \( \alpha s_c \geq \rho^{TR} - \varphi^{TR} \) is more stringent than the condition \( s_c \geq \rho^{TR} - \varphi^{TR} \). Consequently, the chance of implementing \( \{x_i^1, x_j^1\} \) becomes smaller. It shows that, when \( \rho^{TR} \) and \( \varphi^{TR} \) are local information, the center needs to provide more subsidy (compared to \( \rho^{TR} - \varphi^{TR} \)) to incentivize an exchange. As the center provides the subsidy using a part of its utility gain \( s_c \) from an exchange, the value of the exchange to the center \( s_c \) must be more when \( \rho^{TR} \) and \( \varphi^{TR} \) are local information such that a nonnegative gain \( \pi_c \) can be obtained after deducting the subsidy. Thus, the more stringent requirement on \( s_c \) and, as a result, a smaller chance of implementing \( \{x_i^1\} \) represent the cost of dealing with local information.

It should be noted that the parameter \( \alpha \) must be determined before the center receives \( \varphi \) and \( \rho \). The choice of \( \alpha \) corresponds to a trade-off for the center. A smaller \( \alpha \) leads to a smaller chance of success in implementing \( \{x_i^1, x_j^1\} \) but a larger \( \pi_c/s_c \). By contrast, a larger \( \alpha \) increases the chance of success yet leads to a smaller \( \pi_c/s_c \) for the center.

Remark 2: The SEM provides an equal and fair utility gain, i.e., \( \varphi^{TR} - \rho^{TR} + \alpha s_c \), to both users in every successful exchange.

Remark 3: The benchmark allocation \( \{x_i^\star\} \) can be either the solution to problem 1 or any other solution accepted by both users as long as at least one user prefers \( x_i^1 \) to \( x_i^\star \) (otherwise Lemma 4 applies).

The next step is to consider a general multiuser case. Unfortunately, SEM cannot be straightforwardly extended. Generally, it is impossible to determine whether \( \{x_i^1\} \) can be implemented or not in one shot due to the coupling between \( \nu(x) \) and \( \nu_i(x_i) \). Therefore, an iterative algorithm is proposed, as an extension of SEM, in which the center coordinates the users to gradually migrate from \( \{x_i^1\} \) toward \( \{x_i^\star\} \) through a sequence of subsidized exchanges. The proposed extended subsidized exchange mechanism (ESEM) is given in Algorithm 2. The algorithm ends up with either the implementation of \( \{x_i^1\} \) or a solution no worse than \( \{x_i^\star\} \) to

![Fig. 3: Illustration of variables and their relations in ESEM in iteration l](image)

all users as well as the center (both with the subsidies taken into account).

Let \( l \) represent the iteration index, and \( x_i^l \) represent \( x_i \) in iteration \( l \). Initialize \( x_0 = x_i^\star, \forall i \). Define two sets \( S_1 = \{i| x_i^1 < x_i^\star, i \in U \} \) and \( S_2 = \{i| x_i^1 > x_i^\star, i \in U \} \). In the \( l \)-th iteration, user \( i, \forall i \in S_1 \) reports \( \rho_i^l \), its loss from implementing \( x_i^1 - \delta_l \) compared to implementing \( x_i^1 \), and user \( j, \forall j \in S_2^l \) reports \( \varphi_j^l \), its gain from implementing \( x_j^1 + \delta_l \) compared to implementing \( x_j^1 \). For each combination of \( i \in S_1^l \) and \( j \in S_2^l \), the center calculates its valuation of the tentative exchange between users \( i \) and \( j \) as

\[
\theta_{ij}^l = \nu(\bar{x}^{l+1,ij}) - \nu(x^l),
\]

where \( \bar{x}^{l+1,ij} = [x_i^{l+1,ij}, \ldots, x_N^{l+1,ij}] \) with

\[
x_k^{l+1,ij} = \begin{cases} x_k^1 - \delta_l & \text{if } k = i \\ x_k^1 + \delta_l & \text{if } k = j \\ x_k^1 & \text{otherwise} \end{cases}
\]

Define \( \Theta^l = \{\theta_{ij}^l\} \). Define

\[
\psi_{ij}^l = \alpha^l \theta_{ij}^l + \varphi_{ij}^l - \rho_i^l
\]

and \( \Psi^l = \{\psi_{ij}^l\} \). For any given \( i \in S_1^l \) and \( j \in S_2^l \) in iteration \( l \), define the following sets

\[
\Omega_i^l = \{j| j \in S_2^l, \psi_{ij}^l > 0\} \tag{10a}
\]

\[
\Omega_j^l = \{i| i \in S_1^l, \psi_{ij}^l > 0\} \tag{10b}
\]

For simplicity, define a \( |S_1^l| \times |S_2^l| \) matrix \( W^l \) with its elements determined by

\[
W_{ij}^l = \begin{cases} 1 & \text{if } j \in \Omega_i^l \\ 0 & \text{otherwise} \end{cases}
\]

Define

\[
R_{ij}^l = \{i| i \in S_1^l, \sum_j W_{ij}^l \neq 0\}
\]

as the set of row indices corresponding to the rows of \( W_{ij}^l \) with at least one nonzero element. With the above definitions,
the ESEM is given in Algorithm 2 in which Line 4 and Line 7 are the steps executed by the users while other steps are executed by the center.

