Refocusing of Moving Targets Based on Low-Bit Quantized SAR Data via Parametric Quantized Iterative Hard Thresholding

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Abstract—Low-bit quantization of echo improves storage and leads to more efficient downlink transmission of spaceborne synthetic aperture radar (SAR) systems. In this paper, a new parametric quantized iterative hard thresholding (PQIHT) algorithm is proposed to refocus the images of moving targets with low-bit quantized SAR data, based on the combination of quantized iterative hard thresholding (QIHT) and the parametric sparse representation. The blurred and quantization-error-involved subimage of the region of interest (ROI) containing the moving target is represented in a sparse fashion through an adaptive parametric dictionary. The QIHT with a pruned searching method is performed for efficiently estimating the motion-adaptive parameter inside the dictionary, refocusing the ROI image and suppressing the quantization-induced error in an iterative way. Different from the conventional QIHT algorithm with a fixed dictionary that can only represent stationary targets, the proposed method exploits a parametric dictionary with a parameter related to target motion status, which is capable of adaptively representing the radar echo from a moving target with unknown motion status and, therefore, is suitable for moving target refocusing. Simulations and experiments on real GF-3 satellite SAR data demonstrate that, compared with the conventional parametric sparse representation framework for moving target refocusing based on purely precise data, the proposed algorithm can provide satisfactory quality of moving target refocusing with remarkably reduced data volume.

I. INTRODUCTION

Conventional algorithms for synthetic aperture radar (SAR) imaging are designed for a stationary target focusing based on matched filtering. However, the conventional methods cannot focus moving targets because of the motion-induced errors. The motions of moving targets result in position offset and blurring in regular SAR images obtained by a matched-filtering-based method [1]. Generally speaking, the down-range velocity of a moving target will lead to the azimuth image shift, while the cross-range velocity and down-range acceleration of a moving target will result in the cross-range defocus [1].

Refocusing of moving targets has recently attracted much attention in the fields of SAR imaging and target detection and classification [2]–[8], and algorithms have been proposed to solve this problem [9]–[23]. In [9]–[12], autofocusing procedures are performed on the blurred images of moving targets cropped from the SAR image of the entire scene. Alternatively, the raw echo data can be first used to estimate the motion parameters, and subsequently, the images of moving targets can be refocused [13]–[15]. Basically, these conventional methods achieve moving target refocusing without getting rid of the constraints of matched filtering techniques. Recently, with the development of sparse recovery theory, sparsity-aware methods have been applied to SAR moving target refocusing [16]–[19]. The key idea is to use an overcomplete dictionary for mapping the SAR echo into a sparse signal. These methods operate on the entire SAR echo, which may involve interferences from static targets. In recent works, the regions containing moving targets are cropped from the conventional SAR images as the regions of interest (ROIs), and then, motion compensation is carried out on each ROI data using sparsity-driven methods [20]–[22]. In this way, the signal-to-clutter ratio is enhanced, the interferences from static targets are suppressed, and the computational burden is reduced. Chen et al. [23] proposed a moving target imaging algorithm based on the parametric sparse representation of the ROI data. In [23], the motion-adaptive parameter inside the dictionary and the image of the moving target are well retrieved in an iterative fashion.

All the methods mentioned above assume precise SAR data without consideration of the effect of quantization on the imaging quality. However, in spaceborne SAR systems, the SAR echo is first sampled onboard and then transmitted to the ground through the downlink channel, and quantized data are widely used in SAR imaging. Low-bit quantization of the spaceborne SAR echo helps storage saving and efficient downlink transmission, at the cost of the loss of image quality. Thus, it is worth investigating how to reduce the quantized data volume without remarkably sacrificing the SAR image quality. A number of methods have been proposed for SAR imaging using low-bit quantized data.
and have been proved to perform well for static targets in reducing data volume while keeping satisfactory images [24]–[28]. In [29]–[32], some sparse recovery algorithms with fixed dictionaries are applied on low-bit quantized data to further improve the data recovery performance. Especially, the quantized iterative hard thresholding (QIHT) algorithm, proposed in [32] by extending the original iterative hard thresholding (IHT) algorithm [41] to solve the problem of sparse signal recovery from quantized measurements, performs well in increasing the recovery accuracy while suppressing the sidelobes for signal reconstruction from low-bit quantized and even 1-bit coded data. However, these methods cannot deal with the moving target refocusing problem, because the fixed dictionaries designed for stationary targets cannot represent echoes from moving targets, and the uncertainty arising from target motion is not captured.