**Algorithm 2 Extended Subsidized Exchange Mechanism**

**Input:** \( S_0, S_1, x, x^* \)

**Output:** \( x^* \)

**Initialization:** Set \( l = 0, x^0 = x^*, \forall i. \) Set \( \delta^0. \)

1. **while** (true) **do**
2. **Determine** \( \alpha^l \) and announces \( \delta^l \) **to users**
3. **for** User \( i = 1 \) to \( N \) **do**
4. **if** \( i \in S_1^l \) **then**
5. **Report** \( \rho^l_i = v_i(x^l_i) - v_i(x^l_i + \delta_l) \) to the center
6. **else if** \( i \in S_2^l \) **then**
7. **Report** \( \varphi^l_j = v_j(x^l_j + \delta_l) - v_j(x^l_j) \) to the center
8. **end if**
9. **end for**
10. **Calculate** \( \theta^l_{i,j} \) and \( \psi^l_{i,j}, \forall i \in S_1^l, \forall j \in S_2^l \) using (7) and (9). **Calculate** \( W^l \) using (10) and (11).
11. **if** \( W^l = 0 \) **then**
12. **exit with** \( x^l \)
13. **else**
14. **Set** \( R^l_{i+} \) **as in** (12) and \( R^l_{j+} = S_2^l \)
15. **while** \((R^l_{i+} \neq \emptyset \text{ and } R^l_{j+} \neq \emptyset)\) **do**
16. **Select** an \( i \), denoted as \( \hat{i} \), from \( R^l_{i+} \) based on a uniform distribution
17. **Select** a \( j \), denoted as \( \hat{j} \), from \( S_2^l \) based on a uniform distribution
18. **if** \( W_{i,j}^l = 0 \) **then**
19. **Set** \( R^l_{i+} = R^l_{i+} / \{ \hat{i} \} \)
20. **Set** \( R^l_{j+} = R^l_{j+} / \{ \hat{j} \} \)
21. **else**
22. **Charge user** \( \hat{j} \) **the amount** \( \rho_{\hat{i}} - \alpha^l \theta^l_{i,j} \)
23. **Compensate user** \( \hat{i} \) **the amount** \( \varphi^l_{i,j} \)
24. **Set** \( x^{l+1}_i = x^l_i, S_1^{l+1} = S_1^l \) and \( S_2^{l+1} = S_2^l \)
25. **Update** \( x^{l+1}_j = x^l_j - \delta_l \) **and** \( x^{l+1}_j = x^l_j + \delta_l \)
26. **Set** \( S_1^{l+1} = S_1^l / \{ \hat{i} \} \) **if** \( x^{l+1}_i = x^l_i \)
27. **Set** \( S_2^{l+1} = S_2^l / \{ \hat{j} \} \) **if** \( x^{l+1}_j = x^l_j \)
28. **break** the inner while loop
29. **end else**
30. **end while**
31. **if** \( R^l_{i+} \neq \emptyset \text{ or } R^l_{j+} \neq \emptyset \text{ or } S_1^{l+1} = \emptyset \) **then**
32. **Set** \( l = l + 1 \) **and exit with** \( x^l \)
33. **end if**
34. **Set** \( l = l + 1 \) and update \( \delta_l \)
35. **end while**
36. **return** \( x^l \)

**Theorem 2:** The ESEM in Algorithm 2 is EPIC and individually rational if \( \{ \alpha^l \}_{i,j} \) is chosen such that \( \{ \alpha^l \theta^l_{i,j} \}_{i,j} \) is a nonincreasing sequence.

**Proof:** See Section $F$ in Appendix.

It is worth noting that there exist more forms of untruthful reporting in an iterative procedure than in a one-shot exchange. Proved to be incentive compatible, ESEM can prevent all of the following cases of untruthful report including: (i) untruthfully report a larger \( \rho^l_i \) or \( \varphi^l_j \) in iteration \( l \) to achieve larger benefit, (ii) untruthfully report a smaller \( \rho^l_i \) or \( \varphi^l_j \) in iteration \( l \) to achieve larger benefit, and (iii) skipping an iteration to achieve a larger benefit when \( \delta_l \) changes.

VI. APPLICATION ON UNDERLAY D2D COMMUNICATIONS

In this section, the UCP and the NCP in the preceding sections will be re-modeled in an application of underlay D2D communications. Note that the D2D scenario is just one application example. Other networks featuring a distributed/semi-distributed structure, in which users occupy both computation capability and local information, could also be modeled and solved using the proposed approach.

Consider a cellular network in an urban environment with dense microcells as shown in Fig. 4. An underlay-based D2D communications is adopted, and resource blocks (RBs) for cellular communications are reused by D2D communications. A microcell is divided into D2D reuse areas, and RBs can be reused by different D2D links in different reuse areas. For cross-area D2D links, e.g., link \( e \) in Fig. 4, the D2D link can communicate when the microcell BS (microBS) can assign an RB that is available in both areas. D2D links associated with different microBS cannot communicate directly. To control the interference from D2D to cellular and other D2D communications, two assumptions are made. First, the maximum transmit power allowed for D2D communications, denoted as \( P_{max} \), is smaller than that allowed for cellular communications. Second, the total transmit power of all D2D communications reusing the same RB in each microcell is limited by \( P_{max} \).

Denote the \( k \)th D2D RB and the set of D2D RBs as \( b_k \) and \( B \), respectively, and the size of \( B \) as \( N_B \). Also denote \( L_m(b_k), k \in \{1, \ldots, N_B \} \) as the set of D2D links allocated with RB \( b_k \) in all areas of microcell \( m \) and the size of \( L_m(b_k) \) as \( L_{mk} \). Let \( C_m \) represent all neighboring microcells of microcell \( m \). A D2D link in \( L_m(b_k) \) may receive interference from four neighboring microcells.

Fig. 4: The underlay D2D application scenario. The abbreviation DRA stands for D2D reuse area.

\[ \text{untruthfully report a larger } \rho^l_i \text{ or } \varphi^l_j \text{ in iteration } l \text{ to achieve larger benefit, (ii) untruthfully report a smaller } \rho^l_i \text{ or } \varphi^l_j \text{ in iteration } l \text{ to achieve larger benefit, and (iii) skipping an iteration to achieve a larger benefit when } \delta_l \text{ changes.} \]
sources: other D2D links in $\mathcal{L}_m(b_k)$, D2D links in $\mathcal{L}_s(b_k)$, $s \in \mathcal{L}_m$, cellular links using $b_k$ in microcell $m$, and cellular links using $b_k$ in microcells in $\mathcal{L}_m$. The received signal of a D2D link in $\mathcal{L}_m(b_k)$ is

$$y_{m,i} = h_{m,i}b_k \cdot x_{m,i} + \sum_{j \in \mathcal{L}_m(b_k), j \neq i} \sqrt{p_m,j} h_{m,j,i} x_{m,j} + \sum_{s \in \mathcal{L}_m} \sum_{j \in \mathcal{L}_s(b_k)} \sqrt{p_s,j} h_{s,j,s} x_{s,j} + \sum_{s \in \mathcal{C}_m} \sqrt{p_s,s} h_{s,s,i} x_{s,i} + n_{m,i},$$

(13)

where $x_{m,i}$, $x_{m,j}$, and $x_{s,j}$ represent the transmitted signal of the $i$th D2D link using $b_k$ in microcell $m$, the $j$th D2D link using $b_k$ in microcell $m$, the $j$th D2D link using $b_k$ in microcell $s$, respectively. The variables $p_{m,i}$, $p_{m,j}$, and $p_{s,j}$ represent the transmit power of the $i$th D2D link using $b_k$ in microcell $m$, the $j$th D2D link using $b_k$ in microcell $m$, and the $j$th D2D link using $b_k$ in microcell $s$, respectively. The variables $h_{m,i}$, $h_{m,j}$, and $h_{s,j}$ represent the channel coefficients from the transmitter of the $i$th D2D link using $b_k$ in microcell $m$, the $j$th D2D link using $b_k$ in microcell $m$, and the $j$th D2D link using $b_k$ in microcell $s$, respectively.

The received signal of a D2D link is given by

$$r_{m,i}(p_{m,i}) = \log \left( 1 + \frac{y_{m,i}}{\sigma_0^2 + p_{m,i}} \right),$$

(14)

where $h_{m,i}b_k$ represents the channel power gain, and

$$P_{m,i} = \sum_{j \in \mathcal{L}_m(b_k), j \neq i} p_{m,j} g_{m,j,i} + \sum_{s \in \mathcal{C}_m} \sum_{j \in \mathcal{L}_s(b_k)} p_{s,j} g_{s,j,s,i} + p_{m} g_{m,i} + \sum_{s \in \mathcal{C}_m} p_{s} g_{s,i},$$

(15)

is the overall interference power at the receiver of the target D2D link. The variables $g_{m,j,i}$, $g_{s,j,s,i}$, $g_{m,i}$, and $g_{s,i}$ represent the channel power gains of $h_{m,j,i}$, $h_{s,j,s,i}$, $h_{m,i}$, and $h_{s,i}$, respectively. Considering the significant difference between the coverage of cellular and D2D links, the smaller maximum transmit power for a D2D link, and the total power constraint for D2D links, it is assumed that cellular communications are the major interferer of D2D communications. The assumption is appropriate in an urban scenario with dense microcells and small microcell radius. The interference is in turn modeled as a Gaussian random variable $\mathcal{N}(0, \sigma_0^2)$.