In this paper, a new parametric quantized iterative hard thresholding (PQIHT) algorithm is proposed for refocusing of moving targets based on low-bit quantized SAR echo, by combining the QIHT algorithm and the parametric sparse representation framework. This paper is an extension of our previous work [42]. It is assumed that the low-bit quantized SAR data are collected on the radar system by a uniform quantizer and transmitted to ground through the downlink channel. The defocused complex subimage of the ROI containing the moving target is cropped from the regular SAR image formed with low-bit quantized data by the matched-filtering-based method and is represented in a sparse fashion through a parametric dictionary that introduces a parameter adapting to the unknown target motions. The refocusing of the ROI image is accomplished by performing the QIHT algorithm for suppressing the quantization-induced error, together with a motion-related parameter estimation procedure. To efficiently and accurately estimate the motion-adaptive parameter inside the sparse dictionary, we present a pruned searching method and embed it into the iterative process of the QIHT procedure. With a good estimation of the motion-adaptive parameter, the corresponding dictionary is constructed. Based on this motion-adaptive dictionary, the ROI refocusing procedure is carried out by performing QIHT for suppressing the quantization-induced error. When processing low-bit quantized data, the framework detailed in [23] will result in accumulation of the effects of the quantization-induced error, leading to bad parameter estimation performance, since the method in [23] is designed for precise data. Different from the framework in [23], the proposed PQIHT method can avoid the accumulation of the quantization-induced error and thus provide better moving target refocusing performances, since the effect of quantization is considered in the refocusing procedure. Compared with the original QIHT algorithm in [32], the proposed PQIHT method in this paper significantly extends the QIHT method to the case of unknown dictionary to adaptively capture unknown target motions and achieve moving target refocusing together with a pruned searching procedure for motion-related parameter estimation. Simulations and experiments on real SAR data collected by the GF-3 satellite are employed to validate the proposed method. It is demonstrated that, compared to the refocused images of moving targets with precise data, the proposed PQIHT method can provide a comparable image quality for moving target refocusing with low-bit quantized data and even 1-bit quantized data.

The remainder of this paper is organized as follows. Section II presents the quantized signal model and data preprocessing procedures. The proposed PQIHT algorithm based on low-bit quantized data is formulated in Section III. In Section IV, experimental results based on simulated data and real SAR data are demonstrated. Section V gives the conclusion.

II. PROBLEM FORMULATION

In this section, we briefly review the signal model of the side-looking SAR and present the preprocessing of the quantized data to get the regular SAR image.

A. Signal Model

In this paper, we consider the geometry of the side-looking SAR, as shown in Fig. 1. The x and r axes denote the cross-range and down-range directions, respectively. The initial location of a moving target (denoted by the blue point in Fig. 1) is represented by \((x_0, r_0)\).

Assume that the SAR platform moves along the cross-range direction at a constant velocity \(v\). The velocities of the ground target along down-range and cross-range directions are denoted by \(v_x\) and \(v_r\), respectively. The transmitted signal of the SAR system is supposed to be a linear-frequency-modulated (LFM) signal with a modulation rate \(K_r\), expressed as

\[
s_t(t, t_s) = \text{rect}\left(\frac{t}{T_p}\right) \cdot \exp(j\pi K_r t^2 + j2\pi f_0 t) \tag{1}
\]

where \(\text{rect}(\cdot)\) represents the rectangular function, \(T_p\) denotes the pulse duration time, and \(f_0\) is the carrier frequency. Then, the baseband radar echo from the isotropy moving scatterer
with a scattering coefficient $\sigma$ can be written as

$$s_r(t, t_s) = \sigma \cdot \text{rect} \left( \frac{t - 2 R(t_s)/c}{T_p} \right) \cdot \text{rect} \left( \frac{t_s}{\tau_d} \right) \cdot \exp \left\{ j \pi K_r \left( \frac{t - 2 R(t_s)/c}{c} \right) - j 4 \pi f_o R(t_s)/c \right\} \tag{2}$$

where $T_p$ is the synthetic aperture time, $c$ is the speed of light, and $R(t_s) = \sqrt{(r_0 + v_x t_s)^2 + (v_x t_s - x_0 - v_y t_s)^2}$ is the slant range from the radar to the target at slow time $t_s$. In this paper, the scattering coefficient $\sigma$ is assumed to be constant during the observation time duration.

It is assumed that the analog SAR signal is quantized by a $b$-bit quantizer $Q_b(\cdot)$ at the satellite, expressed as

$$Q_b(x) = \begin{cases} l_1, & \text{if } x \in (r_1, r_2) \\ l_i, & \text{if } x \in [r_i, r_{i+1}) \text{ for } i = 2, \ldots, 2^b \end{cases} \tag{3}$$

where $\{r_i\}_{i=1}^{2^b-1}$ is the quantization level and $\{l_i\}_{i=1}^{2^b}$ is the quantization value sequence. In this paper, a uniform quantizer with a constant quantization interval $\Delta_q = \frac{2^b}{2^b-1}$ is considered for simplicity, where $\{-A, A\}$ is the dynamic range of the quantizer. Thus, we have $r_i = -A + (i - 1)\Delta_q$ and $l_i = \frac{1}{2}(r_{i+1} - r_i)$ in (3). With the quantizer $Q_b(\cdot)$, the quantized data of the received signal in (2) can be expressed as

$$s'_r(t, t_s) = Q_b(s_r(t, t_s)) = s_r(t, t_s) + n(t, t_s) \tag{4}$$

where $n(t, t_s)$ denotes the quantization error. In this paper, the low-bit quantized data are used for data preprocessing and moving target refocusing.

**B. Data Preprocessing and Problem Formulation**

Before extracting the ROIs containing the moving targets, conventional imaging algorithms, such as the back-projection algorithm [37], the range-Doppler algorithm [38], the chirp scaling algorithm [39], and the Omega-K algorithm [40], can be performed on the quantized echo data to obtain the focused image of the static scene. In what follows, the Omega-K algorithm [40] is applied on the quantized signal reflected from a moving target, i.e., $s'_r(t, t_s)$ in (4).