The corresponding energy efficiency is given by

$$\epsilon_{m,i}(p_{m,i}) = \frac{1}{p_{m,i} + p_{m,i}} \log \left( 1 + \frac{\mu_{m,i} p_{m,i}}{\sigma_0^2 + p_{m,i}} \right),$$

(16)

where $p_{m,i}$ represents circuit power consumption. Although the energy efficiency function is nonconvex, it is unimodal and concave in $[0, p_{m,i}^*]$, where $p_{m,i}$ maximizes $\epsilon_{m,i}.$

With the above setup, each microBS can independently coordinate its D2D links. Accordingly, we can focus on just one microcell. The microcell index $m$ is neglected when appropriate. Correspondingly, $p_{m,i}$, $r_{m,i}$, and $\epsilon_{m,i}$ will be denoted as $p_i$, $r_i$, and $\epsilon_i$, respectively.

As mentioned in Section III, each link determines its own objective $o_i$, which can be either rate or energy efficiency in this case, without needing to notify the microBS. Correspondingly, $b_i(x_i)$ in Section III becomes $r_i(p_i)$ if link $i$ has a rate objective or $e_i(p_i)$ otherwise. The value of a specific rate $r_i$ or energy efficiency $e_i$ to link $i$, depending on the objective, is determined by $v_i(r_i)$ or $v_i(e_i)$.

A microBS may coordinate its D2D links to maximize the sum-valuation, i.e.,

$$\max_{\{p_i, i \in \mathcal{L}_m(b_k)\}} \sum_{i \in \mathcal{L}_m(b_k)} v_i(p_i) \quad \text{s.t.} \quad 0 \leq p_i \leq p^{\max}, \forall i \in \mathcal{L}_m(b_k)$$

(17a)

$$\sum_{i \in \mathcal{L}_m(b_k)} p_i < p^{\max}. \quad \text{s.t.}$$

(17b)

The above problem could accommodate more constraints, e.g., a minimum rate constraint for D2D links with an energy efficiency objective, while the results in previous sections can still apply. Nevertheless, the above basic form is used since the focus is on incentive instead of optimization. Problem (17) corresponds to the UCP, i.e., problem (1). However, when the D2D links adopt the solution to (17), it is possible that the microBS has preference for a power allocation solution to a different problem, e.g.,

$$\max_{\{p_i, i \in \mathcal{L}_m(b_k)\}} \nu(p) \quad \text{s.t.} \quad 0 \leq p_i \leq p^{\max}, \forall i \in \mathcal{L}_m(b_k)$$

(18a)

$$\sum_{i \in \mathcal{L}_m(b_k)} p_i < p^{\max}. \quad \text{s.t.}$$

(18b)

where $\nu(p)$ with $p = [p_1, \ldots, p_{\mathcal{L}_m(b_k)}]$ is the microBS’s valuation representing a network preference. Problem (18) corresponds to the NCP, i.e., problem (3).

The next section will demonstrate the results obtained in the UCP and NCP for this application using the above models.

VII. SIMULATION EXAMPLES

With the D2D application as an example, this section demonstrates the analysis on dual pricing as well as the proposed SEM and ESEM in the preceding sections. The
common setup for all simulation examples are as follows. The radius of a cell is 500 meters, and the distance between D2D transceivers is in [5, 25] meters. The noise spectral density is -174dB/Hz, and noise figure is 6dB. The close-in (CI) model is used for Large-scale propagation [46], where non-line-of-sight (NOS) channels are assumed. The small-scale fading is modeled by a Rayleigh fading channel with the channel response following $\mathcal{CN}(0, 1)$. The valuation function used as an example here is $v_i(b_i) = \Gamma - e^{-\epsilon_i b_i}$, where $b_i$ can be $r_i$ (given by equation (14)) or $\epsilon_i$ (given by equation (16)). Fig. 5 demonstrates the transmission rate and energy efficiency versus transmit power when eight D2D links reuse an RB in a microcell in the aforementioned setup.

**Example 1:** An illustration of the results regarding dual pricing. Given $v_i(b_i) = \Gamma - e^{-\epsilon_i b_i}$, it is assumed that links report based on a truthful $\epsilon_i$ when reporting truthfully. Eight curves, each corresponding to one D2D link, are shown in Figs. 6 and 7. The curve for each link $i$ represents the resulting utility (Fig. 6) and power allocation (Fig. 7) under dual pricing versus the $\epsilon_i$ used by link $i$ for reporting when other links report truthfully, i.e., $\epsilon_j = \epsilon_j, \forall j \neq i$. The solid hexagram marker on each curve marks the utility (Fig. 6) and power allocation (Fig. 7) when the corresponding link also reports truthfully, i.e., $\epsilon_i = \epsilon_i$. From Fig. 6, it can be seen that reporting truthfully does not yield the maximum utility for a link. This illustrates the fact that reporting truthfully is not EPIC, as suggested by Theorem 1. Fig. 6 and Fig. 7 jointly show that any utility larger than the utility given by truthful reporting corresponds to an allocated power smaller than the allocated power in the case of truthful reporting. This validates the result in Lemma 3.

**Example 2:** An illustration of SEM for two D2D links. In this simulation, link 1 adopts a valuation based on energy efficiency, and link 2 adopts a valuation based on rate. The parameter $\epsilon_i$ is generated from a uniform distribution in [0.1, 0.3] for each link. The valuation of the network used as an example here is $\nu(p) = ae^{-\|p-p^*\|^2/\sigma}$. The overall interference power from all sources at each D2D receiver is generated randomly in the range of 5dB to 20dB of noise power. For each choice of the parameter $\alpha$ in SEM, 100 samples are used. The first plot in Fig. 8 shows the chance of success exchange versus the center’s choice of $\alpha$. The second plot in Fig. 8 shows the average utility gain from an exchange for a microBS versus $\alpha$. The figure illustrates the aforementioned trade-off for the center, i.e., a larger $\alpha$ corresponds to a larger chance of success but a smaller utility gain.

**Example 3:** Illustration of ESEM. In this simulation, 20
D2D links are used, five of which use energy efficiency as their objective, and the rest use data rate as their objective. The parameter $\varepsilon_i$ is randomly generated from a uniform distribution in $[0.1, 0.3]$. The valuation of the microBS used as an example is $\nu = a \exp(-\|p - p^\dagger\|/\sigma)$. The ESEM is implemented 50 times with $a = 2, \sigma = 0.01$. The traces of $\nu$ and $\|p - p^\dagger\|^2$ are shown in Fig 9. A step size of $\delta_l = 10^{-2}$ is used. Due to the randomness in choosing a candidate link in the ESEM, the traces from different runs can be different. In Fig 9, the utility gain for the microBS from an exchange is limited due to a small $a$ and, as a result, it cannot provide enough subsidy for $p$ to converge to $p^\dagger$ (this can be seen from the bottom subplot in Fig 9). However, despite of the difference in the traces, $\nu$ and $\|p - p^\dagger\|$ at the output are almost identical for each run. For example, the average $\nu$ at the final iteration in Fig 9 over the 50 runs is 1.9695 while the difference between the maximum and the minimum $\nu$ is only 3.1% of this average. The progress of utility gain for all 20 D2D links in one run of ESEM is shown in Fig 10. It can be seen that the utilities of all links are nondecreasing in ESEM, which illustrates the individual rationality.

VIII. CONCLUSION

Incentive for truthful reporting is studied for utility optimization problems with local information. The proposed model allows each user to adopt an individual objective and a valuation function, which can incorporate local information. Dual pricing is shown to be not EPIC and thus not able to guarantee truthful reporting while finding solutions to the considered problems. With dual pricing, reporting truthfully is shown to be optimal for a user only when the resource is oversupplied or when the user cannot afford any resource. If the network intends to adopt a solution different from the one accepted by the users, the solution needs to be also preferred by at least one user. Otherwise, no incentive mechanism can specify a condition to guarantee the implementation of the solution and at the same time guarantee truthful reporting. In the two-user case, whether such a solution can be implemented is determined in one shot. In the multiuser case, the switch of solutions must go through an iterative process. The proposed SEM and ESEM achieve incentive compatibility and provide fair and nonnegative utility gains to the users and also a nonnegative gain to the center. Simulations for a D2D application scenario validate the analysis for dual pricing and demonstrate the effectiveness of the proposed SEM and ESEM.

APPENDIX

A. Proof of Lemma 1

When $s_i = s_i^{TR}$, $x_i^* = x_i^*$ and $\lambda^* = \lambda^*$. From the primal optimality, the following result holds:

$$u_i(s_i^{TR}, x_i^{TR}) = v_i(x_i^*) - \lambda^* x_i^* = \sup_{s_i} (v_i(x_i) - \lambda^* x_i)$$

$$\geq v_i(x_i^*) - \lambda^* x_i^* = v_i(x_i^*) - \lambda^* x_i^* - (\lambda^* - \lambda) x_i^*$$

where $\lambda^*$ and $x_i^*$ are the final price and the allocation to user $i$ when $s_i \neq s_i^{TR}$, respectively. If $\lambda^* < \lambda^*$, i.e., user
i unilaterally drives the price higher, then it follows that 
\(- (\lambda^* - \Delta^*) \mathbf{z}^*_i \geq 0 \) and
\[
u_i(s^*_{1TR}, s^*_{-i}) = v_i(x^*_i) - \lambda^* x^*_i \geq \nu_i(x^*_i) - \lambda^* x^*_i = w_i (s^*_{1TR}), \tag{20}
\]
where the strict equality holds when \( x^*_i > 0 \). Accordingly, user i achieves a less utility with such \( s_i \neq s^*_{1TR} \) that renders \( \lambda^* (s^*_{1TR}, s^*_{-i}) > \lambda^* (s^*_{1TR}, s^*_{-i}) \).