After performing matched filtering and the modified Stolt interpolation on the quantized data in (4) in the two-dimensional (2-D) frequency domain, as the Omega-K algorithm operates [40], the 2-D frequency signal can be obtained as

$$S'_1(f_r, f_a) = \sigma \cdot W_r(f_r) \cdot W_a(f_a) \cdot \exp \left\{ -j \frac{2 \pi f_a \theta}{v_c^2} \right\} \cdot \exp \left\{ -j \frac{4 \pi c f_a v_x}{v_c^2} \left( \frac{f_r}{2} + \frac{f_a}{2} \right) \cdot \left( \frac{1}{v_x^2} - \frac{1}{v_c^2} \right) \right\} \times \sqrt{\left( f_0 + f_r \right)^2 + \left( \frac{c f_a}{2} \right)^2 \left( \frac{1}{v_x^2} - \frac{1}{v_c^2} \right)} \cdot \text{FT}_2D(f(n(t, t_s))) \tag{5}$$

where $v_c^2 = (v - v_x)^2 + v_y^2$, $\theta = x_0(v - v_x) - r_0 v_x$; $f_r$ and $f_a$ denote the down-range and cross-range frequencies, respectively; $W_r$ and $W_a$ are the down-range and cross-range envelope functions, respectively; $\text{FT}_2D(\cdot)$ denotes the operation of taking 2-D Fourier transform; the function $f(\cdot)$ contains the operations of the Omega-K algorithm on the quantization error $n(t, t_s)$.

Note that while the first item in (5) is similar to the 2-D frequency signal obtained by the Omega-K algorithm based on precise data, as detailed in [23], the second item in (5) is the result produced by the quantization error $n(t, t_s)$ after Omega-K operation, which leads to higher sidelobes in the focused SAR image. Taking 2-D inverse Fourier transform of (5), the regular SAR image $s'_1(t, t_s)$ can be obtained as

$$s'_1(t, t_s) = \sigma \cdot W_r \left( \frac{t - 2 R(t_s)/c}{c} \right) \cdot W_a \left( t - \frac{\theta}{v_c^2} \right) \cdot \exp \left\{ -j 4 \pi f_0 R(t_s)/c \right\} \cdot \exp \left\{ f(n(t, t_s)) \right\} \tag{6}$$

where

$$R(t_1) = \sqrt{\left( x_0 v_x + r_0 (v - v_x) \right)^2 - \left( t - \frac{\theta}{v_c^2} \right)^2} \cdot \left( \frac{1}{v_x^2} - \frac{1}{v_c^2} \right). \tag{7}$$

We can see from (5) and (6) that, when the target is stationary, the phase of the signal in the 2-D frequency domain only contains the first-order terms of $f_r$ and $f_a$, and the coupling term of the two frequencies is eliminated. Thus, the image based on quantized data can be focused well except for higher sidelobes induced by the quantization-induced error. As for the moving target, because of the motion-induced error, high-order phase residual still exists after conventional Omega-K processing, which can lead to blurring and position offset in the reconstructed image, as shown in (6). Hence, extra phase compensation is required for moving target refocusing.

According to (5) and analysis in [23], the filter used to compensate the motion-induced error of the moving target should be designed as [23]

$$H_a(f_r, f_a) = \exp \left\{ -j \frac{4 \pi R_{ref} f_r}{c} \left( f_0 + f_r \right)^2 + \left( \frac{c f_a}{2} \right)^2 \cdot \left( \frac{1}{v_x^2} - \alpha \right) \right\} \cdot \exp \left\{ -j \frac{4 \pi R_{ref} (f_0 + f_r)}{c} \right\} \tag{8}$$

where $\alpha = 1/(v_c^2 + (v - v_x)^2)$ and $R_{ref}$ is the reference slant range. The filter in (8) is dependent on the parameter $\alpha$, which is related to target motions. By multiplying (5) by (8) and taking the 2-D inverse Fourier transform, the well-focused image of a moving target can be achieved when the filter used to compensate the motion-induced phase error in (8) is well designed with an accurate estimation of $\alpha$ and the received echo from the moving target is precise, as discussed in [21] and [23]. However, in most cases of moving target refocusing, the true value of the parameter $\alpha$ is unknown and, thus, should be estimated during the refocusing procedure.
In addition, with low-bit quantized echo data, even if the motion-adaptive parameter \( \alpha \) is estimated correctly, the ROI image that is refocused well with low sidelobes cannot be achieved using the above process because of the quantization-induced error. To reconstruct the desired SAR image of the moving target with suppressed blurring and quantization-induced sidelobes, a quantized sparsity driven algorithm is established in Section III to process the low-bit quantized ROI data extracted from the regular SAR image of the entire scene.

### III. PROPOSED PQIHT ALGORITHM

In this section, we formulate the PQIHT algorithm for moving target refocusing based on low-bit quantized SAR data.

#### A. Signal Model of Low-Bit Quantized ROI data

The ROI subimage containing the moving target is extracted from \( s^{(q)}_1(t, t_s) \) in (6) and is referred to as \( s^{(q)}_{1, \text{ROI}}(\bar{t}, \bar{t}_s) \), where \( \bar{t} \) and \( \bar{t}_s \) denote the equivalent fast time and slow time of ROI data, respectively. By taking the 2-D Fourier transform subsequently, the refocused ROI image of the moving target can be obtained as

\[
S_{1, \text{ROI}}(\bar{f}_r, \bar{f}_a) = \sigma \cdot W_r(\bar{f}_r) \cdot W_a(\bar{f}_a) \cdot \exp \left\{ -j \frac{2\pi f_{0} \bar{t}}{v_s} \right\} \exp \left\{ -j \frac{4\pi (x_0 v_r + r_0 (v - v_s))}{c v_s} \right\} \\
\times \left\{ \left( f_0 + \bar{f}_r \right)^2 + \left( \frac{c \bar{f}_a}{2} \right)^2 \cdot \left( \frac{1}{v^2} - \frac{1}{v_s^2} \right) \right\} + \text{IFT}_{2-D}(n^{(q)}_{\text{ROI}}(\bar{t}, \bar{t}_s))
\]