B. Proof of Lemma 2

Define \( D_i = [0, \infty) \) if \( v_i(x_i) \) is nondecreasing and concave, and \( D_i = \{0, x^*_i\} \) if \( v_i(x_i) \) is unimodal and concave in \( [0, x^*_i] \), where \( x^*_i \) maximizes \( v_i(x_i) \). For any given \( v_i(x_i) \) that satisfies the aforementioned properties (i.e., nondecreasing and concave in \( D_i \), twice-differentiable, and \( v_i(0) = 0 \)), a \( \nu_i(x_i) \) can be found such that \( \nu_i(x_i) < v_i(x_i), \forall x_i \in D_i \), and \( \nu_i(x_i) < v_i(x_i), \forall x_i \in D_i \). Denote strategy \( s_i \neq s^*_{1TR} \) as the strategy of reporting based on the KKT conditions. It contradicts with

\( \lambda^* (s^*_{1TR}, s^*_{-i}) \geq \lambda^* (s^*_{1TR}, s^*_{-i}) \).

Summarizing the above four cases concludes the proof.

C. Proof of Lemma 3

Since it is impossible that \( x^*_i = 0, \forall i \), there must exist at least one user, denoted as user \( i \), and a corresponding \( s_i \neq s^*_{1TR} \) so that \( \lambda^* (s^*_{1TR}, s^*_{-i}) < \lambda^* (s^*_{1TR}, s^*_{-i}) \) according to Lemma 2. Moreover, based on the four cases in the proof of Lemma 2, the impact of \( s_i \) satisfies \( x^*_i > x^*_i \) in all possible cases. Given that \( \lambda^* > \lambda^* \), \( x^*_i > x^*_i, \forall j \neq i \) and the strict inequality holds if \( x^*_i \neq 0 \). Two cases are possible.

Case i: max \( v'_j(0) < \lambda^* \). As a result, \( \exists s_i \) such that \( \lambda^* (s^*_{1TR}, s^*_{-i}) < \lambda^* (s^*_{1TR}, s^*_{-i}) \). Accordingly, the following result holds

\[
u_i(s^*_{1TR}) = v_i(x^*_i) - \lambda^* x^*_i = v_i(x^*_i) - \lambda^* x^*_i > v_i(x^*_i) - \lambda^* x^*_i = u_i(s^*_{1TR}, s^*_{-i}). \tag{28}
\]

Case ii: max \( v'_j(0) \geq \lambda^* \). As a result, \( \exists j \neq i \) such that \( x^*_j > x^*_j, \forall \lambda^* (s^*_{1TR}, s^*_{-i}) < \lambda^* \). Define

\[ \mathcal{I}_{-i} = \{ j | j \in U \setminus \{i\}, x^*_j > x^*_j \}, \tag{29a} \]

\[ \mathcal{U}_{-i} = \{ j | j \in U \setminus \{i\}, x^*_j = x^*_j \}. \tag{29b} \]

Since \( x^*_j > x^*_j, \forall j \neq i \), \( \mathcal{I}_{-i} \cup \mathcal{U}_{-i} = U \setminus \{i\} \). Given \( x^*_j < x^*_j \), \( x^*_j > x^*_j \) for \( j \in \mathcal{L}_{-i} \), and \( x^*_j = x^*_j \) for \( j \in \mathcal{U}_{-i} \), denote \( x^*_j = x^*_j - \Delta x_j \) and, accordingly, \( x^*_j = x^*_j + \Delta x_j, \forall j \in \mathcal{I}_{-i} \), where \( \Delta x_j > 0, \Delta x_j > 0, \Delta x_i = \sum_{j \in \mathcal{L}_{-i}} \Delta x_j. \) Pick any \( j \) with
λ∗ = ν′(x∗) from I−i. Suppose that Lemma 3 is not true. The following result must hold:

\[
\begin{align*}
u_i(s_i, s_{-i}^{TR}) &= ν_i(x_i^*) - ν_j(x_j^*)x_i^* \\
&≥ ν_i(x_i^* - ∆x_i) - ν_j(x_j^* + ∆x_j^*)(x_i^* - ∆x_i) \\
&= u_i(s_i, s_{-i}^{TR}) \tag{30}
\end{align*}
\]

for all possible ∆x_i such that x_i^* = x_i^* - ∆x_i falls in D_i. Let ∆x_i → 0, reformat (30), and take limit on both sides. The following result holds:

\[
\begin{align*}
limit_{\Delta x_i \to 0} \frac{v_i(x_i^*) - v_i(x_i^* - ∆x_i)}{∆x_i} \\
≥ limit_{\Delta x_i \to 0} \left(\frac{v_j'(x_j^*) - v_j'(x_j^* + ∆x_j)}{∆x_j} \frac{∆x_j}{x_i^*} + v_j'(x_j^* + ∆x_j^*) \right).
\end{align*}
\]

(31)

Since v_i are twice differentiable in D_i, the above result can be rewritten as

\[
v_j'(x_j^*) ≥ -γ_{ji} ∆x_j/∆x_i + v_j'(x_j^*) > v_j'(x_j^*),
\]

where γ_{ji} = limit_{\Delta x_i \to 0} ∆x_j/∆x_i. The second inequality is due to the fact that v_i, ∀i is strictly concave, and x_i^* and γ_{ji} are both positive values. However, (32) contradicts with v_i'(x_i^*) = λ^* = ν′(x∗).