(9)

where \( \bar{f}_r \in [-0.5 f_s/n_r, 0.5 f_s/n_r] \) denotes the down-range frequency and \( \bar{f}_a \in [-0.5 \text{PRF}/n_a, 0.5 \text{PRF}/n_a] \) denotes the cross-range frequency of the ROI data, with \( f_s \) and PRF represent the sample frequency and the pulse repetition rate, respectively; \( n^{(q)}_{\text{ROI}}(\bar{t}, \bar{t}_s) \) is the ROI noise extracted from \( f(n(t, t_s)) \) in (6). Thus, the filter used to compensate the motion-induced error of the ROI data should be designed as [23]

\[
H^{(q)}_a(\bar{f}_r, \bar{f}_a) = \exp \left\{ j \left\{ \frac{4\pi R_{\text{ref}}}{c} \sqrt{\left( f_0 + \bar{f}_r \right)^2 + \left( \frac{c \bar{f}_a}{2} \right)^2 \left( \frac{1}{v^2} - \frac{1}{v_s^2} \right)} \\
- \frac{4\pi R_{\text{ref}} (f_0 + \bar{f}_r)}{c} \right\} \right\}.
\]

(10)

By multiplying (9) by \( H^{(q)}_a \) in (10) with an accurate estimation of \( \alpha \), and performing the 2-D inverse Fourier transform subsequently, the refocused ROI image \( \Theta \) of the moving target can be obtained as

\[
\Theta = \sigma \cdot \exp \left\{ B_a \left( \bar{t} - \frac{x_0 (v - v_s) - r_0 v_r}{v_s^2} \right) \right\} \\
\cdot \exp \left\{ B_r \left( \bar{t} - \frac{2 x_0 v_r + r_0 (v - v_s)}{v_s} \right) \right\} \\
\cdot \exp \left\{ -j 4\pi f_0 x_0 v_s + r_0 (v - v_s) \right\} \\
\cdot \text{IFT}_{2-D} \left[ \text{IFT}_{2-D} \left( n^{(q)}_{\text{ROI}}(\bar{t}, \bar{t}_s) \right) \right] \\
\cdot H^{(q)}_a(\bar{f}_r, \bar{f}_a)
\]

(11)

where \( B_a \) and \( B_r \) represent the bandwidths in down-range and cross-range directions, respectively; \( \text{IFT}_{2-D}(\cdot) \) denotes the operation of taking the 2-D inverse Fourier transform; the operator \( \circ \) denotes the Hadamard product. It can be seen from (11) that the well-focused ROI image of a moving target can be achieved when the filter used to compensate the motion-induced phase error in (10) is well designed with an accurate estimation of \( \alpha \), despite the sidelobes induced by the quantization error.

It can be seen from (10) and (11) that two operations are required for moving target refocusing: 1) suppression of the quantization-induced error and 2) accurate estimation of the motion-adaptive parameter \( \alpha \). In the following subsections, the PQIHT algorithm accomplishes these goals by embedding the procedure of refining the estimation of \( \alpha \) into the iterative fashion of QIHT. The parametric sparse representation has been applied on precise radar data for moving target imaging and motion compensation, as shown in [33]–[36]. In the following subsections, the parametric dictionary is extended to fit quantized data based on a quantized sparse reconstruction framework. At each iteration of QIHT, the ROI images obtained with different candidate values of \( \alpha \) are updated, and the candidate value set is modified after each iteration based on the updated ROI image qualities.

#### B. Moving Target Refocusing via PQIHT

From (9)–(11), the image formation of moving target from the ROI data \( s^{(q)}_{1, \text{ROI}}(\bar{t}, \bar{t}_s) \) extracted from \( s^{(q)}_1(t, t_s) \) in (6) with a certain value of \( \alpha \) can be expressed as the following formulation:

\[
\Theta = G_\alpha(s^{(q)}_{1, \text{ROI}}) = \Psi^{-1}_r \cdot \left( (\Psi_r \cdot s^{(q)}_{1, \text{ROI}} \cdot \Psi_a) \circ H^{(q)}_a(\bar{f}_r, \bar{f}_a) \right) \cdot \Psi^{-1}_a
\]

(12)

where \( \Theta \) denotes the reconstructed subimage of the moving target; \( \Psi_r \in \mathbb{C}^{n_r \times n_r} \) and \( \Psi_a \in \mathbb{C}^{n_a \times n_a} \) are the Fourier transform matrices in down-range and cross-range directions, respectively; \( \Psi^{-1}_r \) and \( \Psi^{-1}_a \) are the inverse matrices of \( \Psi_r \) and \( \Psi_a \), respectively; and \( G_\alpha(\cdot) \) is called a sparse transform. From (12), it is clear that the transform \( G_\alpha(\cdot) \) is invertible. We formulate the parametric sparse representation by taking the inverse transform of \( G_\alpha(\cdot) \), expressed as

\[
s^{(q)}_{1, \text{ROI}} = G^{-1}_\alpha(\Theta) = \Psi^{-1}_r \cdot \left( (\Psi_r \cdot \Theta \cdot \Psi_a) \circ (H^{(q)}_a)^* \right) \cdot \Psi^{-1}_a
\]