This completes the proof of Lemma 3. ■

D. Proof of Lemma 4

A mechanism is either one-shot or iterative. The following cases are possible.

Case i: one-shot, center initiates. In this case, center proposes a transfer t_i, user i decides to accept it or not based on the comparison between v_i(x_i^*) - v_i(x_i^*) and t_i. Truthful reporting is optimal for user i. However, since the center has no information on v_i, it is impossible to guarantee t_i > v_i(x_i^*) - v_i(x_i^*) in one shot. Consequently, no condition can be found to guarantee that ν(x^1) - ν(x^*) ≥ \sum t_i.

Case ii: one-shot, user initiates. In this case, user i proposes a t_i based on v_i(p_i^*) - v_i(x_i^*), center decides to accept it or not based on c - \sum t_i = v_i(x_i^*) - ν(x^*) - \sum t_i. Truthful reporting is optimal for user i only if the maximum t_i acceptable to the center, denoted as t_i^*, satisfies t_i^* ≤ v_i(x_i^*) - v_i(x_i^*). However, x^1 cannot be implemented if t_i^* < v_i(x_i^*) - v_i(x_i^*) while t_i happens to equal t_i^* in one shot is impossible given that user i has no information on v.

Case iii: iterative, center initiates. In such case, truthful reporting is optimal only if the center uses a fixed total t^T = i t_i = i (v_i(x_i^*) - v_i(x_i^*)) and adjusts the division of t^T to t_1 and t_2 through the iterations. If the users know t^T = i t_i = i (v_i(x_i^*) - v_i(x_i^*)) in all iterations, truthful reporting is optimal for everyone. However, it is impossible for the center to figure out such t^T and use it in every iteration since v_i is local information. Any t^T > \sum t_i leads to untruthful reporting. Any t^T < \sum t_i cannot implement x^1. A varying t^T across iterations leads to either untruthful reporting or fail of implementing x^1.

Case iv: iterative, user initiates. Reporting is not truthful in such case because user i proposes different t_i through the iterations, which cannot all be equal to v_i(x_i^*) - v_i(x_i^*).

In summary, a mechanism cannot simultaneously be incentive compatible and guarantee the implementation of x^1 when x_i^* ≤ x_i^*, ∀i.

E. Proof of Lemma 5

The utility gain for the users, as a function of quotes ρ and ϕ, from participating in the procedure in Algorithm 1 is

\[
\begin{align*}
π_1(ϕ, ρ) &= ((ϕ + αs_c) - ρ^{TR})^+ \tag{33a} \\
π_2(ϕ, ρ) &= (ϕ^{TR} - (ϕ + αs_c))^+ \tag{33b}
\end{align*}
\]

where ρ^{TR} = v_1(p_1^*) - v_1(p_1^*) and ϕ^{TR} = v_2(p_2^*) - v_2(p_2^*). The superscript (·)^+ represents the projection onto the nonnegative orthant. The values ρ^{TR} and ϕ^{TR} represent the true cost for user 1 and the true gain of user 2 in the exchange, respectively. The projection reflects the fact that an exchange is made only if ϕ + αs_c ≥ ρ. The reported and the true values satisfy ρ ≥ ρ^{TR} and ϕ ≤ ϕ^{TR} for any rational users.

DSIC is achieved if π_1(ϕ, ρ) ≤ π_1(ϕ, ρ^{TR}), ∀ρ, ϕ and π_2(ϕ, ρ) ≤ π_2(ϕ^{TR}, ρ), ∀ϕ, ∀ρ. From (33a) and (33b), it can be seen that π_1(ϕ, ρ) is in fact independent on ρ, and π_2(ϕ, ρ) is independent on ϕ. Therefore, when ϕ + αs_c ≥ ρ, it holds that π_1(ϕ, ρ) = π_1(ϕ, ρ^{TR}), ∀ϕ and π_2(ϕ, ρ) = π_2(ϕ^{TR}, ρ), ∀ϕ. Meanwhile, from the perspective of user 1, the chance that ϕ + αs_c ≥ ρ is smaller than the chance that ϕ + αs_c ≥ ρ^{TR} for any ρ > ρ^{TR}. Therefore, reporting a ρ with ρ ≥ ρ^{TR} does not increase π_1(ϕ, ρ) when an exchange is made but increases the chance of no exchange, in which case π_1(ϕ, ρ) = 0 ≤ π_1(ϕ, ρ^{TR}). Similarly, reporting a ϕ with ϕ < ϕ^{TR} does not increase π_2(ϕ, ρ) when an exchange is made but increases the chance of no exchange, in which case π_2(ϕ, ρ) = 0 ≤ π_2(ϕ^{TR}, ρ). Therefore, π_1(ϕ, ρ) ≤ π_1(ϕ, ρ^{TR}), ∀ϕ, ∀ρ and π_2(ϕ, ρ) ≤ π_2(ϕ^{TR}, ρ), ∀ϕ, ∀ρ regardless of whether of the condition ϕ + αs_c ≥ ρ is satisfied or not.