(13)
where $(\cdot)^*$ represents the conjugate matrix. The first operation required to achieve ROI image refocusing is to suppress the quantization-induced error. According to discussions in [32], the quantization error can be well suppressed by forcing consistent reconstruction, which requires that the entries of the parametric sparse representation of the refocused ROI image $\hat{\Theta}$ should fall in the same quantization levels as the corresponding entries of $s_{1,ROI}^{(q)}$. By taking the unknown key parameter $\alpha$ into consideration, the desired solution of the parametric sparse representation in (13) with the quantization-induced error eliminated can be obtained by an optimization framework, expressed as

$$
(\hat{\Theta}, \hat{\alpha}) = \arg \min_{\Theta, \alpha} \{ E + \lambda_1 \| \Theta \|_1 \} \tag{14}
$$

where $\hat{\alpha}$ is the final estimation of the motion-adaptive parameter, $\lambda_1$ is the regularization parameter, $\| \cdot \|_1$ is L1-norm, and $E$ corresponds to the quantization consistency between $G^{-1}_\alpha(\Theta)$ and $s_{1,ROI}^{(q)}$, described as [32]

$$
E = \sum_{i=2}^{s^0} \sum_{m} \left\{ r_i \left\{ \text{sign} \left[ \left( G^{-1}_\alpha(\Theta) \right)_m - w_i \right] \right\} \times \left( \left[ s_{1,ROI}^{(q)} \right]_m - w_i \right) \right\} \tag{15}
$$

where $w_i = l_i - l_{i-1}$, the operator $(\cdot)_-$ is defined to project any positive component to zero while maintaining the negative ones, and the representation $[\cdot]_m$ means selecting the $m$th element of the vector.

An objective function $E$ in (15) is similar to that in the original QIHT algorithm. However, different from QIHT that uses a fixed dictionary, which can only handle stationary targets, a parametric dictionary $G^{-1}_\alpha(\cdot)$ in terms of a motion-related parameter $\alpha$ is developed to represent echoes from moving targets in a sparse fashion. In this way, the PQIHT framework in (14) is able to adaptively achieve moving target refocusing, with a motion-related parameter optimization procedure involved. Note that when the target is stationary, the PQIHT framework will reduce to the original QIHT algorithm with a fixed and complete dictionary as a special case.

A correct value of $\alpha$ enables this parametric dictionary to completely capture the motion of the target. It can be shown that the solution $\hat{\Theta}$ of (14) is a corresponding reconstruction of the ROI image with the quantization-induced error eliminated, by the similar justifications in the stationary target case in [32].

Define the reconstructed scene in the $k$th iteration of PQIHT algorithm as $\hat{\Theta}^k$. Given a candidate value of $\alpha$, calculating the subgradient of the objective function in (15) and thresholding the obtained refocused ROI image gives the update of the solution:

$$
\hat{\Theta}^{k+1} = P_K \left[ \hat{\Theta}^k + \mu G_{\alpha} \left( s_{1,ROI}^{(q)} - Q_{\alpha} \left( G^{-1}_\alpha(\hat{\Theta}^k) \right) \right) \right] \tag{16}
$$

where $\mu > 0$ controls the gradient step size, and the thresholding operator $P_K(\cdot)$ is defined to maintain the $K$ biggest coefficients while setting others to zero. It can be seen from (16) that the update procedure is a function of $\alpha$, and the reconstructed scene varies with different values of $\alpha$.

C. Optimal Dictionary Parameter Estimation

According to (14) and (16), different candidate values of $\alpha$ will generate different dictionaries and then achieve different qualities of the refocused ROI image. Due to the fact that the correct value of $\alpha$ is always unknown, the fixed dictionary with a certain value of $\alpha$ is incomplete for an adaptive moving target refocusing procedure. Thus, a key issue for the new PQIHT framework is to obtain a good estimation of the parameter $\alpha$.

Since the velocities of the ground moving target are much smaller than the velocity $v$ of the radar platform in spaceborne or airborne SAR systems, the true value of the parameter $\alpha$ should be close to $1/v^2$ based on the expression of $\alpha = 1/(v^2 + (v - v_x)^2)$. Thus, the correct value of $\alpha$ can be estimated by searching around $1/v^2$ with the criteria of image contrast maximization, which can be expressed as

$$
\hat{\alpha} = \arg \max_{\alpha} \left\{ \frac{\text{mean} \left[ \left( \hat{\Theta}(\alpha) \right)^2 \right] - \left( \text{mean} \left[ \hat{\Theta}(\alpha) \right] \right)^2}{\left( \text{mean} \left[ \hat{\Theta}(\alpha) \right] \right)^2} \right\} \tag{17}
$$

where $\hat{\Theta}(\alpha)$ is the refocused ROI image obtained by the QIHT algorithm with the certain candidate value of $\alpha$, and the function $\text{mean}[\cdot]$ calculates the mean value.

In this paper, we employ a pruned searching method for efficiently and accurately estimating the motion-adaptive parameter. It can be seen from (16) and (17) that the image contrast calculated with a candidate value of $\alpha$ can, to some extent, reflect the correctness of the candidate. That is, the image contrast will be larger when the candidate value of $\alpha$ is closer to the true value. Thus, the pruned searching method can obtain the correct estimation of the true value of $\alpha$ when the candidate set is complete enough.

In addition, we embed this parameter estimation process into the iterative operations of the QIHT algorithm to reduce computational burden and efficiently refocus the ROI image. In each iteration, some candidate values of $\alpha$ that lead to small image contrasts can be removed from the candidate set, since they are less likely to be around the true value. We define that there are $n_k$ candidates in the $k$th iteration of QIHT, and only $n_k/2$ candidate values of $\alpha$ are maintained and plugged into the next iteration after the pruning process based on image contrast comparison. The pruning operation is followed until the optimal estimation of $\alpha$ is selected, and the QIHT algorithm reaches the maximum iteration number. Thus, the whole QIHT iteration is only performed for the candidate of $\alpha$ that is very close to the true value. In this way, the computation complexity is remarkably reduced, compared to the conventional full search method for parameter estimation, which performs the whole QIHT iteration for each candidate value and compares the final imaging results to find the good estimation corresponding to the highest image contrast. With the optimal estimation of $\alpha$, a refocused ROI image of the
moving target can be obtained. The PQIHT algorithm is summarized in Algorithm 1.

IV. EXPERIMENTAL RESULTS

In this section, simulations and experiments on real data collected by spaceborne SAR on the GF-3 satellite are carried out to demonstrate the effectiveness of the proposed PQIHT algorithm.

A. Simulations

In our simulations, the point-like scattering model is adopted, and an airborne radar with 300-MHz bandwidth centered at 10 GHz is used. The pulse duration time of the transmitted LMF signal is 2.2 μs, and the pulse repetition frequency is set to be 3 kHz. The radar system is assumed to move along the cross-range direction with a constant velocity of 150 m/s. The simulated imaging scene is shown in Fig. 2, where the scattering coefficients of scattering points denoted by circle and triangle are supposed to be 1 and 0.5, respectively. The scene to be observed consists of two stationary point targets S1 and S2, and a moving target is comprised of 12 scattering points denoted by V1–V12 with constant velocities of 1 and 2 m/s along down-range and cross-range directions, respectively, i.e., \( v_x = 1 \) m/s and \( v_x = 2 \) m/s. The reference slant range is 10 km, and the two stationary targets are at the same range line with 40 m separated at the cross-range direction. The center of the moving target lies between the two stationary targets. We set the compression ratios (the ratio of the number of measurements to the size of ROI) of both the down-range and cross-range directions to 0.7, which means 0.49 × \( n_r × n_d \) measurements are used for ROI refocusing in simulations. Before quantization, Gaussian noise is added to the measurements, wherein the signal-to-noise ratio (SNR) is 15 dB. Since the velocities of the ground moving target are relatively small compared to the velocity of the radar platform in airborne and spaceborne SAR systems, the true value of the motion-related parameter is close to the value of \( \bar{\alpha} \) for \( \alpha = 1/(v^2 + (v - v_x)^2) \). Therefore, we set the initial candidate value set of \( \alpha \) to contain 100 candidate values around 1 

\[
\hat{\alpha} = \frac{\alpha}{\alpha} = \frac{1/(v^2 + (v - v_x)^2)}{1/(v^2 + (v - v_x)^2)}
\]

The regular image of the entire scene obtained by the Omega-K algorithm based on 2-bit quantized data, with a white dashed box denoting the ROI containing the moving target.
1) Comparisons With the Method in [23]: We first compare the proposed PQIHT method with the method based on purely precise data in [23]. For comparison, the refocused image obtained by the method in [23] with precise data (64-bit quantized) is given in Fig. 4(a), which serves as ground truth. The ROI image extracted from Fig. 3 is shown in Fig. 4(b), which is to be refocused. The refocusing results of ROI data obtained by method proposed in [23] and the method proposed in this paper with 2-bit quantized data are shown in Fig. 4(c) and (d), respectively. It is evident from Fig. 4(c) and (d) that the new PQIHT method can provide a better focused ROI image with lower sidelobes based on 2-bit quantized data. Obvious sidelobes along both down-range and cross-range directions corresponding to scattering point V12 are visible in Figs. 5 and 6, respectively. The refocused subimage produced by the PQIHT method has asymmetric sidelobes lower than $-20$ dB, which illustrates that effective quantization-induced error suppression can be achieved by PQIHT.

As for the computational complexity, taking into consideration of the quantization-induced error in PQIHT increases the computational complexity, compared to the method in [23]. The computational complexity of the PQIHT is related to the QIHT iteration number and the initial size of the candidate value set of $\alpha$. Based on the initial candidate value set described above, the computational times of the proposed PQIHT algorithm and the method in [23] are 1728.408 and 114.573 s, respectively. Since the quantization-induced error can be well compensated by the
The estimation of $\alpha$ corresponding to Fig. 4(d) obtained by PQIHT is $4.5690e - 5$ s$^2$/m$^3$, which is closer to the true value in the simulated scene, which can be calculated as $4.5652e - 5$ s$^2$/m$^3$. It is obvious that the proposed PQIHT algorithm can obtain a good estimation of $\alpha$ based on 2-bit quantized data. However, in our PQIHT framework, the cross-range and down-range velocities cannot be calculated based on the estimation of $\alpha$ because of the lack of prior information about the relationship between the down-range and cross-range velocities.