Individual rationality follows from the fact that π_1(ϕ, ρ) ≥ 0 and π_2(ϕ, ρ) ≥ 0 based on (33a) and (33b). ■

F. Proof of Theorem 2

The proof for individual rationality follows the same logic as that in the proof of Lemma 5 and is omitted. Proof of incentive compatibility is given as follows. Truthful Report means that

\[
\begin{align*}
ρ_i^1 &= v_i(x_i^1) - v_i(x_i^1 - δ^1), i ∈ S_1^1 \tag{34a} \\
ϕ_j^1 &= v_j(x_j^1 + δ^1) - v_j(x_j^1), j ∈ S_2^1 \tag{34b}
\end{align*}
\]

We prove that it is optimal for user i, i ∈ S_1^1, to report truthfully assuming other users do the same. The proof for j ∈ S_2^1 follows the same logic. Denote a report not necessarily truthful as ρ_i[15]. All users report truthfully. Denote the resulting matrix W^1 when user i reports ρ_i[15] as W_i(ρ_i[15]). Denoted the events that user i = i (i.e., row i of W^1) is selected in the

15It is assumed by default that i ∈ S_1^1 in the rest of the proof.
fifth step of Algorithm 2) and $i \neq ˜i$ with report $\rho^i$ as $C_i(\rho^i)$ and $C_i(\rho^i)$, respectively. Two cases are possible.  

Case i: $\rho^i \neq \rho^i$ but $W^i(\rho^i) = W^i(\rho^i)$. In such case, the probabilities of $C_i(\rho^i)$ and $C_i(\rho^i)$ are equal. The selection of $j$ given the event $C_i(\rho^i)$ is independent on the $\rho_j$ given that $W^i(\rho^i) = W^i(\rho^i)$. Moreover, the utility gain

$$
\pi_t^i(\rho^i) = \alpha \theta^i + \varphi^i - \rho^i
$$

is independent on $\rho^i$. Therefore, $\pi_t^i(\rho^i) = \pi_t^i(\rho)$ assuming $C_i(\rho^i)$ and $\pi_t^i(\rho^i) = \pi_t^i(\rho)$ assuming $C_i(\rho^i)$. It can also be observed that $\rho_j$ in such a case will not have an effect that propagates into any future iteration.

Case ii: $\rho^i \neq \rho^i$ and $W^i(\rho^i) \neq W^i(\rho^i)$. In this case, three sub-cases are possible.  

Sub-case ii-1: $i = ˜i$ with both $\rho^i$ and $\rho_j$, and $W_{i,j}^i(\rho_j) = \rho^i$, i.e., the selected element is not affected by untruthful report. In this sub-case, $\pi_t^i(\rho^i) = 0 \leq \pi_t^i(\rho_j)$. Moreover, as user $i$ makes the center believe that $\rho^i$ is a truthful report, the following result holds

$$
\psi_{t,j}^i(\rho_j) = \varphi^i + \alpha \theta^i - \rho^i,
$$

$$
\psi_{t,j}^i(\rho_j) = \varphi^i + \alpha \theta^i - \rho^i 
$$

Sub-case ii-2: $W_{i,j}^i(\rho^i) = 1$ and $W_{i,j}^i(\rho^i) = 0$. In this sub-case, $\pi_t^i(\rho^i) = 0 \leq \pi_t^i(\rho^i)$. Moreover, as user $i$ makes the center believe that $\rho^i$ is a truthful report, the following result holds

$$
\psi_{t,j}^i(\rho^i) = \varphi^i + \alpha \theta^i - \rho^i 
$$

Sub-case ii-3: $W_{i,j}^i(\rho^i) = 0$ and $W_{i,j}^i(\rho^i) = 1$. In this sub-case,

$$
\pi_t^i(\rho^i) = \alpha \theta^i + \varphi^i - \rho^i < 0.
$$

The impact of $\rho^i$ will not propagate into any future iteration since $W_{i,j}^i(\rho^i)$ due to untruthful reporting leads to the same or less utility gain for user $i$ in Case ii. Combining Case i and Case ii proves Theorem II. 

REFERENCES