In Table I, the normalized scattering coefficients of V1–V12 calculated by PQIHT are investigated. The values are obtained by averaging 50 random trails. It is demonstrated in Table I that the relative strengths of these scattering points can be well recovered by PQIHT based on 2-bit quantized data, with lower values for V6–V9 while higher values for the rest scattering points. The value of V9 is much smaller than its true scattering coefficient due to the influence of coarse quantization and other strong scattering points. The image contrasts based on quantized data with different quantization levels are investigated in Table II. It is clear that the PQIHT method can provide image contrast higher than 37 based on low-bit quantized data, which is closer to that of method in [23] with 40.9436 based on precise data, compared to the image contrast obtained by method in [23] based on low-bit quantized data. Thus, the conclusion can be reached that, compared with the result formed from precise data, the results of the proposed PQIHT algorithm can achieve comparable image quality with much less data volume.

Next, we evaluate the performance of the new PQIHT method and the method in [23] versus compression ratio and SNR, whose results are shown in Figs. 7 and 8, respectively. The image contrasts of the refocused ROI images obtained by the proposed PQIHT method (denoted by the red curve) and the method in [23] (denoted by the black curve), respectively.
and the SNR increase. Specifically, as shown in Fig. 7, the image contrast obtained by the proposed PQIHT method tends to increase much slower from 37.7 to 37.9 when the total compression ratio is larger than 0.3. As demonstrated in Fig. 8, the proposed PQIHT method can achieve satisfactory image quality even under a coarse condition with SNR = 0 dB.

2) Comparisons With QIHT: The proposed PQIHT algorithm extends the QIHT algorithm to fit the cases of unknown dictionaries, which can be used for moving target refocusing. In order to clarify the ability of the proposed PQIHT method to deal with moving target refocusing, we compare the results with the original QIHT method. Since the PQIHT method and the original QIHT method have similar performance on suppressing the quantization-induced error, we only take 2-bit quantized data for example in simulations. The refocused image using the conventional QIHT method is demonstrated in Fig. 4(e).

It can be obviously seen from Fig. 4(d) and (e) that the conventional QIHT algorithm cannot achieve moving target refocusing, since the motion-induced error is not considered in the signal model and the fixed dictionary, which is a special case of PQIHT method with fixed $\alpha = 1/\nu^2$. As for the proposed PQIHT algorithm, however, a procedure of estimating the motion-adaptive parameter is implemented simultaneously with the procedure of suppressing quantization-induced error, as illustrated in (14), which takes into consideration of unknown target motions. The image contrasts of the conventional QIHT method are shown in Table II. The fixed value of $\alpha$ in the conventional QIHT framework, which is equivalent to the estimation of $\alpha$ for stationary target, is far from the true value. Such mismatch will lead to incompleteness of the sensing dictionary and subsequently result in worse performance of ROI refocusing in Fig. 4(e).

Since the proposed PQIHT method introduces a parametric dictionary, the procedure of parameter searching will definitely increase the computational complexity, compared to the original QIHT method. The computational time for the QIHT method to obtain the result in Fig. 4(e) is 47.319 s, which is much less than that of the PQIHT method to obtain the result in Fig. 4(d). However, the imaging quality of the PQIHT method, as demonstrated in Fig. 4(d), is much better than that of the QIHT method.

B. Experiments on GF-3 SAR Data

The real data used in this paper are collected by the spaceborne SAR on the GF-3 satellite. The bandwidth and the pulsewidth of the transmitted signal are 60 MHz and 35.01 $\mu$s, respectively, and the PRF is 2.362 KHz. The down-range and cross-range resolutions are both 5 m under the GF-3 working mode in this paper. The equivalent velocity of the SAR platform is calculated as 7.1463e3 m/s. The low-bit quantized data are regenerated from the raw data before the imaging procedures. The regular SAR image of the sea surface obtained by the conventional Omega-K algorithm, containing three moving ships, is shown in Fig. 9. The three moving ships, indicated by white boxes P1–P3, are defocused, while the background is well focused, except higher sidelobes induced by the quantization error. In experiments, we assume that the maximum velocities of the moving ships along down-range and cross-range directions are both 30 kn, i.e., 15.432 m/s. Therefore, the true value of $\alpha$ should fall in the region of $[1.9497e-8, 1.9666e-8]$. Since the equivalent velocity of the SAR platform is an approximate calculation result, the region of the candidate value of $\alpha$ is extended to $[1.9e-8, 2e-8]$. The initial candidate value set is set to consist of 1000 candidate values ranging from $1.9e-8$ to $2e-8$ s$^2$/m$^2$.

Different methods are used to obtain the refocused ROI images of P1–P3, shown in Figs. 10–12, respectively, with a compression ratio of 1, which means full measurements of ROIs are used for refocusing procedures. For comparison,
the subimages refocused by the method in [23] based on the precise data (64-bit quantized) are shown in subfigures (a) in Figs. 10–12, which serve as ground truths. And the ROI images extracted from the regular SAR image in Fig. 9 with 2-bit quantized data are demonstrated in subfigures (b) in Figs. 10–12 as the inputs of the refocusing procedures.

1) Comparisons With the Method in [23]: The refocused P1–P3 ROI images obtained by the method in [23] based on 2-bit quantized data are shown in subfigures (c) in Figs. 10–12, respectively. The subfigures (d) in Figs. 10–12 are the results of P1–P3 obtained by the proposed PQIHT method, respectively.

It can be seen from Figs. 10–12 that compared with subimages (c), the results in (d) show better focused ROI images with lower sidelobes based on low-bit quantized data. The method in [23] cannot achieve moving target focusing based on low-bit quantized echo because the quantization-induced error is not considered in the signal model and will be accumulated during the iterative fashion. In contrast, the proposed PQIHT algorithm can produce satisfactory image quality using 2-bit quantized data, compared with that...
Fig. 12. ROI subimages of moving target P3 based on different methods. (a) Refocused ROI image obtained using the method in [23] with precise data (64-bit quantized), which is denoted as the ground truth. (b) ROI subimage to be refocused, which is extracted from the regular SAR image in Fig. 9. (c) Refocused ROI image obtained using the method in [23] with 2-bit quantized SAR data. (d) Refocused ROI image obtained using PQIHT based on 2-bit quantized data. (e) Result obtained by PQIHT with 1-bit quantized data. (f) Reconstructed ROI image obtained by the conventional QIHT method with 2-bit quantized data.

The computational times of the proposed PQIHT method and the method in [23] are compared. The refocusing procedures of PQIHT take 45.463, 28.604, and 28.594 s to obtain the subimages (d) of P1–P3, respectively, while the computational times for the method in [23] to obtain the subimages (c) of P1–P3 are 3.573, 3.092, and 2.993 s, respectively. It is obvious that the computational burden of the proposed PQIHT method is higher due to the quantization-induced error suppression procedure.

<table>
<thead>
<tr>
<th>Method</th>
<th>Data</th>
<th>Data Volume ((n_r \times n_a))</th>
<th>P1 Image Contrast</th>
<th>P2 Image Contrast</th>
<th>P3 Image Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. [23]</td>
<td>Precise (64-bit)</td>
<td>64</td>
<td>16.7787</td>
<td>13.6834</td>
<td>12.7505</td>
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<tr>
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<td>1</td>
<td>11.7802</td>
<td>11.4633</td>
<td>11.5331</td>
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<tr>
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<td>2</td>
<td>11.9938</td>
<td>11.4684</td>
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<tr>
<td></td>
<td>4-bit</td>
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<td>12.1379</td>
<td>11.4926</td>
<td>11.7091</td>
</tr>
<tr>
<td></td>
<td>8-bit</td>
<td>8</td>
<td>12.1495</td>
<td>11.5052</td>
<td>11.7123</td>
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<tr>
<td>Ref. [23]</td>
<td>1-bit</td>
<td>1</td>
<td>0.1913</td>
<td>0.0779</td>
<td>0.0964</td>
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<tr>
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<tr>
<td></td>
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<td>0.2461</td>
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</table>

The quantitative comparisons of the proposed PQIHT algorithm and the method in [23] in terms of the image contrasts and the estimation of \(\alpha\) are given in Tables III and IV, respectively. We can see that, compared with the imaging result obtained by the purely precise data of \(64 \times n_r \times n_a\) data volume, the proposed PQIHT algorithm can well preserve the imaging quality based on low-bit and even 1-bit quantized data.

2) Comparisons With QIHT: The reconstructed P1–P3 ROI images using the conventional QIHT algorithm are demonstrated in subfigures (f) in Figs. 10–12, respectively. Compared with the refocused ROI images (d) in Figs. 10–12 using the proposed PQIHT method, the results in (f) of Figs. 10–12 have worse performances for moving target refocusing with remarkably higher sidelobes. Thus, the conclusion can be reached that the proposed PQIHT algorithm outperforms the conventional QIHT algorithm in moving target refocusing, due to its parametric signal model in (11). The computational times of the QIHT method are 3.181, 1.879, and 1.823 s for P1–P3, respectively. It is obvious that the proposed PQIHT method has higher computational burden because of the pruned searching method embedded in the QIHT iterations for parameter estimation. Comparisons of the image contrasts and the values of the motion-adaptive parameter \(\alpha\) are shown in Tables III and IV, respectively. Worse estimations of \(\alpha\) via the conventional QIHT method are demonstrated in Table IV, which corresponds to the worse performances illustrated in subfigures (f) in Figs. 10–12. In practice, we cannot obtain estimations of the cross-range and down-range velocities.
based on the estimation of $\alpha$, because the true velocity of the SAR platform and the relationship between the down-range and cross-range velocities are always unknown.

V. CONCLUSION

In this paper, we present a new PQIHT algorithm for moving target refocusing based on low-bit quantized SAR data, by combining the QIHT algorithm and a motion-adaptive parametric dictionary. The ROI image based on low-bit quantized data is represented in a parametric sparse representation, with a motion-adaptive parameter embedded into the sparse dictionary to deal with the phase error induced by target motion. Then, the proposed PQIHT framework achieves moving target refocusing by performing the QIHT method and a simultaneously pruned searching process that is used to efficiently and accurately estimate the motion-adaptive parameter. The pruned searching scheme is embedded into the QIHT iteration procedure. The simulation and experimental results in Section IV illustrate the effectiveness in parameter estimation and its influences in the subsequent refocusing procedure. It is demonstrated that the proposed PQIHT method can provide satisfactory ROI refocusing quality with well-reconstructed scattering strengths of scatterers in a moving target based on low-bit quantized data. Higher image contrasts can be obtained by PQIHT compared to the method in [23] and the original QIHT method, which illustrates the effectiveness of the proposed PQIHT method on suppressing the motion-induced error as well as the quantization-induced error. In addition, the refocused ROI image contrast provided by the proposed PQIHT method rises as the compression ratio and the SNR increase. As demonstrated in simulations, the performance of the proposed PQIHT method is still satisfactory even under the coarse condition with low SNR and compression ratio. Thus, the conclusion can be reached that the proposed PQIHT algorithm can achieve moving target refocusing for low-bit quantized and even 1-bit data, with little sacrifice of the image quality compared to that obtained from precise data.

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